Small Ramsey Numbers

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Submitted: June 11, 1994; Accepted: July 3, 1994
Revision #5: July 9, 1998

ABSTRACT: We present data which, to the best of our knowledge, includes all known nontrivial values and bounds for specific graph, hypergraph and multicolor Ramsey numbers, where the avoided graphs are complete or complete without one edge. Many results pertaining to other more studied cases are also presented. We give references to all cited bounds and values, as well as to previous similar compilations. We do not attempt complete coverage of asymptotic behavior of Ramsey numbers, but concentrate on their specific values.

Mathematical Reviews Subject Number 05C55.

1. Scope and Notation

There is a vast literature on Ramsey type problems starting in 1930 with the original paper of Ramsey [Ram]. Graham, Rothschild and Spencer in their book [GRS] present an exciting development of Ramsey Theory. The subject has grown amazingly, in particular with regard to asymptotic bounds for various types of Ramsey numbers (see the survey paper [GrRödl]), but the progress on evaluating the basic numbers themselves has been very unsatisfactory for a long time. In the last decade, however, considerable progress has been obtained in this area, mostly by employing computer algorithms. The few known exact values and several bounds for different numbers are scattered among many technical papers. This compilation is a fast source of references for the best results known for specific numbers. It is not supposed to serve as a source of definitions or theorems, but these can be easily accessed via the references gathered here.

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* - This paper updates and extends a technical report RIT-TR-93-009 by the author [Ra4].

Ramsey Theory studies conditions when a combinatorial object contains necessarily some smaller given objects. The role of Ramsey numbers is to quantify some of the general existential theorems in Ramsey Theory.

Let $G_1, G_2, \ldots, G_m$ be graphs or s-uniform hypergraphs (s is the number of vertices in each edge). $R(G_1, G_2, \ldots, G_m; s)$ denotes the m-color *Ramsey number* for s-uniform graphs/hypergraphs, avoiding G_i in color i for $1 \le i \le m$. It is defined as the least integer n such that, in any coloring with m colors of the s-subsets of a set of n elements, for some i the s-subsets of color i contain a sub-(hyper)graph isomorphic to G_i (not necessarily induced). The value of $R(G_1, G_2, \ldots, G_m; s)$ is fixed under permutations of the first m arguments.

If $s = 2$ (standard graphs) then s can be omitted. If G_i is a complete graph K_k, then we can write k instead of G_i, and if $G_i = G$ for all i we can use the abbreviation $R_m(G)$ (or $R_m(G; s)$). For $s = 2$, $K_k - e$ denotes a K_k without one edge, and for $s = 3$, $K_k - t$ denotes a K_k without one triangle (hyperedge). P_i is a path on i vertices, C_i is a cycle of length i, and W_i is a wheel with $i-1$ spokes, i.e. a graph formed by some vertex x, connected to all vertices of some cycle C_{i-1}. $K_{n,m}$ is a complete n by m bipartite graph, in particular $K_{1,n}$ is a star graph. The book graph $B_i = K_2 + \overline{K}_i = K_1 + K_{1,i}$ has $i+2$ vertices, and can be seen as i triangular pages attached to a single edge. For a graph G, $n(G)$ and $e(G)$ denote the number of vertices and edges, respectively. Finally let $\chi(G)$ be the chromatic number of G, and let nG denote n disjoint copies of G.

Section 2 contains the data for the classical two color Ramsey numbers $R(k,l)$ for complete graphs, and section 3 for the two color case when the avoided graphs are complete or have the form $K_k - e$, but not both are complete. Section 4 lists the most studied two color cases for other graphs. The multicolor and hypergraph cases are gathered in sections 5 and 6, respectively. If some new bound has been not yet published, we also give a reference to the best published previous result, if any. Finally, section 7 gives pointers to cumulative data and to some previous surveys, especially those containing data not subsumed by this compilation.

2. Classical Two Color Ramsey Numbers

We split the data into the table of values and a table with corresponding references (Table I). Known exact values appear as centered entries, lower bounds as top entries, and upper bounds as bottom entries. All the lower bounds for higher numbers listed in Table II, except $237 \le R(5,15)$, were obtained by construction of cyclic graphs.

