# Small Ramsey Numbers

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**ABSTRACT:** We present data which, to the best of our knowledge, includes all known nontrivial values and bounds for specific graph, hypergraph and multicolor Ramsey numbers, where the avoided graphs are complete or complete without one edge. Many results pertaining to other more studied cases are also presented. We give references to all cited bounds and values, as well as to previous similar compilations. We do not attempt complete coverage of asymptotic behavior of Ramsey numbers, but concentrate on their specific values.

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## 1. Scope and Notation

There is a vast literature on Ramsey type problems starting in 1930 with the original paper of Ramsey [Ram]. Graham, Rothschild and Spencer in their book [GRS] present an exciting development of Ramsey Theory. The subject has grown amazingly, in particular with regard to asymptotic bounds for various types of Ramsey numbers (see the survey paper [GrRö]), but the progress on evaluating the basic numbers themselves has been very unsatisfactory for a long time. In the last decade, however, considerable progress has been obtained in this area, mostly by employing computer algorithms. The few known exact values and several bounds for different numbers are scattered among many technical papers. This compilation is a fast source of references for the best results known for specific numbers. It is not supposed to serve as a source of definitions or theorems, but these can be easily accessed via

* - This paper updates and extends a technical report RIT-TR-93-009 by the author [Ra4].

the references gathered here.

Ramsey Theory studies conditions when a combinatorial object contains necessarily some smaller given objects. The role of Ramsey numbers is to quantify some of the general existential theorems in Ramsey Theory.

Let $G_1, G_2, \ldots, G_m$ be graphs or $s$-uniform hypergraphs ($s$ is the number of vertices in each edge). $R(G_1, G_2, \ldots, G_m; s)$ denotes the $m$-color *Ramsey number* for $s$-uniform graphs/hypergraphs, avoiding $G_i$ in color $i$ for $1 \le i \le m$. It is defined as the least integer $n$ such that, in any coloring with $m$ colors of the $s$-subsets of a set of $n$ elements, for some $i$ the $s$-subsets of color $i$ contain a sub-(hyper)graph isomorphic to $G_i$ (not necessarily induced). If $s = 2$ (standard graphs) then $s$ can be omitted. If $G_i$ is a complete graph $K_k$, then we can write $k$ instead of $G_i$, and if $G_i = G$ for all $i$ we can use the abbreviation $R_m(G)$ (or $R_m(G; s)$). For $s = 2$, $K_k - e$ denotes a $K_k$ without one edge, and for $s = 3$, $K_k - t$ denotes a $K_k$ without one triangle (hyperedge). $P_i$ is a path on $i$ vertices, $C_i$ is a cycle of length $i$, and $W_i$ is a wheel with $i-1$ spokes, i.e. a graph formed by some vertex $x$, connected to all vertices of some cycle $C_{i-1}$. The book graph $B_i = K_2 + \overline{K}_i$ has $i+2$ vertices, and can be seen as $i$ triangular pages attached to a single edge. $K_{n,m}$ is a complete $n$ by $m$ bipartite graph, in particular $K_{1,n}$ is a star graph. For a graph $G$, $n(G)$ and $e(G)$ denote the number of vertices and edges, respectively, in $G$.

Section 2 contains the data for the classical two color Ramsey numbers $R(k, l)$ for complete graphs, and section 3 for the two color case when the avoided graphs are complete or have the form $K_k - e$, but not both are complete. Section 4 lists the most studied two color cases for other graphs. The multicolor and hypergraph cases are gathered in sections 5 and 6, respectively. If some new bound has been not yet published, we also give a reference to the best published previous result, if any. Finally, section 7 gives pointers to cumulative data and to some previous surveys, especially those containing data not subsumed by this compilation.

## 2. Classical Two Color Ramsey Numbers

We split the data into the table of values and a table with corresponding references. Known exact values appear as centered entries, lower bounds as top entries, and upper bounds as bottom entries.

All the critical graphs for the numbers $R(k, l)$ (graphs on $R(k, l) - 1$ vertices without $K_k$ and without $K_l$ in the complement) are known for $k = 3$ and $l = 3$, 4, 5, 6 [Ka2], 7 [RK3, MZ], and there are 1, 3, 1, 7 and 191 of them, respectively. There exists a unique critical graph for $R(4,4)$ [Ka2]. There are 4 such graphs known for $R(3,8)$ [RK2], 1 for $R(3,9)$ [Ka2] and 350904 for $R(4, 5)$ [MR5], but there might be more of them.

| $k$ \ $l$ | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 3 | 6 | 9 | 14 | 18 | 23 | 28 | 36 | 40<br>43 | 46<br>51 | 51<br>60 | 60<br>69 | 66<br>78 | 73<br>89 |
| 4 | | 18 | 25 | 35<br>41 | 49<br>62 | 53<br>85 | 69<br>116 | 80<br>151 | 93<br>191 | 98<br>238 | 112<br>291 | 119<br>349 | 128<br>417 |
| 5 | | | 43<br>49 | 58<br>87 | 80<br>143 | 95<br>216 | 114<br>317 | 445 | | | | | |
| 6 | | | | 102<br>165 | 300 | 497 | 784 | 1180 | | | | | |
| 7 | | | | | 205<br>545 | 1035 | 1724 | 2842 | | | | | |
| 8 | | | | | | 282<br>1874 | 3597 | 6116 | | | | | |
| 9 | | | | | | | 565<br>6680 | 12795 | | | | | |
| 10 | | | | | | | | 798<br>23981 | | | | | |

Table I. Known nontrivial values and bounds for two color Ramsey numbers $R(k,l) = R(k,l;2)$.

| $k$ \ $l$ | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 3 | GG | GG | GG | Ka2 | Ka2<br>GY | GR<br>MZ | Ka2<br>GR | Ex2<br>RK2 | Ka2<br>RK2 | Ex8<br>RK2 | XZ<br>RK2 | Ex7<br>RK2 | WW<br>Ra1 |
| 4 | | GG | Ka1<br>MR5 | Ex8<br>MR4 | Ex3<br>Spe/HZ | Ex3<br>Spe | RK1<br>Spe | Piw1<br>Spe | Piw1<br>Spe | CET<br>Spe | Piw1<br>Spe | Piw1<br>Spe | CET<br>Spe |
| 5 | | | Ex4<br>MR4 | Ex8<br>Spe/HZ | CET<br>Spe | Piw1<br>Spe | CET<br>Spe | Spe | | | | | |
| 6 | | | | Ka1<br>Mac | Spe/HZ | Spe/HZ | Spe | Spe | | | | | |
| 7 | | | | | Ma/S1<br>Mac | Spe/HZ | HZ | Spe | | | | | |
| 8 | | | | | | BR<br>Mac | Ea1 | HZ | | | | | |
| 9 | | | | | | | Ma/S1<br>Mac | Ea1 | | | | | |
| 10 | | | | | | | | Ma/S1<br>Mac | | | | | |

References for Table I.

Chokbowornpaishand and Longani [ChLo] established $R(3,20) \geq 98$. Wang and Wang in [WW] showed that $R(3,16) \geq 79$, and [WWY] gives $R(3,17) \geq 92$, $R(3,18) \geq 98$, $R(3,19) \geq 106$,

$R(3,20) \geq 109$, $R(3,21) \geq 122$, $R(3,22) \geq 125$ and $R(3,23) \geq 136$. Most of the lower bounds for $R(4,n)$ presented by Bannani in [Ba], and two other results $R(3,13) \geq 58$ [Ka2] and $R(5,8) \geq 94$ [RK1], were improved in [Piw1] by Piwakowski and in [XZ].

The best published upper bounds for $(k,l)$ = (4, 6), (5, 5) and (5, 6) of 43, 52 and 94 can be found in [MR2, MR3] and [Wa2], respectively. The bound $R(6,6) \leq 166$ is an immediate consequence of theorem 1 in [Wa1] and $R(4,6) \leq 41$, in this case the best published bound of 169 is due to Giraud [Gi3]. Recently, T. Spencer [Spe], Mackey [Mac], and Huang and Zhang [HZ], using the bounds for minimum and maximum number of edges in (4,5) Ramsey graphs found in [MR2, MR4], were able to establish new upper bounds for several higher Ramsey numbers, improving all the previous longstanding results of Giraud [Gi1, Gi3, Gi4]. Spencer and Mackey keep outdoing each other on these bounds, and when the situation stabilizes we will try to report the outcome of this competition. We have recomputed the bounds marked [HZ] using the method from the paper [HZ], because the bounds in the paper relied on an overly optimistic personal communication from Spencer. The upper bounds implied by $R(k,l) \leq R(k-1,l) + R(k,l-1)$, or by its slight improvement with strict inequality when both terms on the right hand side are even, are marked [Ea1].

For a more in depth study of triangle-free graphs in relation to the case of $R(3,k)$, for which considerable progress has been obtained in recent years, see also [AKS, FL, Fra1, Fra2, Gri, Loc, RK3, RK4, S2, Stat, Yu]. Good asymptotic bounds for $R(k,k)$ can be found, for example, in [Chu3] (lower bound) and [Tho] (upper bound), and for many other asymptotic bounds in the general case of $R(k,l)$ consult [GRS, GrRö].

In 1995, Kim [Kim] proved that $R(3,k)$ has order of magnitude exactly $\Theta(k^2/\log k)$.

## 3. Two Colors - Dropping One Edge from Complete Graph

| $G$ \ $H$ | $K_3-e$ | $K_4-e$ | $K_5-e$ | $K_6-e$ | $K_7-e$ | $K_8-e$ | $K_9-e$ | $K_{10}-e$ |
|---|---|---|---|---|---|---|---|---|
| $K_3-e$ | 3 | 5 | 7 | 9 | 11 | 13 | 15 | 17 |
| $K_3$ | 5 | 7 | 11 | 17 | 21 | 25 | 31 | 36-39 |
| $K_4-e$ | 5 | 10 | 13 | 17 | 28 | | | |
| $K_4$ | 7 | 11 | 19 | | | | | |
| $K_5-e$ | 7 | 13 | 22 | | | | | |
| $K_5$ | 9 | 16 | 30-34 | | | | | |
| $K_6-e$ | 9 | 17 | | | | | | |
| $K_6$ | 11 | | | | | | | |

Table II. Two types of Ramsey numbers $R(G,H)$, includes all known nontrivial values.

| $H$ / $G$ | $K_3-e$ | $K_4-e$ | $K_5-e$ | $K_6-e$ | $K_7-e$ | $K_8-e$ | $K_9-e$ | $K_{10}-e$ |
|---|---|---|---|---|---|---|---|---|
| $K_3-e$ | Tr | Tr | Tr | Tr | Tr | Tr | Tr | Tr |
| $K_3$ | Tr | CH2 | Clan | FRS1 | GH | Ra2 | Ra2 | Ea3-Ra2 |
| $K_4-e$ | Tr | CH1 | FRS2 | McR | McR | | | |
| $K_4$ | Tr | CH2 | EHM1 | | | | | |
| $K_5-e$ | Tr | FRS2 | CEHMS | | | | | |
| $K_5$ | Tr | BH | Ex7-Ea1 | | | | | |
| $K_6-e$ | Tr | McR | | | | | | |
| $K_6$ | Tr | | | | | | | |

References for Table II.

For the following numbers it was established that the critical graphs are unique: $R(K_3, K_l - e)$ for $l = 3$ [Tr], 6 and 7 [Ra2], $R(K_4-e, K_4-e)$ [FRS2], $R(K_5-e, K_5-e)$ [Ra3] and $R(K_4-e, K_7-e)$ [McR].

## 4. General Graph Numbers in Two Colors

This section includes data with respect to general graph results. We tried to include all nontrivial values and identities regarding exact results (or references to them), but only those out of general bounds and other results which, in our opinion, have a direct connection to the evaluation of specific numbers. If some small value cannot be found below, it may be covered by the cumulative data gathered in section 7, or be a special case of a general result listed in this section. Note finally that $B_1 = C_3 = W_3 = K_3$, $B_2 = K_4 - e$, $W_4 = K_4$ and $C_4 = K_{2,2}$ imply other identities not mentioned explicitly.

Paths:

$R(P_n, P_m) = n + \lfloor m/2 \rfloor - 1$ for all $n \geq m \geq 2$ [GeGy]

Cycles:

$R(C_3, C_3) = 6$ [GG]

$R(C_4, C_4) = 6$ [CH1]

Result obtained independently in [Ros] and [FS1]

$$R(C_n, C_m) = \begin{cases} 2n-1 & \text{for } 3 \le m \le n,\ m \text{ odd, } (n,m) \ne (3,3) \\ n-1+m/2 & \text{for } 4 \le m \le n,\ m \text{ and } n \text{ even, } (n,m) \ne (4,4) \\ \max\{n-1+m/2, 2m-1\} & \text{for } 4 \le m < n,\ m \text{ even and } n \text{ odd} \end{cases}$$

$R(nC_3, mC_3) = 3n + 2m$ for $n \ge m \ge 1$, $n \ge 2$ [BES]
Unions of cycles [MS]

Wheels:

$R(W_3, W_5) = 11$ [Clan]
$R(W_3, W_n) = 2n-1$ for all $n \ge 6$ [BE2]
All critical colorings for $R(W_3, W_n)$ for all $n \ge 3$ [RJ]
$R(W_4, W_5) = 17$ [He3]
$R(W_5, W_5) = 15$ [HM2, He2]
$R(W_4, W_6) = 19$, $R(W_5, W_6) = 17$ and $R(W_6, W_6) = 17$,
and all critical colorings (2, 1 and 2) for these numbers [FM]

Books:

$R(B_1, B_n) = 2n + 3$ for all $n > 1$ [RS1]
$R(B_3, B_3) = 14$ [RS1, HM2]
$R(B_2, B_5) = 16$, $R(B_3, B_5) = 17$, $R(B_5, B_5) = 21$,
$R(B_4, B_4) = 18$, $R(B_4, B_6) = 22$, $R(B_6, B_6) = 26$,
in general $R(B_n, B_n) = 4n + 2$ for $4n + 1$ a prime power,
and some other general equalities and bounds for $R(B_n, B_m)$ [RS1].

Complete bipartite graphs:

$R(K_{2,2}, K_{1,15}) = 20$ [La2]
$R(K_{2,3}, K_{1,7}) = 13$ [Par4]
$R(K_{2,3}, K_{2,3}) = 10$ [Bu4]
$R(K_{2,3}, K_{2,4}) = 12$ [ER]
$R(K_{2,3}, K_{3,3}) = 13$ and $R(K_{3,3}, K_{3,3}) = 18$ [HM3]
$R(nK_{1,3}, mK_{1,3}) = 4n + m - 1$ for $n \ge m \ge 1$, $n \ge 2$ [BES]
$R(K_{1,n}, K_{1,m}) = n + m - \varepsilon$, where $\varepsilon = 1$ if both $n$ and $m$ are even and $\varepsilon = 0$ otherwise [Ha1]. It is also a special case of multicolor numbers for stars obtained in [BuRo1].
$R(K_{2,n}, K_{2,n}) \le 4n - 2$ for all $n \ge 2$, exact values 6, 10, 14, 18, 21, 26, 30, 33, 38, 42, 46, 50, 54, 57 and 62 of $R(K_{2,n}, K_{2,n})$ for $2 \le n \le 16$, respectively. The first open case is $65 \le R(K_{2,17}, K_{2,17}) \le 66$ [EHM2].

Special cases:

$R(W_5, K_5 - e) = 17$ [He2][YH]
$R(B_3, K_4) = 14$ [He3]
$R(C_5, W_6) = 13$ [CS]
$R(C_4, K_6) \geq 18$ [Ex9]
Exact values 7, 9, 11, 12, 13, 16 and 16 of $R(C_4, B_n)$ for $2 \leq n \leq 8$, respectively [FRS6]
$26 \leq R(K_{2,2,2}, K_{2,2,2})$ (octahedron) [Ex7]

General cases (exact results and bounds):

Paths versus stars [Par2, BEFRS2]
Paths versus books [RS2]
Paths versus cycles [FLPS, BEFRS2]
Paths versus $K_n$ [Par1]
Paths versus $K_{n,m}$ [Häg]
Paths and cycles versus trees [FSS]
Books and $K_1$ + *tree* versus $K_n$ [LR]
Cycles versus stars [La1, Clark, see Par5]
Cycles versus books [FRS5]
Cycles versus $K_n$ [BoEr]
Unicyclic graphs [Gro1, Köh]
$C_4$ versus some stars [Par3]
$C_4$ versus books [FRS6]
$C_4$ versus trees [EFRS3, Bu6]
$nC_3$ versus $mK_4$ [LorMu]
$K_{2,n}$ versus some stars [Par4]
$R(nK_4, nK_4) = 7n + 4$ for large $n$ [Bu7]
Stars versus trees [Bu1, GV]
Stars versus stripes, stripes [CL, Lor]
Stars versus books [CRSPS, RS2]
Stars versus $K_{n,m}$ [Stev, Par3]
Stars versus $K_n - tK_2$ [Hua]
Union of two stars [Gro2]
Double stars* [GHK]
Trees [EG, GRS, FSS, GV]
Trees versus $K_n$ [Chv]
Trees versus $K_n + \overline{K}_m$ [RS2, FSR]
Trees versus bipartite graphs [EFRS5]
Trees versus almost complete graphs [GJ2]

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* - A double star is a union of two stars with their centers joined by an edge.

Trees versus small ($n(G) \le 5$) connected $G$ [FRS4]
Linear forests [BuRo2, FS3]
Forests versus $K_n$ [Stahl]
Forests versus almost complete graphs [CGP]
Sparse graphs versus paths and cycles [BEFRS2]
Multipartite complete graphs [BEFRS3, EFRS3, FRS3, Stev]
Multipartite complete graphs versus trees [EFRS6, BEFRSGJ]
Disconnected graphs versus any graph [GJ1]
Graphs with long tails [Bu2, BG]
Brooms[+] [EFRS2]

General results:

[Kim] $R(3,k) = \Theta(k^2/\log k)$.

[CCD] Explicit construction for $R(3,4k+1) \ge 6R(3,k+1) - 5$, for all $k \ge 1$.

[Alon2] Explicit construction of triangle-free graphs with independence $k$ on $\Omega(k^{3/2})$ vertices.

[Wa1] $R(k,k) \le 4R(k,k-2) + 2$.

[Chv] $R(K_n, T_m) = (n-1)(m-1) + 1$ for any tree $T$ on $m$ vertices.

[CH2] $R(G,H) \ge (\chi(G)-1)(c(H)-1) + 1$, where $\chi(G)$ is the chromatic number of $G$, and $c(H)$ is the size of the largest connected component of $H$.

[BE1] $R(G,G) \ge \lfloor (4n(G)-1)/3 \rfloor$ for any connected $G$.

[BE2] Graphs yielding $R(K_n, G) = (n-1)(n(G)-1) + 1$ and related results (see also [EFRS4]).

[BES] Study of Ramsey numbers for multiple copies of graphs (see also [Bu1, LorMu]).

[Zeng] $R(nK_3, nG)$ for all isolate-free graphs $G$ on 4 vertices.

[Bu7, Bu8] Study of Ramsey numbers for large disjoint unions of graphs, in particular $R(nK_k, nK_l) = n(k+l-1) + R(K_{k-1}, K_{l-1}) - 2$ for $n$ large enough.

[BEFRS1] $R(K_3, G) = 2n(G) - 1$ for any connected $G$ on at least 4 vertices and with at most $(17n(G)+1)/15$ edges. In particular it holds for $G = P_i$ and $G = C_i$, for all $i \ge 4$.