All the critical graphs for the numbers $R(k,l)$ (graphs on $R(k,l) - 1$ vertices without K_k and without K_l in the complement) are known for $k = 3$ and $l = 3$, 4, 5, 6 [Ka2], 7 [RK3, MZ], and there are 1, 3, 1, 7 and 191 of them, respectively. There exists a unique critical graph for $R(4,4)$ [Ka2]. There are 4 such graphs known for $R(3,8)$ [RK2], 1 for $R(3,9)$ [Ka2] and 350904 for $R(4,5)$ [MR5], but there might be more of them. In [MR4] evidence is given for the conjecture that $R(5,5) = 43$ and that there exist 656 critical graphs on 42 vertices.

k \ l	3	4	5	6	7	8	9	10	11	12	13	14	15
3	6	9	14	18	23	28	36	40 43	46 51	52 60	59 69	66 78	73 89
4		18	25	35 41	49 61	55 84	69 115	80 149	96 191	128 238	131 291	136 349	145 417
5			43 49	58 87	80 143	95 216	116 316	141 442	153	181	193	221	237
6				102 165	109 298	122 495	153 780	167 1171	203	224	242	258	338
7					205 540	1031	1713	2826					
8						282 1870	3583	6090					
9							565 6625	12715					
10								798 23854					

Table I. Known nontrivial values and bounds for two color Ramsey numbers $R(k,l) = R(k,l;2)$.

k \ l	3	4	5	6	7	8	9	10	11	12	13	14	15
3	GG	GG	GG	Ka2	Ka2 GY	GR MZ	Ka2 GR	Ex2 RK2	Ka2 RK2	Ex12 RK2	Piw3 RK2	Ex7 RK2	WW Ra1
4		GG	Ka1 MR5	Ex8 MR4	Ex3 Mac	Ex12 Mac	RK1 Mac	Piw3 Mac	Piw3 Spe	SLL1 Spe	Ea3 Spe	Ea3 Spe	Ea3 Spe
5			Ex4 MR4	Ex8 Spe/HZ	CET Spe	Piw3 Spe	Ex12 Mac	Ex12 Mac	Ex12	Ex12	Ex12	Ex12	Ex12
6				Ka1 Mac	Ex12 Mac	Ex12 Mac	Ex12 Mac	Ex12 Mac	Ea3	LSL	SLZ3	LSL	LSL
7					Ma/S1 Mac	Mac	HZ	Mac					
8						BR Mac	Ea1	HZ					
9							Ma/S1 Mac	Ea1					
10								Ma/S1 Mac					

References for Table I.

k \ l	15	16	17	18	19	20	21	22	23
3	73 WW	79 WW	92 WWY	98 WWY	106 WWY	109 WWY	122 WWY	125 WWY	136 WWY
4	145 Ea3				194 SLZ1	230 SLZ1	242 SLZ1	258 LSW2	
5	237 Ex12		252 LSW1	282 SLZ1	338 SLZ1		374 SLZ2	380 LSZ	
6	338 LSL		420 LS1						
7			548 SLZ1		618 SLL2	648 SLL2	674 SLL2		
8			602 SLZ3	642 SLZ3	684 SLZ3	762 SLZ3			

Table II. Known nontrivial lower bounds for higher two color Ramsey numbers $R(k,l)$, with references.

Yu [Yu2] constructed a special class of triangle-free cyclic graphs establishing the following lower bounds: $R(3,61) \geq 479$, $R(3,83) \geq 697$, $R(3,103) \geq 955$, $R(3,161) \geq 1253$, $R(3,188) \geq 1591$ and $R(3,256) \geq 1969$.

Most of the lower bounds for $R(4,n)$ presented by Bannani in [Ba], and two other results $R(3,13) \geq 58$ [Ka2] and $R(5,8) \geq 94$ [RK1], were improved in [Piw1, Piw3] by Piwakowski. The bound $R(3,13) \geq 60$ [XZ] cited in the 1995 version of this survey was shown to be incorrect in [Piw3]. The previously best published upper bound for $(k,l) = (5,6)$ of 94 can be found in [Wa2]. The graphs constructed by Exoo in [Ex12], and some others, are available electronically from http://isu.indstate.edu/ge/RAM/.

By taking a disjoint union of two critical graphs one can easily see that $R(k,p) \geq s$ and $R(k,q) \geq t$ imply $R(k,p+q-1) \geq s+t-1$. For example, this gives trivially a lower bound $R(4,15) \geq 145$ with $p=4$, $q=12$. Higher lower bounds implied this way are not shown. Some upper bounds implied by $R(k,l) \leq R(k-1,l) + R(k,l-1)$, or by its slight improvement with strict inequality when both terms on the right hand side are even, are marked [Ea1]. There are obvious generalizations of these inequalities for graphs other than complete.