[BEFRS4] Graphs $H$ yielding $R(G,H) = (\chi(G)-1)(n(H)-1) + s(G)$, where $s(G)$ is a chromatic surplus of $G$, defined as the minimum number of vertices in some color class under all vertex colorings in $\chi(G)$ colors (such $H$'s are called $G$-good). This idea, initiated in [Bu2], is a basis of a number of

+ - A broom is a star with a path attached to its center.

exact results for $R(G,H)$ for large and sparse graphs $H$ [BG, BEFRS2, Bu5, FS, EFRS3, FRS3, BEFSRGJ]. A survey of this area appeared in [FRS7].

[BEFS] Bounds for the difference between consecutive Ramsey numbers.

[Par3, Par4] Relations between some Ramsey graphs and block designs.

[CSRT] $R(G,G) \le c_d n(G)$ for all $G$, where constant $c_d$ depends only on the maximum degree $d$ in $G$.

[GS] $R(G,G) \le c_d n$ for all $d$-arrangeable graphs $G$ on $n$ vertices.

[Alon1] $R(G,G) \le 12n$ for all $n$-vertex graphs $G$, in which no two vertices of degree at least 3 are adjacent.

[Sid3] $R(G,K_3) \le 2e(G)+1$ for any graph $G$ without isolated vertices.

[Sid2] $R(G,K_3) \le n(G)+e(G)$ for all $G$, a conjecture.

[FSS] Discussion of the conjecture $R(T_1,T_2) \le n(T_1)+n(T_1)-2$ for any trees $T_1$ and $T_2$.

[FM] Counterexample $G = W_6$ to the Erdös conjecture (see [GRS]) $R(G,G) \ge R(K_{\chi(G)}, K_{\chi(G)})$.

[-] Special cases of multicolor results listed in section 5.

[-] See also surveys listed in section 7.

## 5. Multicolor Graph Numbers

The only known value of a multicolor classical Ramsey number:

| | |
|---|---|
| $R(3,3,3) = R(3,3,3;2) = 17$ | [GG] |
| 2 critical colorings | [KS, LayMa] |

Bounds for multicolor classical numbers:

| | |
|---|---|
| $51 \le R(3,3,3,3) \le 64$ | [Chu1] [San] |
| $162 \le R(3,3,3,3,3) \le 317$ | [Ex11] [Wh, Ea1] |
| $500 \le R(3,3,3,3,3,3)$ | [Ex11] |
| $128 \le R(4,4,4) \le 242$ | [HI] [Ea2] |
| $30 \le R(3,3,4) \le 32$ | [Ka2] [Piw2] |
| $45 \le R(3,3,5) \le 59$ | [Ex9, KLR] [Ea2] |
| $55 \le R(3,4,4) \le 81$ | [KLR] [Ea2] |
| $84 \le R(3,3,3,4) \le 159$ | [Ex9] [Ea2] |

The result by Sanchez-Flores [San], 1995, improved very old bound $R(3,3,3,3) \le 65$ obtained by Folkman [Fo] in 1974.

Multicolor general graphs:

| | |
|---|---|
| $R(C_4, C_4, C_4) = 11$ | [BS, see also Clap] |
| $R(C_4, C_4, K_3) = 12$ | [Schu] |
| $R(C_4, K_3, K_3) = 17$ | [ER] |
| $R(C_5, C_5, C_5) = 17$ | [YR1] |
| $R(C_6, C_6, C_6) = 12$ | [YR3] |
| $R(K_4 - e, K_4 - e, P_3) = 11$ | [Ex6] |
| $R(C_4, C_4, C_4, T) = 16$ for $T = P_4$ and $T = K_{1,3}$ | [ER] |
| $28 \le R(K_4 - e, K_4 - e, K_4 - e) \le 32$ | [Ex6] [Ex6] |
| $R_4(C_4) \ge 18$ and $R_5(C_4) \ge 25$ | [Ex9] |

General multicolor results:

- General bounds for $R_k(G)$ [CH3].
- Bounds for $R_k(3)$ [Fre, Chu1, Chu2, ChGri, GrRö].
- Formulas for $R_k(G)$ for $G$ being $P_3$, $2K_2$ and $K_{1,3}$ for all $k$, and for $P_4$ if $k$ is not divisible by 3 [Ir]. Wallis [Wal] showed $R_6(P_4) = 13$, which already implied $R_{3t}(P_4) = 6t + 1$, for all $t \ge 2$. Independently, the case $R_k(P_4)$ for $k \ne 3^m$ was completed by Lindström in [Lin], and later Bierbrauer proved $R_{3^m}(P_4) = 2 \cdot 3^m + 1$ for all $m \ge 1$.
- $R_k(C_4) \le k^2 + k + 1$ for all $k \ge 1$, and $R_k(C_4) \ge k^2 - k + 2$ for all $k - 1$ a prime power [Ir, Chu2, ChGra]. For small $k$ some improvements on the latter are known: $R_3(C_4) = 11$ [BS], $R_4(C_4) \ge 18$ and $R_5(C_4) \ge 25$ [Ex9].
- Bounds for the bipartite graphs $R_k(K_{s,t})$, in particular for $K_{2,2} = C_4$ [ChGra].
- Formulas for $R(C_n, C_m, C_k)$ and $R(C_n, C_m, C_k, C_l)$ for $n$ sufficiently large [EFRS1].
- Formulas for $R(P_{n_1}, \cdots, P_{n_k})$, except few cases [FS2].
- Monotone paths and cycles [Lef].
- Formulas for $R(S_1, \cdots, S_k)$, where $S_i$'s are arbitrary stars [BuRo1].
- Formulas for $R(S_1, \cdots, S_k, K_n)$, where $S_i$'s are arbitrary stars [Jac].
- Formulas for $R(S_1, \cdots, S_k, nP_2)$, where $S_i$'s are arbitrary stars [CL2].
- Formulas for $R(pP_3, qP_3, rP_3)$ and $R(pP_4, qP_4, rP_4)$ [Scob].
- Cockayne and Lorimer [CL1] found the exact formula for $R(n_1P_2, \cdots, n_kP_2)$, and later Lorimer [Lor] extended it to a more general case of $R(K_m, n_1P_2, \cdots, n_kP_2)$. Still more general cases of the latter, with multiple copies of the complete graph and forests, were studied in [Stahl, LorSe, LorSo].

- If $G$ is connected and $R(K_k, G) = (k-1)(n(G)-1)+1$, in particular if G is any tree, then $R(K_{n_1}, \cdots, K_{n_k}, G)$ = $(R(K_{n_1}, \cdots, K_{n_k})-1)(n(G)-1)+1$ [BE2]. A generalization for connected $G_1, \ldots, G_n$ in place of $G$ appeared in [Jac].
- Study of $R(S, G_1, \cdots, G_k)$ for large sparse $S$ [EFRS1, Bu3].
- Bounds for trees $R_k(T)$ and forests $R_k(F)$ [EG, GRS, BB, GT, Bra].
- See also surveys listed in section 7.

## 6. Hypergraph Numbers

The only known value of a classical Ramsey number for hypergraphs:

| | |
|---|---|
| $R(4,4;3) = 13$ | [MR1] |
| more than 200000 critical colorings | |

Other hypergraph cases:

| | |
|---|---|
| $R(K_4-t, K_4-t; 3) = 7$ | [Ea4] |
| $R(K_4-t, K_4; 3) = 8$ | [So, Ex1, MR1] |
| $14 \le R(K_4-t, K_5; 3)$ | [Ex1] |
| $63 \le R(5,5;3)$ | [Ea1] |
| $32 \le R(4,5;3)$ | [Ex7] |
| $33 \le R(5,5;4)$ | [Ex10] |
| $56 \le R(4,4,4;3)$ | [Ex7] |
| $13 \le R(K_4-t, K_4-t, K_4-t; 3) \le 17$ | [Ex1] [Ea1] |

The computer evaluation of $R(4,4;3)$ in [MR1] consisted of an improvement of the upper bound from 15 to 13, which followed an extensive theoretical study of this number in [Gi2, Is1, Sid1]. Exoo in [Ex1] announced the bounds $R(4,5;3) \ge 30$ and $R(5,5;4) \ge 27$ without presenting the constructions. The best published bound of $R(4,5;3) \ge 24$ was obtained by Isbell [Is2]. Shastri in [Sha] shows a weak bound $R(5,5;4) \ge 19$, nevertheless his lemmas and those in [Ka3, Abb, GRS] can be used to derive other lower bounds for higher numbers. Study of lower bounds for $R_m(k;s)$ can be found in [DLR].

Theoretical results on hypergraph numbers are gathered in [GrRö, GRS].

## 7. Cumulative Data and Surveys

Cumulative data for two colors:

[CH1] $R(G,G)$ for all graphs $G$ without isolates on at most 4 vertices.

[CH2] $R(G,H)$ for all graphs $G$ and $H$ without isolates on at most 4 vertices.

[Clan] $R(G,H)$ for all graphs $G$ on at most 4 vertices and $H$ on 5 vertices, except five entries.

[He4] All critical colorings for $R(G,H)$, for isolate-free graphs $G$ and $H$ as in [Clan] above.

[Bu4] $R(G,G)$ for all graphs $G$ without isolates and with at most 6 edges.

[He1] $R(G,G)$ for all graphs $G$ without isolates and with at most 7 edges.

[HM2] $R(G,G)$ for all graphs $G$ on 5 vertices and with 7 or 8 edges.

[He2] $R(G,H)$ for all graphs $G$ and $H$ on 5 vertices without isolates, except 7 entries.

[FRS1] $R(K_3,G)$ for all connected graphs $G$ on 6 vertices.

[Jin] $R(K_3,G)$ for all connected graphs $G$ on 7 vertices.

[FRS4] $R(G,T)$ for all connected graphs $G$ on at most 5 vertices and all (except some cases) trees $T$.

Cumulative data for three colors:

[YR2] $R_3(G)$ for all graphs $G$ with at most 4 edges and no isolates.

[YR1] $R_3(G)$ for all graphs $G$ with 5 edges and no isolates, except $K_4-e$.