The bound $R(6,6) \leq 166$ is an immediate consequence of theorem 1 in [Wa1] and $R(4,6) \leq 41$, in this case the best published bound of 169 is due to Giraud [Gi3]. T. Spencer [Spe], Mackey [Mac], and Huang and Zhang [HZ], using the bounds for minimum and maximum number of edges in (4,5) Ramsey graphs listed in [MR2, MR4], were able to establish new upper bounds for several higher Ramsey numbers, improving all the previous longstanding results of Giraud [Gi1, Gi3, Gi4]. We have recomputed the bounds marked [HZ] using the method from the paper [HZ], because the bounds there relied on an overly optimistic personal communication from Spencer.

For a more in depth study of triangle-free graphs in relation to the case of $R(3,k)$, for which considerable progress has been obtained in recent years, see also [AKS, FL, Fra1, Fra2, Gri, Loc, KM, RK3, RK4, S2, Stat, Yu1]. In 1995, Kim [Kim] obtained a breakthrough by proving that $R(3,k)$ has order of magnitude exactly $\Theta(k^2/\log k)$. Good asymptotic bounds for $R(k,k)$ can be found, for example, in [Chu3, McS] (lower bound) and [Tho] (upper bound), and for many other asymptotic bounds in the general case of $R(k,l)$ consult [GRS, GrRö].

3. Two Colors - Dropping One Edge from Complete Graph

For the following numbers it was established that the critical graphs are unique: $R(K_3,K_l-e)$ for $l=3$ [Tr], 6 and 7 [Ra2], $R(K_4-e,K_4-e)$ [FRS2], $R(K_5-e,K_5-e)$ [Ra3] and $R(K_4-e,K_7-e)$ [McR]. The marks "$\geq n$" in the Table III indicate lower bounds.

G \ H	K_3-e	K_4-e	K_5-e	K_6-e	K_7-e	K_8-e	K_9-e	$K_{10}-e$
K_3-e	3	5	7	9	11	13	15	17
K_3	5	7	11	17	21	25	31	37-38
K_4-e	5	10	13	17	28			
K_4	7	11	19	≥ 27	≥ 35			
K_5-e	7	13	22	≥ 30				
K_5	9	16	30-34	≥ 43				
K_6-e	9	17	≥ 30					
K_6	11	21	≥ 35					

Table III. Two types of Ramsey numbers $R(G,H)$, includes all known nontrivial values.

G \ H	K_3-e	K_4-e	K_5-e	K_6-e	K_7-e	K_8-e	K_9-e	$K_{10}-e$
K_3-e	Tr	Tr	Tr	Tr	Tr	Tr	Tr	Tr
K_3	Tr	CH2	Clan	FRS1	GH	Ra2	Ra2	MR6
K_4-e	Tr	CH1	FRS2	McR	McR			
K_4	Tr	CH2	EHM1	Ex10	Ea1			
K_5-e	Tr	FRS2	CEHMS	Ea1				
K_5	Tr	BH	Ex7-Ea1	Ea1				
K_6-e	Tr	McR	Ea1					
K_6	Tr	McN	Ea1					

References for Table III.

4. General Graph Numbers in Two Colors

This section includes data with respect to general graph results. We tried to include all nontrivial values and identities regarding exact results (or references to them), but only those out of general bounds and other results which, in our opinion, have a direct connection to the evaluation of specific numbers. If some small value cannot be found below, it may be covered by the cumulative data gathered in section 7, or be a special case of a general result listed in this section. Note that $B_1 = C_3 = W_3 = K_3$, $B_2 = K_4 - e$, $P_3 = K_3 - e$, $W_4 = K_4$ and $C_4 = K_{2,2}$ imply other identities not mentioned explicitly.