[YY] $R_3(G)$ for all graphs $G$ with 6 edges and no isolates, except 10 cases.

[AKM] $R(F,G,H)$ for most triples of isolate-free graphs with at most 4 vertices.

Chvátal and Harary [CH1, CH2] formulated several simple but very useful observations how to discover values of some numbers. All five missing entries in the tables of Clancy [Clan] have been solved. The open entries of [He2] with $G \neq K_4$ and $H = K_5$ still stand, but may yield soon. The case of $R_3(K_4-e)$ is also open.

Surveys:

[Bu1] A general survey of results in Ramsey graph theory by S.A. Burr (1974)

[Par5] A general survey of results in Ramsey graph theory by T.D. Parsons (1978)

[Ha2] Summary of progress by Frank Harary (1981)

[ChGri] A general survey of bounds and values by F.R.K. Chung and C.M. Grinstead (1983)

[JGT] Special volume of *the Journal of Graph Theory* (1983)

[Rob] Nice textbook-type review of Ramsey graph theory for newcomers (1984)

[Bu6] What can we hope to accomplish in generalized Ramsey Theory ? (1987)

[GrRö] Survey of asymptotic problems by R.L. Graham and V. Rödl (1987)

[GRS] An excellent book by R.L. Graham, B.L. Rothschild and J.H. Spencer, second edition (1990)

[FRS7] Survey of goodness results of the type $R(G,H) = (\chi(G)-1)(n(H)-1)+s(G)$ (1991)

[Neš] A chapter in *Handbook of Combinatorics*, to appear (1994)

The surveys by S.A. Burr [Bu1] and T.D. Parsons [Par5] contain extensive chapters on general exact results in graph Ramsey theory. F. Harary presented the state of the theory in 1981 in [Ha2], where he also gathered many references including seven to other survey papers. A decade ago, Chung and Grinstead in their survey paper [ChGri] gave less data than in this note, but included a broad discussion of different methods used in Ramsey computations in the classical case. S.A. Burr, one of the most experienced researchers in Ramsey graph theory, formulated in [Bu6] seven conjectures on Ramsey numbers for sufficiently large and sparse graphs, and reviewed the evidence for them found in the literature.

For newer extensive presentations see [GRS, GrRö, FRS7, Neš], though these focus on asymptotic theory not on the numbers themselves. Finally, this compilation could not pretend to be complete without mentioning a special volume of the Journal of Graph Theory [JGT] dedicated entirely to Ramsey theory. Besides a number of research papers, it includes historical notes and presents to us Frank P. Ramsey (1903-1930) as a person.

## 8. Concluding Remarks

This compilation does not include information on numerous variations of Ramsey numbers, nor related topics, like size Ramsey numbers, zero-sum Ramsey numbers, irredundant Ramsey numbers, local Ramsey numbers, connected Ramsey numbers, chromatic Ramsey numbers, avoiding sets of graphs in some colors, coloring graphs other than complete, or the so called Ramsey multiplicities. Interested reader can find such information in the surveys listed in section 7 here.

The author apologizes for any omissions or other errors in reporting results belonging to the scope of this work. Suggestions for any kind of corrections and/or additions will be greatly appreciated.

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**References**

We mark the papers containing results obtained with the help of computer algorithms with stars. We identify two categories of such papers: marked with * involving some use of computers, where the results are easily verifiable with some computations, and those marked with **, where cpu intensive algorithms have to be implemented to replicate or verify the results. The first category contains mostly constructions done by algorithms, while the second mostly nonexistence results or claims of complete enumerations of special kinds of graphs.

[Abb] H.L. Abbot, A Theorem Concerning Higher Ramsey Numbers, in *Infinite and Finite Sets,* (A. Hajnal, R. Rado and V.T. Sós eds.) North Holland, (1975) 25-28.

[AKS] M. Ajtai, J. Komlós and E. Szemerédi, A Note on Ramsey Numbers, *Journal of Combinatorial Theory,* Series A, 29 (1980) 354-360.

[Alon1] N. Alon, Subdivided Graphs Have Linear Ramsey Numbers, *Journal of Graph Theory,* 18 (1994) 343-347.

[Alon2] N. Alon, Explicit Ramsey Graphs and Orthonormal Labelings, *The Electronic Journal of Combinatorics,* http://ejc.math.gatech.edu:8080/Journal/ejc-wce.html, #R12, 1 (1994) 8 pages.

[AKM] J. Arste, K. Klamroth and I. Mengersen, Three Color Ramsey Numbers for Small Graphs, *preprint,* Technische Universität Braunschweig (1993).

[Ba]* F. Bannani, Bounds on Classical Ramsey Numbers, *Ph.D. thesis,* Carleton University, Ottawa, November 1988.

[BS] A. Bialostocki and J. Schönheim, On Some Turán and Ramsey Numbers for $C_4$, in *Graph Theory and Combinatorics* (ed. B. Bollobás), Academic Press, London, (1984) 29-33.

[Bier] J. Bierbrauer, Ramsey Numbers for the Path with Three Edges, *European Journal of Combinatorics,* 7 (1986) 205-206.

[BB] J. Bierbrauer and A. Brandis, On Generalized Ramsey Numbers for Trees, *Combinatorica,* 5 (1985) 95-107.

[BH] R. Bolze and H. Harborth, The Ramsey Number $r(K_4-x, K_5)$, in *The Theory and Applications of Graphs,* (Kalamazoo, MI, 1980), John Wiley & Sons, New York, (1981) 109-116.

[BoEr] J.A. Bondy and P. Erdös, Ramsey Numbers for Cycles in Graphs, *Journal of Combinatorial Theory,* Series B, 14 (1973) 46-54.

[-] A. Brandis, see [BB].

[Bra] S. Brandt, Subtrees and Subforests in Graphs, *Journal of Combinatorial Theory*, Series B, 61 (1994) 63-70.

[BR]* J.P. Burling and S.W. Reyner, Some Lower Bounds of the Ramsey Numbers $n(k,k)$, *Journal of Combinatorial Theory*, Series B, 13 (1972) 168-169.

[Bu1] S.A. Burr, Generalized Ramsey Theory for Graphs - a Survey, in *Graphs and Combinatorics* (R. Bari and F. Harary eds.), Springer LNM 406, Berlin, (1974) 52-75.

[Bu2] S.A. Burr, Ramsey Numbers Involving Graphs with Long Suspended Paths, *Journal of the London Mathematical Society* (2), 24 (1981) 405-413.

[Bu3] S.A. Burr, Multicolor Ramsey Numbers Involving Graphs with Long Suspended Path, *Discrete Mathematics*, 40 (1982) 11-20.

[Bu4] S.A. Burr, Diagonal Ramsey Numbers for Small Graphs, *Journal of Graph Theory*, 7 (1983) 57-69.

[Bu5] S.A. Burr, Ramsey Numbers Involving Powers, *Ars Combinatoria*, 15 (1983) 163-168.

[Bu6] S.A. Burr, What Can We Hope to Accomplish in Generalized Ramsey Theory?, *Discrete Mathematics*, 67 (1987) 215-225.

[Bu7] S.A. Burr, On the Ramsey Numbers $r(G,nH)$ and $r(nG,nH)$ When $n$ Is Large, *Discrete Mathematics*, 65 (1987) 215-229.

[Bu8] S.A. Burr, On Ramsey Numbers for Large Disjoint Unions of Graphs, *Discrete Mathematics*, 70 (1988) 277-293.

[BE1] S.A. Burr and P. Erdös, Extremal Ramsey Theory for Graphs, *Utilitas Mathematica*, 9 (1976) 247-258.

[BE2] S.A. Burr and P. Erdös, Generalizations of a Ramsey-Theoretic Result of Chvátal, *Journal of Graph Theory*, 7 (1983) 39-51.

[BEFRS1] S.A. Burr, P. Erdös, R.J. Faudree, C.C. Rousseau and R.H. Schelp, An Extremal Problem in Generalized Ramsey Theory, *Ars Combinatoria*, 10 (1980) 193-203.

[BEFRS2] S.A. Burr, P. Erdös, R.J. Faudree, C.C. Rousseau and R.H. Schelp, Ramsey Numbers for the Pair Sparse Graph-Path or Cycle, *Transactions of the American Mathematical Society*, 269 (1982) 501-512.

[BEFRS3] S.A. Burr, P. Erdös, R.J. Faudree, C.C. Rousseau and R.H. Schelp, On Ramsey Numbers Involving Starlike Multipartite Graphs, *Journal of Graph Theory*, 7 (1983) 395-409.

[BEFRS4] S.A. Burr, P. Erdös, R.J. Faudree, C.C. Rousseau and R.H. Schelp, The Ramsey Number for the Pair Complete Bipartite Graph-Graph of Limited Degree, in *Graph Theory with Applications to Algorithms and Computer Science*, (Y. Alavi et al. eds.), John Wiley & Sons, New York, (1985) 163-174.

[BEFRSGJ] S.A. Burr, P. Erdös, R.J. Faudree, C.C. Rousseau, R.H. Schelp, R.J. Gould and M.S. Jacobson, Goodness of Trees for Generalized Books, *Graphs and*

*Combinatorics,* 3 (1987) 1-6.

[BEFS] S.A. Burr, P. Erdös, R.J. Faudree and R.H. Schelp, On the Difference between Consecutive Ramsey Numbers, *Utilitas Mathematica,* 35 (1989) 115-118.

[BES] S.A. Burr, P. Erdös and J.H. Spencer, Ramsey Theorems for Multiple Copies of Graphs, *Transactions of the American Mathematical Society,* 209 (1975) 87-99.

[BG] S.A. Burr and J.W. Grossman, Ramsey Numbers of Graphs with Long Tails, *Discrete Mathematics,* 41 (1982) 223-227.

[BuRo1] S.A. Burr and J.A. Roberts, On Ramsey Numbers for Stars, *Utilitas Mathematica,* 4 (1973) 217-220.

[BuRo2] S.A. Burr and J.A. Roberts, On Ramsey Numbers for Linear Forests, *Discrete Mathematics,* 8 (1974) 245-250.

[CET]* N.J. Calkin, P. Erdös and C.A. Tovey, New Ramsey Bounds from Cyclic Graphs of Prime Order, *in preparation* (1995).

[CGP] G. Chartrand, R.J. Gould and A.D. Polimeni, On Ramsey Numbers of Forests versus Nearly Complete Graphs, *Journal of Graph Theory,* 4 (1980) 233-239.

[CRSPS] G. Chartrand, C.C. Rousseau, M.J. Stewart, A.D. Polimeni and J. Sheehan, On Star-Book Ramsey Numbers, in *Proceedings of the Fourth International Conference on the Theory and Applications of Graphs,* (Kalamazoo, MI 1980), John Wiley & Sons, (1981) 203-214.

[ChLo]* V. Chokbowornpaishand and V. Longani, Lower Bounds for Some Ramsey Numbers, *Southeast Asian Bulletin of Mathematics,* 15 (1991) 105-107.

[Chu1] F.R.K. Chung, On the Ramsey Numbers $N(3,3,...,3;2)$, *Discrete Mathematics,* 5 (1973) 317-321.

[Chu2] F.R.K. Chung, On Triangular and Cyclic Ramsey Numbers with $k$ Colors, in *Graphs and Combinatorics* (R. Bari and F. Harary eds.), Springer LNM 406, Berlin, (1974) 236-241.

[Chu3] F.R.K. Chung, A Note on Constructive Methods for Ramsey Numbers, *Journal of Graph Theory,* 5 (1981) 109-113.

[CCD] F.R.K. Chung, R. Cleve and P. Dagum, A Note on Constructive Lower Bounds for the Ramsey Numbers $R(3,t)$, *Journal of Combinatorial Theory,* Series B, 57 (1993) 150-155.

[ChGra] F.R.K. Chung and R.L. Graham, On Multicolor Ramsey Numbers for Complete Bipartite Graphs, *Journal of Combinatorial Theory,* Series B, 18 (1975) 164-169.

[ChGri] F.R.K. Chung and C.M. Grinstead, A Survey of Bounds for Classical Ramsey Numbers, *Journal of Graph Theory,* 7 (1983) 25-37.

[Chv] V. Chvátal, Tree-Complete Graph Ramsey Numbers, *Journal of Graph Theory,* 1 (1977) 93.

[CH1] V. Chvátal and F. Harary, Generalized Ramsey Theory for Graphs, II. Small Diagonal Numbers, *Proceedings of American Mathematical Society,* 32 (1972) 389-394.

[CH2] V. Chvátal and F. Harary, Generalized Ramsey Theory for Graphs, III. Small Off-Diagonal Numbers, *Pacific Journal of Mathematics*, 41 (1972) 335-345.

[CH3] V. Chvátal and F. Harary, Generalized Ramsey Theory for Graphs, I. Diagonal Numbers, *Periodica Mathematica Hungarica*, 3 (1973) 115-124.

[CRST] V. Chvátal, V. Rödl, E. Szemerédi and W.T. Trotter Jr., The Ramsey Number of a Graph with Bounded Maximum Degree, *Journal of Combinatorial Theory*, Series B, 34 (1983) 239-243.

[CS] V. Chvátal and A. Schwenk, On the Ramsey Number of the Five-Spoked Wheel, in *Graphs and Combinatorics* (R. Bari and F. Harary eds.), Springer LNM 406, Berlin, (1974) 247-261.

[Clan] M. Clancy, Some Small Ramsey Numbers, *Journal of Graph Theory*, 1 (1977) 89-91.

[Clap] C. Clapham, The Ramsey Number $r(C_4, C_4, C_4)$, *Periodica Mathematica Hungarica*, 18 (1987) 317-318.

[CEHMS] C. Clapham, G. Exoo, H. Harborth, I. Mengersen and J. Sheehan, The Ramsey Number of $K_5 - e$, *Journal of Graph Theory*, 13 (1989) 7-15.

[Clark] L. Clark, On Cycle-Star Graph Ramsey Numbers, *Congressus Numerantium*, 50 (1985) 187-192.

[-] R. Cleve, see [CCD].

[CL1] E.J. Cockayne and P.J. Lorimer, The Ramsey Number for Stripes, *Journal of the Australian Mathematical Society*, Series A, 19 (1975) 252-256.

[CL2] E.J. Cockayne and P.J. Lorimer, On Ramsey Graph Numbers for Stars and Stripes, *Canadian Mathematical Bulletin*, 18 (1975) 31-34.

[-] P. Dagum, see [CCD].

[DLR] D. Duffus, H. Lefmann and V. Rödl, Shift Graphs and Lower Bounds on Ramsey Numbers $r_k(l; r)$, *Discrete Mathematics*, 137 (1995) 177-187.

[Ea1] Easy to obtain by simple combinatorics from other results.

[Ea2] Easy to obtain using $R(3,3,4) \leq 32$.

[Ea3] Easy to obtain using graphs (or their disjoint unions) establishing other lower bounds.

[Ea4] Unique 2-(6,3,2) design gives lower bound 7, upper bound is easy.

[EFRS1] P. Erdös, R.J. Faudree, C.C. Rousseau and R.H. Schelp, Generalized Ramsey Theory for Multiple Colors, *Journal of Combinatorial Theory*, Series B, 20 (1976) 250-264.

[EFRS2] P. Erdös, R.J. Faudree, C.C. Rousseau and R.H. Schelp, Ramsey Numbers for Brooms, *Congressus Numerantium*, 35 (1982) 283-293.

[EFRS3] P. Erdös, R.J. Faudree, C.C. Rousseau and R.H. Schelp, Multipartite Graph-Sparse Graph Ramsey Numbers, *Combinatorica*, 5 (1985) 311-318.

[EFRS4] P. Erdös, R.J. Faudree, C.C. Rousseau and R.H. Schelp, A Ramsey Problem of Harary on Graphs with Prescribed Size, *Discrete Mathematics,* 67 (1987) 227-233.

[EFRS5] P. Erdös, R.J. Faudree, C.C. Rousseau and R.H. Schelp, Extremal Theory and Bipartite Graph-Tree Ramsey Numbers, *Discrete Mathematics,* 72 (1988) 103-112.

[EFRS6] P. Erdös, R.J. Faudree, C.C. Rousseau and R.H. Schelp, Multipartite Graph-Tree Graph Ramsey Numbers, in *Graph Theory and Its Applications: East and West, Proceedings of the First China-USA International Graph Theory Conference,* Annals of the New York Academy of Sciences, 576 (1989) 146-154.

[EG] P. Erdös and R.L. Graham, On Partition Theorems for Finite Sets, in *Infinite and Finite Sets,* (A. Hajnal, R. Rado and V.T. Sós eds.) North Holland, (1975) 515-527.

[-] P. Erdös, see also [BoEr, BE1, BE2, BEFRS1, BEFRS2, BEFRS3, BEFRS4, BEFRSGJ, BEFS, BES, CET].

[Ex1]* G. Exoo, Ramsey Numbers of Hypergraphs, *Journal of Combinatorial Mathematics and Combinatorial Computing,* 2 (1987) 5-11.

[Ex2]* G. Exoo, On Two Classical Ramsey Numbers of the Form $R(3,n)$, *SIAM Journal of Discrete Mathematics,* 2 (1989) 488-490.

[Ex3]* G. Exoo, Applying Optimization Algorithm to Ramsey Problems, in *Graph Theory, Combinatorics, Algorithms, and Applications* (Y. Alavi ed.), SIAM Philadelphia, (1989) 175-179.

[Ex4]* G. Exoo, A Lower Bound for $R(5,5)$, *Journal of Graph Theory,* 13 (1989) 97-98.

[Ex5]* G. Exoo, A Lower Bound for $r(K_5-e,K_5)$, *Utilitas Mathematica,* 38 (1990) 187-188.

[Ex6]* G. Exoo, On the Three Color Ramsey Number of $K_4-e$, *Discrete Mathematics,* 81 (1991) 301-305.

[Ex7]* G. Exoo, Indiana State University, *personal communication* (1992).

[Ex8]* G. Exoo, Announcement: On the Ramsey Numbers $R(4,6)$, $R(5,6)$ and $R(3,12)$, *Ars Combinatoria,* 35 (1993) 85.

[Ex9]* G. Exoo, Constructing Ramsey Graphs with a Computer, *Congressus Numerantium,* 59 (1987) 31-36.

[Ex10]* G. Exoo, Indiana State University, *personal communication* (1993).

[Ex11]* G. Exoo, A Lower Bound for Schur Numbers and Multicolor Ramsey Numbers of $K_3$, *The Electronic Journal of Combinatorics,* http://ejc.math.gatech.edu:8080/Journal/ejc-wce.html, #R8, 1 (1994) 3 pages.

[EHM1] G. Exoo, H. Harborth and I. Mengersen, The Ramsey Number of $K_4$ versus $K_5-e$, *Ars Combinatoria,* 25A (1988) 277-286.

[EHM2] G. Exoo, H. Harborth and I. Mengersen, On Ramsey Number of $K_{2,n}$, in *Graph Theory, Combinatorics, Algorithms, and Applications* (Y. Alavi, F.R.K. Chung, R.L. Graham and D.F. Hsu eds.), SIAM Philadelphia, (1989) 207-211.

[ER]* G. Exoo and D.F. Reynolds, Ramsey Numbers Based on $C_5$-Decompositions, *Discrete Mathematics*, 71 (1988) 119-127.

[-] G. Exoo, see also [CEHMS].

[FLPS] R.J. Faudree, S.L. Lawrence, T.D. Parsons and R.H. Schelp, Path-Cycle Ramsey Numbers, *Discrete Mathematics*, 10 (1974) 269-277.

[FM]** R.J. Faudree and B.D. McKay, A Conjecture of Erdös and the Ramsey Number $r(W_6)$, *Journal of Combinatorial Mathematics and Combinatorial Computing*, 13 (1993) 23-31.