Paths:

$R(P_n, P_m) = n + \lfloor m/2 \rfloor - 1$ for all $n \geq m \geq 2$ [GeGy]

Cycles:

$R(C_3, C_3) = 6$ [GG]
$R(C_4, C_4) = 6$ [CH1]

Result obtained independently in [Ros] and [FS1], new simple proof in [KR]:

$$R(C_n, C_m) = \begin{cases} 2n-1 & \text{for } 3 \leq m \leq n,\ m \text{ odd},\ (n,m) \neq (3,3) \\ n-1+m/2 & \text{for } 4 \leq m \leq n,\ m \text{ and } n \text{ even},\ (n,m) \neq (4,4) \\ \max\{n-1+m/2, 2m-1\} & \text{for } 4 \leq m < n,\ m \text{ even and } n \text{ odd} \end{cases}$$

$R(nC_3, mC_3) = 3n + 2m$ for $n \geq m \geq 1$, $n \geq 2$ [BES]
Unions of cycles [MS, Den]

Wheels:

$R(W_3, W_5) = 11$ [Clan]
$R(W_3, W_n) = 2n - 1$ for all $n \geq 6$ [BE2]
All critical colorings for $R(W_3, W_n)$ for all $n \geq 3$ [RaJi]
$R(W_4, W_5) = 17$ [He3]
$R(W_5, W_5) = 15$ [HM2, He2]
$R(W_4, W_6) = 19$, $R(W_5, W_6) = 17$ and $R(W_6, W_6) = 17$,
and all critical colorings (2, 1 and 2) for these numbers [FM]

Books:

$R(B_1, B_n) = 2n + 3$ for all $n > 1$ [RS1]
$R(B_3, B_3) = 14$ [RS1, HM2]
$R(B_2, B_5) = 16$, $R(B_3, B_5) = 17$, $R(B_5, B_5) = 21$,
$R(B_4, B_4) = 18$, $R(B_4, B_6) = 22$, $R(B_6, B_6) = 26$,
in general $R(B_n, B_n) = 4n + 2$ for $4n + 1$ a prime power,
and some other general equalities and bounds for $R(B_n, B_m)$ [RS1].

Complete bipartite graphs:

$R(K_{2,3}, K_{2,3}) = 10$ [Bu4]
$R(K_{2,3}, K_{2,4}) = 12$ [ER]
$R(K_{2,3}, K_{1,7}) = 13$ [Par4]
$R(K_{2,3}, K_{3,3}) = 13$ and $R(K_{3,3}, K_{3,3}) = 18$ [HM3]
$R(K_{2,2}, K_{2,8}) = 15$ and $R(K_{2,2}, K_{2,11}) = 18$ [HM]
$R(K_{2,2}, K_{1,15}) = 20$ [La2]
$R(nK_{1,3}, mK_{1,3}) = 4n + m - 1$ for $n \geq m \geq 1$, $n \geq 2$ [BES]
Asymptotics for $K_{2,m}$ versus K_n [CLZ]

$R(K_{1,n}, K_{1,m}) = n + m - \varepsilon$, where $\varepsilon = 1$ if both n and m are even and $\varepsilon = 0$ otherwise [Ha1]. It is also a special case of multicolor numbers for stars obtained in [BuRo1].

$R(K_{2,n}, K_{2,n}) \leq 4n - 2$ for all $n \geq 2$, exact values 6, 10, 14, 18, 21, 26, 30, 33, 38, 42, 46, 50, 54, 57 and 62 of $R(K_{2,n}, K_{2,n})$ for $2 \leq n \leq 16$, respectively. The first open case is $65 \leq R(K_{2,17}, K_{2,17}) \leq 66$ [EHM2].

Triangle versus other graphs:

$R(3, k) = \Theta(k^2 / \log k)$ [Kim]

Explicit construction for $R(3, 4k + 1) \geq 6R(3, k + 1) - 5$, for all $k \geq 1$ [CCD]

Explicit triangle-free graphs with independence k on $\Omega(k^{3/2})$ vertices [Alon2]

$R(K_3, G) = 2n(G) - 1$ for any connected G on at least 4 vertices and with at most $(17n(G) + 1)/15$ edges, in particular for $G = P_i$ and $G = C_i$, for all $i \geq 4$ [BEFRS1]

$R(K_3, G) \leq 2e(G) + 1$ for any graph G without isolated vertices [Sid3]
$R(K_3, G) \leq n(G) + e(G)$ for all G, a conjecture [Sid2]
$R(K_3, K_n)$, see section 2
$R(K_3, K_n - e)$, see section 3
$R(K_3, G)$ for all connected G up to 9 vertices, see section 7
See also [AKS, FL, Fra1, Fra2, Gri, Loc, KM, RK3, RK4, S2, Stat, Yu1]