[FRS1] R.J. Faudree, C.C. Rousseau and R.H. Schelp, All Triangle-Graph Ramsey Numbers for Connected Graphs of Order Six, *Journal of Graph Theory*, 4 (1980) 293-300.

[FRS2] R.J. Faudree, C.C. Rousseau and R.H. Schelp, Studies Related to the Ramsey Number $r(K_5 - e)$, in *Graph Theory and Its Applications to Algorithms and Computer Science*, (Y. Alavi et al. eds.), John Wiley and Sons, New York, (1985) 251-271.

[FRS3] R.J. Faudree, C.C. Rousseau and R.H. Schelp, Generalizations of the Tree-Complete Graph Ramsey Number, in *Graphs and Applications*, (F. Harary and J.S. Maybee eds.), John Wiley and Sons, New York, (1985) 117-126.

[FRS4] R.J. Faudree, C.C. Rousseau and R.H. Schelp, Small Order Graph-Tree Ramsey Numbers, *Discrete Mathematics*, 72 (1988) 119-127.

[FRS5] R.J. Faudree, C.C. Rousseau and J. Sheehan, Cycle-Book Ramsey Numbers, *Ars Combinatoria*, 31 (1991) 239-248.

[FRS6] R.J. Faudree, C.C. Rousseau and J. Sheehan, More from the Good Book, in *Proceedings of the Ninth Southeastern Conference on Combinatorics, Graph Theory, and Computing*, Utilitas Mathematica Publ., (1978) 289-299.

[FRS7] R.J. Faudree, C.C. Rousseau and R.H. Schelp, A Good Idea in Ramsey Theory, in *Graph Theory, Combinatorics, Algorithms, and Applications* (San Francisco, CA 1989), SIAM Philadelphia, PA (1991) 180-189.

[FS1] R.J. Faudree and R.H. Schelp, All Ramsey Numbers for Cycles in Graphs, *Discrete Mathematics*, 8 (1974) 313-329.

[FS2] R.J. Faudree and R.H. Schelp, Path Ramsey Numbers in Multicolorings, *Journal of Combinatorial Theory*, Series B, 19 (1975) 150-160.

[FS3] R.J. Faudree and R.H. Schelp, Ramsey Numbers for All Linear Forests, *Discrete Mathematics*, 16 (1976) 149-155.

[FSR] R.J. Faudree, R.H. Schelp and C.C. Rousseau, Generalizations of a Ramsey Result of Chvátal, in *Proceedings of the Fourth International Conference on the Theory and Applications of Graphs*, (Kalamazoo, MI 1980), John Wiley & Sons,

(1981) 351-361.

[FSS] R.J. Faudree, R.H. Schelp and M. Simonovits, On Some Ramsey Type Problems Connected with Paths, Cycles and Trees, *Ars Combinatoria,* 29A (1990) 97-106.

[FS] R.J. Faudree and M. Simonovits, Ramsey Problems and Their Connection to Turán-Type Extremal Problems, *Journal of Graph Theory,* 16 (1992) 25-50.

[-] R.J. Faudree, see also [BEFRS1, BEFRS2, BEFRS3, BEFRS4, BEFRSGJ, BEFS, EFRS1, EFRS2, EFRS3, EFRS4, EFRS5, EFRS6].

[Fo] J. Folkman, Notes on the Ramsey Number $N(3,3,3,3)$, *Journal of Combinatorial Theory,* Series A, 16 (1974) 371-379.

[Fra1] K. Fraughnaugh Jones, Independence in Graphs with Maximum Degree Four, *Journal of Combinatorial Theory,* Series B, 37 (1984) 254-269.

[Fra2] K. Fraughnaugh Jones, Size and Independence in Triangle-Free Graphs with Maximum Degree Three, *Journal of Graph Theory,* 14 (1990) 525-535.

[FL] K. Fraughnaugh Jones and S.C. Locke, Lower Bounds on the Number of Edges in Triangle-Free Graphs, *to appear.*

[Fre] H. Fredrickson, Schur Numbers and the Ramsey Numbers $N(3,3,...,3;2)$, *Journal of Combinatorial Theory,* Series A, 27 (1979) 376-377.

[GeGy] L. Geréncser and A. Gyárfás, On Ramsey-Type Problems, *Ann. Univ. Sci. Budapest, Eötvös Sect. Math.,* 10 (1967) 167-170.

[Gi1] G. Giraud, Nouvelles majorations des nombres de Ramsey binaires-bicolores, *C.R. Acad. Sc. Paris, A,* 268 (1969) 5-7.

[Gi2] G. Giraud, Majoration du nombre de Ramsey ternaire-bicolore en (4,4), *C.R. Acad. Sc. Paris,* 269, 15 (1969) 620-622.

[Gi3] G. Giraud, Une minoration du nombre de quadrangles unicolores et son application a la majoration des nombres de Ramsey binaires bicolors, *C.R. Acad. Sci., Paris, A,* 276 (1973) 1173-1175.

[Gi4] G. Giraud, Sur le Probleme de Goodman pour les Quadrangles et la Majoration des Nombres de Ramsey, *Journal of Combinatorial Theory,* Series B, 27 (1979) 237-253.

[-] A.M. Gleason, see [GG].

[GJ1] R.J. Gould and M.S. Jacobson, Bounds for the Ramsey Number of a Disconnected Graph Versus Any Graph, *Journal of Graph Theory,* 6 (1982) 413-417.

[GJ2] R.J. Gould and M.S. Jacobson, On the Ramsey Number of Trees Versus Graphs with Large Clique Number, *Journal of Graph Theory,* 7 (1983) 71-78.

[-] R.J. Gould, see also [BEFRSGJ, CGP].

[GrRö] R.L. Graham and V. Rödl, Numbers in Ramsey Theory, in *Surveys in Combinatorics,* (ed. C. Whitehead), Cambridge University Press, 1987.

[GRS] R.L. Graham, B.L. Rothschild and J.H. Spencer, *Ramsey Theory,* John Wiley & Sons, 1990.

[-] R.L. Graham, see also [ChGra, EG].

[GY] J.E. Graver and J. Yackel, Some Graph Theoretic Results Associated with Ramsey's Theorem, *Journal of Combinatorial Theory,* 4 (1968) 125-175.

[GG] R.E. Greenwood and A.M. Gleason, Combinatorial Relations and Chromatic Graphs, *Canadian Journal of Mathematics,* 7 (1955) 1-7.

[GH] U. Grenda and H. Harborth, The Ramsey Number $r(K_3, K_7 - e)$, *Journal of Combinatorics, Information & System Sciences,* 7 (1982) 166-169.

[Gri] J.R. Griggs, An Upper Bound on the Ramsey Numbers $R(3,k)$, *Journal of Combinatorial Theory,* Series A, 35 (1983) 145-153.

[GR]** C. Grinstead and S. Roberts, On the Ramsey Numbers $R(3,8)$ and $R(3,9)$, *Journal of Combinatorial Theory,* Series B, 33 (1982) 27-51.

[-] C. Grinstead, see also [ChGri].

[Gro1] J.W. Grossman, Some Ramsey Numbers of Unicyclic Graphs, *Ars Combinatoria,* 8 (1979) 59-63.

[Gro2] J.W. Grossman, The Ramsey Numbers of the Union of Two Stars, *Utilitas Mathematica,* 16 (1979) 271-279.

[GHK] J.W. Grossman, F. Harary and M. Klawe, Generalized Ramsey Theory for Graphs, X: Double Stars, *Discrete Mathematics,* 28 (1979) 247-254.

[-] J.W. Grossman, see also [BG].

[GS] Guantao Chen and R.H. Schelp, Graphs with Linearly Bounded Ramsey Numbers, *Journal of Combinatorial Theory,* Series B, 57 (1993) 138-149.

[GV] Yubao Guo and L. Volkmann, Tree-Ramsey Numbers, *Australasian Journal of Combinatorics,* 11 (1995) 169-175.

[GT] A. Gyárfás and Z. Tuza, An Upper Bound on the Ramsey Number of Trees, *Discrete Mathematics,* 66 (1987) 309-310.

[-] A. Gyárfás, see also [GeGy].

[Häg] R. Häggkvist, On the Path-Complete Bipartite Ramsey Number, *Discrete Mathematics,* 75 (1989) 243-245.

[Ha1] F. Harary, Recent Results on Generalized Ramsey Theory for Graphs, in *Graph Theory and Applications,* (Y. Alavi et al. eds.) Springer, Berlin (1972) 125-138.

[Ha2] F. Harary, Generalized Ramsey Theory I to XIII: Achievement and Avoidance Numbers, in *Proceedings of the Fourth International Conference on the Theory and Applications of Graphs,* (Kalamazoo, MI 1980), John Wiley & Sons, (1981) 373-390.

[-] F. Harary, see also [CH1, CH2, CH3, GHK].

[HM1] H. Harborth and I. Mengersen, An Upper Bound for the Ramsey Number $r(K_5 - e)$, *Journal of Graph Theory,* 9 (1985) 483-485.

[HM2] H. Harborth and I. Mengersen, All Ramsey Numbers for Five Vertices and Seven or Eight Edges, *Discrete Mathematics,* 73 (1988/89) 91-98.

[HM3] H. Harborth and I. Mengersen, The Ramsey Number of $K_{3,3}$, in *Combinatorics, Graph Theory, and Applications*, Vol. 2 (Y. Alavi, G. Chartrand, O.R. Oellermann and J. Schwenk eds.), John Wiley & Sons, (1991) 639-644.

[-] H. Harborth, see also [BH, CEHMS, EHM1, EHM2, GH].

[He1] G.R.T. Hendry, Diagonal Ramsey Numbers for Graphs with Seven Edges, *Utilitas Mathematica*, 32 (1987) 11-34.

[He2] G.R.T. Hendry, Ramsey Numbers for Graphs with Five Vertices, *Journal of Graph Theory*, 13 (1989) 245-248.