Cycles versus complete graphs:

$R(C_4, K_3) = R(C_4, C_3) = 7$
$R(C_4, K_4) = 10$ [CH2]
$R(C_4, K_5) = 14$ [GG][He2, LRZ]
$R(C_4, K_6) = 18$ [Ex9][RoJa1]
$21 \le R(C_4, K_7) \le 22$ [JR1]
$R(C_5, K_3) = R(C_5, C_3) = 9$
$R(C_5, K_4) = 13$ [He2, JR2]
$R(C_5, K_5) = 17$ [He2, JR2]
$R(C_5, K_6) = 21$ [JR3]
C_4 versus K_n [LRZ]
Cycles versus K_n [BoEr, EFRS2, CLZ]

Cycles versus other graphs:

C_4 versus stars [Par3, Chen]
C_4 versus trees [EFRS4, Bu6, Chen]
C_4 versus $K_{m,n}$ [HM]
C_4 versus all graphs on six vertices [JR3]
$R(C_4, B_n)$ = 7, 9, 11, 12, 13, 16 and 16 for $2 \le n \le 8$, respectively [FRS6]
$R(C_5, W_6) = 13$ [ChvS]
Cycles versus paths [FLPS, BEFRS2]
Cycles versus stars [La1, Clark, see Par5]
Cycles versus trees [FSS]
Cycles versus books [FRS5, FSR6]
Cycles versus wheels [Zhou]
See bipartite graphs for $K_{2,2}$

Mixed special cases:

$R(B_3, K_4) = 14$ [He3]
$R(C_5 + e, K_5) = 17$ [He5]
$R(W_5, K_5 - e) = 17$ [He2][YH]
$20 \le R(B_3, K_5) \le 22$ [He2]
$27 \le R(W_5, K_5) \le 29$ [He2]
$25 \le R(K_5 - P_3, K_5) \le 28$ [He2]
$26 \le R(K_{2,2,2}, K_{2,2,2})$ (octahedron) [Ex7]

General cases (exact results and bounds):

Paths versus stars [Par2, BEFRS2]
Paths versus books [RS2]
Paths versus cycles [FLPS, BEFRS2]

Paths versus K_n [Par1]
Paths versus $K_{n,m}$ [Häg]
Paths and cycles versus trees [FSS]
Books versus trees [EFRS7]
Books and $(K_1 + tree)$ versus K_n [LR1]
Unicyclic graphs [Gro1, Köh]
$K_{2,n}$ versus some stars [Par4]
$K_{2,m}$ and C_{2m} versus K_n [CLZ]
nK_3 versus mK_3 [BES]
nK_3 versus mK_4 [LorMu]
$R(nK_4, nK_4) = 7n + 4$ for large n [Bu7]
Stars versus trees [Bu1, GV]
Stars versus stripes, stripes [CL, Lor]
Stars versus books [CRSPS, RS2]
Stars versus $K_{n,m}$ [Stev, Par3]
Stars versus $K_n - tK_2$ [Hua1, Hua2]
Union of two stars [Gro2]
Double stars* [GHK]
Graphs with bridge versus K_n [Li]
Fans $K_1 + nK_2$ versus K_n [LR2]
Trees [EG, GRS, FSS, GV]
Trees versus K_n [Chv]
Trees versus $K_n + \overline{K}_m$ [RS2, FSR]
Trees versus bipartite graphs [EFRS6]
Trees versus almost complete graphs [GJ2]
Trees versus small $(n(G) \le 5)$ connected G [FRS4]
Linear forests [BuRo2, FS3]
Forests versus K_n [Stahl]
Forests versus almost complete graphs [CGP]
Sparse graphs versus paths and cycles [BEFRS2]
Multipartite complete graphs [BEFRS3, EFRS4, FRS3, Stev]
Multipartite complete graphs versus trees [EFRS8, BEFRSGJ]
Disconnected graphs versus any graph [GJ1]
Graphs with long tails [Bu2, BG]
Brooms+ [EFRS3]

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* - A double star is a union of two stars with their centers joined by an edge.

+ - A broom is a star with a path attached to its center.