[He3] G.R.T. Hendry, The Ramsey Numbers $r(K_2+\overline{K}_3, K_4)$ and $r(K_1+C_4, K_4)$, *Utilitas Mathematica*, 35 (1989) 40-54, addendum in 36 (1989) 25-32.

[He4] G.R.T. Hendry, Critical Colorings for Clancy's Ramsey Numbers, *Utilitas Mathematica*, 41 (1992) 181-203.

[-] G.R.T. Hendry, see also [YH].

[HI]* R. Hill and R.W. Irving, On Group Partitions Associated with Lower Bounds for Symmetric Ramsey Numbers, *European Journal of Combinatorics*, 3 (1982) 35-50.

[Hua] Huang Guotai, Some Generalized Ramsey Numbers (in Chinese), *Mathematics and Its Applications*, 1 (1988) 97-101.

[HZ] Huang Yi Ru and Zhang Ke Min, An New Upper Bound Formula for Two Color Classical Ramsey Numbers, *to appear* (1993).

[Ir] R.W. Irving, Generalised Ramsey Numbers for Small Graphs, *Discrete Mathematics*, 9 (1974) 251-264.

[-] R.W. Irving, see also [HI].

[Is1] J.R. Isbell, $N(4,4;3) \geq 13$, *Journal of Combinatorial Theory*, 6 (1969) 210.

[Is2] J.R. Isbell, $N(5,4;3) \geq 24$, *Journal of Combinatorial Theory*, Series A, 34 (1983) 379-380.

[Jac] M.S. Jacobson, On the Ramsey Number for Stars and a Complete Graph, *Ars Combinatoria*, 17 (1984) 167-172.

[-] M.S. Jacobson, see also [BEFRSGJ, GJ1, GJ2].

[Jin]** Jin Xia, Ramsey Numbers Involving a Triangle: Theory & Applications, *Technical Report RIT-TR-93-019*, MS thesis, Department of Computer Science, Rochester Institute of Technology, 1993.

[-] Jin Xia, see also [RJ].

[JGT] Special volume on Ramsey theory of *Journal of Graph Theory*, Volume 7, Number 1, (1983).

[Ka1] J.G. Kalbfleisch, Construction of Special Edge-Chromatic Graphs, *Canadian Mathematical Bulletin*, 8 (1965) 575-584.

[Ka2]* J.G. Kalbfleisch, Chromatic Graphs and Ramsey's Theorem, *Ph.D. thesis*, University of Waterloo, January 1966.

[Ka3] J.G. Kalbfleisch, On Robillard's Bounds for Ramsey Numbers, *Canadian Mathematical Bulletin,* 14 (1971) 437-440.

[KS] J.G. Kalbfleisch and R.G. Stanton, On the Maximal Triangle-Free Edge-Chromatic Graphs in Three Colors, *Journal of Combinatorial Theory,* 5 (1968) 9-20.

[Kim] Jeong Han Kim, The Ramsey Number $R(3,t)$ has Order of Magnitude $t^2/\log t$, *to appear.*

[-] K. Klamroth, see [AKM].

[-] M. Klawe, see [GHK].

[Köh] W. Köhler, On a Conjecture by Grossman, *Ars Combinatoria,* 23 (1987) 103-106.

[-] J. Komlós, see [AKS].

[KLR]* D.L. Kreher, Wei Li and S.P. Radziszowski, Lower Bounds for Multi-Colored Ramsey Numbers From Group Orbits, *Journal of Combinatorial Mathematics and Combinatorial Computing,* 4 (1988) 87-95.

[-] D.L. Kreher, see also [RK1, RK2, RK3, RK4].

[La1] S.L. Lawrence, Cycle-Star Ramsey Numbers, *Notices of the American Mathematical Society,* 20 (1973) Abstract A- 420.

[La2] S.L. Lawrence, Bipartite Ramsey Theory, *Notices of the American Mathematical Society,* 20 (1973) Abstract A- 562.

[-] S.L. Lawrence, see also [FLPS].

[LayMa] C. Laywine and J.P. Mayberry, A Simple Construction Giving the Two Non-isomorphic Triangle-Free 3-Colored $K_{16}$'s, *Journal of Combinatorial Theory,* Series B, 45 (1988) 120-124.

[Lef] H. Lefmann, Ramsey Numbers for Monotone Paths and Cycles, *Ars Combinatoria,* 35 (1993) 271-279.

[-] H. Lefmann, see also [DLR].

[LR] Yusheng Li and C.C. Rousseau, On Book-Complete Graph Ramsey Numbers, *to appear.*

[Lin] B. Lindström, Undecided Ramsey-Numbers for Paths, *Discrete Mathematics,* 43 (1983) 111-112.

[Loc] S.C. Locke, Bipartite Density and the Independence Ratio, *Journal of Graph Theory,* 10 (1986) 47-53.

[-] S.C. Locke, see also [FL].

[-] V. Longani, see [ChLo].

[Lor] P.J. Lorimer, The Ramsey Numbers for Stripes and One Complete Graph, *Journal of Graph Theory,* 8 (1984) 177-184.

[LorMu] P.J. Lorimer and P.R. Mullins, Ramsey Numbers for Quadrangles and Triangles, *Journal of Combinatorial Theory,* Series B, 23 (1977) 262-265.

[LorSe] P.J. Lorimer and R.J. Segedin, Ramsey Numbers for Multiple Copies of Complete Graphs, *Journal of Graph Theory*, 2 (1978) 89-91.

[LorSo] P.J. Lorimer and W. Solomon, The Ramsey Numbers for Stripes and Complete Graphs 1, *Discrete Mathematics*, 104 (1992) 91-97.

[-] P.J. Lorimer, see also [CL1, CL2].

[Mac]* J. Mackey, University of Hawaii, *personal communication* (1993,1994).

[Ma]* R. Mathon, Lower Bounds for Ramsey Numbers and Association Schemes, *Journal of Combinatorial Theory*, Series B, 42 (1987) 122-127.

[-] J.P. Mayberry, see [LayMa].

[MR1]** B.D. McKay and S.P. Radziszowski, The First Classical Ramsey Number for Hypergraphs is Computed, *Proceedings of the Second Annual ACM-SIAM Symposium on Discrete Algorithms*, SODA'91, San Francisco, (1991) 304-308.

[MR2]** B.D. McKay and S.P. Radziszowski, Linear Programming in Some Ramsey Problems, *Journal of Combinatorial Theory*, Series B, 61 (1994) 125-132.

[MR3]* B.D. McKay and S.P. Radziszowski, New Upper Bound for the Ramsey Number $R(5,5)$, *Australasian Journal of Combinatorics*, 5 (1992) 13-20.

[MR4]** B.D. McKay and S.P. Radziszowski, *results from current research, unpublished* (1993,1994). The best published upper bounds for $R(k,l)$, for $(k,l)$ = (4,6), (5,5) and (5,6) of 43, 52 and 94 can be found in [MR2, MR3] and [Wa2], respectively. The bound $R(6,6) \le 166$ is an immediate consequence of theorem 1 in [Wa1] and $R(4,6) \le 41$, the best published bound of 169 is due to Giraud [Gi3].

[MR5]** B.D. McKay and S.P. Radziszowski, $R(4,5) = 25$, *Journal of Graph Theory*, 19 (1995) 309-322.

[MZ]** B.D. McKay and Zhang Ke Min, The Value of the Ramsey Number $R(3,8)$, *Journal of Graph Theory*, 16 (1992) 183-192.

[-] B.D. McKay, see also [FM].

[McR]** J. McNamara and S.P. Radziszowski, The Ramsey Numbers $R(K_4-e, K_6-e)$ and $R(K_4-e, K_7-e)$, *Congressus Numerantium*, 81 (1991) 89-96.

[-] I. Mengersen, see [AKM, CEHMS, EHM1, EHM2, HM1, HM2, HM3].

[MS] H. Mizuno and I. Sato, Ramsey Numbers for Unions of Some Cycles, *Discrete Mathematics*, 69 (1988) 283-294.

[-] P.R. Mullins, see [LorMu].

[Neš] J. Nešetřil, Ramsey Theory, in *Handbook of Combinatorics, to appear.*

[Par1] T.D. Parsons, The Ramsey Numbers $r(P_m, K_n)$, *Discrete Mathematics*, 6 (1973) 159-162.

[Par2] T.D. Parsons, Path-Star Ramsey Numbers, *Journal of Combinatorial Theory*, Series B, 17 (1974) 51-58.

[Par3] T.D. Parsons, Ramsey Graphs and Block Designs, I, *Transactions of the American Mathematical Society*, 209 (1975) 33-44.

[Par4] T.D. Parsons, Ramsey Graphs and Block Designs, *Journal of Combinatorial Theory,* Series A, 20 (1976) 12-19.

[Par5] T.D. Parsons, Ramsey Graph Theory, in *Selected Topics in Graph Theory,* (L.W. Beineke and R.J. Wilson eds.), Academic Press, (1978) 361-384.

[-] T.D. Parsons, see also [FLPS].

[Piw1]* K. Piwakowski, Technical University of Gdańsk, paper presented at the conference "System-Modeling-Control", Zakopane, Poland (1993).

[Piw2]* K. Piwakowski, On Ramsey Number $R(4,3,3)$ and Triangle-free Edge-chromatic Graphs in Three Colors, paper presented at the "Second Kraków Conference on Graph Theory", Kraków, Poland (1994).

[-] A.D. Polimeni, see [CGP, CRSPS].

[Ra1]* S.P. Radziszowski, used the same method as in [RK2], *unpublished* (1988).

[Ra2]** S.P. Radziszowski, The Ramsey Numbers $R(K_3, K_8 - e)$ and $R(K_3, K_9 - e)$, *Journal of Combinatorial Mathematics and Combinatorial Computing,* 8 (1990) 137-145.

[Ra3]** S.P. Radziszowski, On the Ramsey Number $R(K_5 - e, K_5 - e)$, *Ars Combinatoria,* 36 (1993) 225-232.

[Ra4] S.P. Radziszowski, Small Ramsey Numbers, *Technical Report RIT-TR-93-009,* Department of Computer Science, Rochester Institute of Technology (1993).

[RJ] S.P. Radziszowski and Jin Xia, Paths, Cycles and Wheels in Graphs without Antitriangles, *Australasian Journal of Combinatorics,* 9 (1994) 221-232.

[RK1]* S.P. Radziszowski and D.L. Kreher, Search Algorithm for Ramsey Graphs by Union of Group Orbits, *Journal of Graph Theory,* 12 (1988) 59-72.

[RK2]** S.P. Radziszowski and D.L. Kreher, Upper Bounds for Some Ramsey Numbers $R(3,k)$, *Journal of Combinatorial Mathematics and Combinatorial Computing,* 4 (1988) 207-212.

[RK3]** S.P. Radziszowski and D.L. Kreher, On $R(3,k)$ Ramsey Graphs: Theoretical and Computational Results, *Journal of Combinatorial Mathematics and Combinatorial Computing,* 4 (1988) 37-52.

[RK4] S.P. Radziszowski and D.L. Kreher, Minimum Triangle-Free Graphs, *Ars Combinatoria,* 31 (1991) 65-92.

[-] S.P. Radziszowski, see also [KLR, MR1, MR2, MR3, MR4, MR5, McR].

[Ram] F.P. Ramsey, On a Problem of Formal Logic, *Proceedings of the London Mathematical Society,* 30 (1930) 264-286.

[-] S.W. Reyner, see [BR].

[-] D.F. Reynolds, see [ER].

[Rob] F.S. Roberts, *Applied Combinatorics,* Prentice-Hall, Englewood Cliffs, 1984.

[-] J.A. Roberts, see [BuRo1, BuRo2].

[-] S. Roberts, see [GR].

[-] V. Rödl, see [CRST, DLR, GrRö].

[Ros] V. Rosta, On a Ramsey Type Problem of J.A. Bondy and P. Erdös, I & II, *Journal of Combinatorial Theory,* Series B, 15 (1973) 94-120.

[-] B.L. Rothschild, see [GRS].

[RS1] C.C. Rousseau and J. Sheehan, On Ramsey Numbers for Books, *Journal of Graph Theory,* 2 (1978) 77-87.

[RS2] C.C. Rousseau and J. Sheehan, A Class of Ramsey Problems Involving Trees, *Journal of the London Mathematical Society (2),* 18 (1978) 392-396.

[-] C.C. Rousseau, see also [BEFRS1, BEFRS2, BEFRS3, BEFRS4, BEFRSGJ, CRSPS, EFRS1, EFRS2, EFRS3, EFRS4, EFRS5, EFRS6, FRS1, FRS2, FRS3, FRS4, FRS5, FRS6, FRS7, FSR, LR].

[-] P. Rowlinson, see [YR1, YR2, YR3].

[San] A. Sanchez-Flores, An Improved Bound for Ramsey Number $N(3,3,3,3;2)$, *Discrete Mathematics,* 140 (1995) 281-286.

[-] I. Sato, see [MS].

[-] R.H. Schelp, see [BEFRS1, BEFRS2, BEFRS3, BEFRS4, BEFRSGJ, BEFS, EFRS1, EFRS2, EFRS3, EFRS4, EFRS5, EFRS6, FLPS, FRS1, FRS2, FRS3, FRS4, FRS7, FS1, FS2, FS3, FSR, FSS, GS].

[-] J. Schönheim, see [BS].

[Schu] C. -U. Schulte, Ramsey-Zahlen für Bäume und Kreise, *Ph.D. thesis,* Heinrich-Heine-Universität Düsseldorf, (1992).

[-] A. Schwenk, see [CS].

[Scob] M.W. Scobee, On the Ramsey Number $R(m_1P_3, m_2P_3, m_3P_3)$ and Related Results, ..., *MA thesis,* University of Louisville (1993).

[-] R.J. Segedin, see [LorSe].

[Sha] A. Shastri, Lower Bounds for Bi-Colored Quaternary Ramsey Numbers, *Discrete Mathematics,* 84 (1990) 213-216.

[S1]* J.B. Shearer, Lower Bounds for Small Diagonal Ramsey Numbers, *Journal of Combinatorial Theory,* Series A, 42 (1986) 302-304.

[S2] J.B. Shearer, A Note on the Independence Number of Triangle-free Graphs II, *Journal of Combinatorial Theory,* Series B, 53 (1991) 300-307.

[-] J. Sheehan, see [CRSPS, CEHMS, FRS5, FRS6, RS1, RS2].

[Sid1] A.F. Sidorenko, On Turán Numbers $T(n,5,4)$ and Number of Monochromatic 4-cliques in 2-colored 3-graphs (in Russian), *Voprosy Kibernetiki,* 64 (1980) 117-124.

[Sid2] A.F. Sidorenko, An Upper Bound on the Ramsey Number $R(K_3, G)$ Depending Only on the Size of the Graph $G$, *Journal of Graph Theory,* 15 (1991) 15-17.

[Sid3] A.F. Sidorenko, The Ramsey Number of an $N$-Edge Graph Versus Triangle Is at Most $2N+1$, *Journal of Combinatorial Theory*, Series B, 58 (1993) 185-196.

[-] M. Simonovits, see [FSS, FS].

[So] A. Sobczyk, Euclidian Simplices and the Ramsey Number $R(4,4;3)$, *Technical Report #10, Clemson University* (1967).

[-] W. Solomon, see [LorSo].

[-] J.H. Spencer, see [BES, GRS].

[Spe]* T. Spencer, University of Nebraska at Omaha, *personal communication* (1993), and, Upper Bounds for Ramsey Numbers via Linear Programming, *to appear.*

[Stahl] S. Stahl, On the Ramsey Number $R(F,K_m)$ where $F$ is a Forest, *Canadian Journal of Mathematics,* 27 (1975) 585-589.

[-] R.G. Stanton, see [KS].

[Stat] W. Staton, Some Ramsey-type Numbers and the Independence Ratio, *Transactions of the American Mathematical Society,* 256 (1979) 353-370.

[Stev] S. Stevens, Ramsey Numbers for Stars Versus Complete Multipartite Graphs, *Congressus Numerantium,* 73 (1990) 63-71.

[-] M.J. Stewart, see [CRSPS].

[-] E. Szemerédi, see [AKS, CRST].

[Tho] A. Thomason, An Upper Bound for Some Ramsey Numbers, *Journal of Graph Theory,* 12 (1988) 509-517.

[-] C.A. Tovey, see [CET].

[Tr] Trivial results.

[-] W.T. Trotter Jr., see [CRST].

[-] Z. Tuza, see [GT].

[-] L. Volkmann, see [GV].

[Wa1] K. Walker, Dichromatic Graphs and Ramsey Numbers, *Journal of Combinatorial Theory,* 5 (1968) 238-243.

[Wa2] K. Walker, An Upper Bound for the Ramsey Number $M(5,4)$, *Journal of Combinatorial Theory,* 11 (1971) 1-10.

[Wal] W.D. Wallis, On a Ramsey Number for Paths, *Journal of Combinatorics, Information & System Sciences,* 6 (1981) 295-296.

[-] Wang Gongben, see [WW, WWY].

[WW]* Wang Qingxian and Wang Gongben, New Lower Bounds of Ramsey Numbers $r(3,q)$ (in Chinese), *Acta Scientiarum Naturalium, Universitatis Pekinensis,* 25 (1989) 117-121.

[WWY]* Wang Qingxian, Wang Gongben and Yan Shuda, A Search Algorithm And New Lower Bounds for Ramsey Numbers $r(3,q)$, *to appear*.

[-] Wei Li, see [KLR].

[Wh] E.G. Whitehead, The Ramsey Number $N(3,3,3,3;2)$, *Discrete Mathematics,* 4 (1973) 389-396.

[XZ]* Xie Jiguo and Zhang Xiaoxian, A New Lower Bound for Ramsey Number $r(3,13)$ (in Chinese), *Journal of Lanzhou Railway Institute,* 12 (1993) 87-89.

[-] J. Yackel, see [GY].

[-] Yan Shu-da, see [WWY].

[YY]** Yang Yuansheng, On the Third Ramsey Numbers of Graphs with Six Edges, *Journal of Combinatorial Mathematics and Combinatorial Computing,* 17 (1995) 199-208.

[YH]* Yang Yuansheng and G.R.T. Hendry, The Ramsey Number $r(K_1+C_4, K_5-e)$, *Journal of Graph Theory,* 19 (1995) 13-15.

[YR1]** Yang Yuansheng and P. Rowlinson, On the Third Ramsey Numbers of Graphs with Five Edges, *Journal of Combinatorial Mathematics and Combinatorial Computing,* 11 (1992) 213-222.

[YR2]* Yang Yuansheng and P. Rowlinson, The Third Ramsey Numbers for Graphs with at Most Four Edges, *Discrete Mathematics,* 125 (1994) 399-406.

[YR3]* Yang Yuansheng and P. Rowlinson, On Graphs without 6-Cycles and Related Ramsey Numbers, *Utilitas Mathematica,* 44 (1993) 192-196.

[Yu] Yu Song-Nian, A Computer Assisted Number Theoretical Construction of $(3,k)$-Ramsey Graphs, *Annales Univ. Sci. Budapest., Sect. Comput.,* 10 (1989) 35-44.

[Zeng] Zeng Wei-Bin, Ramsey Numbers for Triangles and Graphs of Order Four with No Isolated Vertex, *Journal of Mathematical Research & Exposition,* 6 (1986) 27-32.

[-] Zhang Ke Min, see [HZ, MZ].

[-] Zhang Xiaoxian, see [XZ].

Out of 220 references gathered above 171 appeared in 33 different periodicals, among which most articles were published in: *Journal of Combinatorial Theory* (old, Series A and B) 32, *Journal of Graph Theory* 29, and *Discrete Mathematics* 28. The results of 48 references depend on computer algorithms.