General results:

[Wa1]	$R(k,k) \leq 4R(k,k-2)+2$.
[Chv]	$R(K_n,T_m) = (n-1)(m-1)+1$ for any tree T on m vertices.
[CH2]	$R(G,H) \geq (\chi(G)-1)(c(H)-1)+1$, where $\chi(G)$ is the chromatic number of G, and $c(H)$ is the size of the largest connected component of H.
[BE1]	$R(G,G) \geq \lfloor (4n(G)-1)/3 \rfloor$ for any connected G.
[BE2]	Graphs yielding $R(K_n,G) = (n-1)(n(G)-1)+1$ and related results (see also [EFRS5]).
[BES]	Study of Ramsey numbers for multiple copies of graphs (see also [Bu1, LorMu]).
[Zeng]	$R(nK_3,nG)$ for all isolate-free graphs G on 4 vertices.
[Bu7, Bu8]	Study of Ramsey numbers for large disjoint unions of graphs, in particular $R(nK_k,nK_l) = n(k+l-1)+R(K_{k-1},K_{l-1})-2$, for n large enough.
[BEFRS4]	Graphs H yielding $R(G,H) = (\chi(G)-1)(n(H)-1)+s(G)$, where $s(G)$ is a chromatic surplus of G, defined as the minimum number of vertices in some color class under all vertex colorings in $\chi(G)$ colors (such H's are called G-good). This idea, initiated in [Bu2], is a basis of a number of exact results for $R(G,H)$ for large and sparse graphs H [BG, BEFRS2, Bu5, FS, EFRS4, FRS3, BEFSRGJ, BF]. A survey of this area appeared in [FRS7].
[BEFS]	Bounds for the difference between consecutive Ramsey numbers.
[Par3, Par4]	Relations between some Ramsey graphs and block designs.
[Bra3]	$R(G,H) > h(G,d)n(H)$ for all nonbipartite G and almost every d-regular H, for some h unbounded in d.
[CSRT]	$R(G,G) \leq c_d n(G)$ for all G, where constant c_d depends only on the maximum degree d in G.
[ChenS]	$R(G,G) \leq c_d n$ for all d-arrangeable graphs G on n vertices. The constant was improved in [Eaton].
[EFRS9]	Study of graphs G for which there exists a constant C such that for all H with no isolates $R(G,H) \leq Ce(H)$.
[Alon1]	$R(G,G) \leq 12n$ for all n-vertex graphs G, in which no two vertices of degree at least 3 are adjacent.
[FSS]	Discussion of the conjecture that $R(T_1,T_2) \leq n(T_1)+n(T_2)-2$ holds for all trees T_1, T_2.
[FM]	$R(W_6,W_6) = 17$ and $\chi(W_6) = 4$. This gives a counterexample $G = W_6$ to the Erdős conjecture (see [GRS]) $R(G,G) \geq R(K_{\chi(G)},K_{\chi(G)})$.
[LR3]	Bounds on $R(H+\overline{K}_n,K_n)$ for general H.

[-] Special cases of multicolor results listed in section 5.

[-] See also surveys listed in section 7.

5. Multicolor Graph Numbers

The only known value of a multicolor classical Ramsey number:

$R(3,3,3) = R(3,3,3;2) = 17$	[GG]
2 critical colorings	[KS, LayMa]

Bounds for multicolor classical numbers:

$51 \le R(3,3,3,3) \le 64$	[Chu1] [San]
$162 \le R(3,3,3,3,3) \le 317$	[Ex11] [Wh, Ea1]
$500 \le R(3,3,3,3,3,3)$	[Ex11]
$128 \le R(4,4,4) \le 236$	[HI] [Ea2]
$458 \le R(4,4,4,4)$	[Su]
$30 \le R(3,3,4) \le 31$	[Ka2] [PR]
$45 \le R(3,3,5) \le 57$	[Ex9, KLR] [Ea2]
$90 \le R(3,3,9)$	[LS2]
$108 \le R(3,3,11)$	[LS3]
$55 \le R(3,4,4) \le 79$	[KLR] [Ea2]
$80 \le R(3,4,5) \le 161$	[Ex12] [Ea2]
$87 \le R(3,3,3,4) \le 155$	[Ex12] [Ea2]

The result by Sanchez-Flores [San], 1995, improved a very old bound $R(3,3,3,3) \le 65$ obtained by Folkman [Fo] in 1974. The result in [PR] improved the bound from [Piw2] by 1. The upper bounds marked [Ea1] and [Ea2] are easy implications of basic inequalities. Out of several lower bounds on $R_k(4)$ in [Su], the one above for $k = 4$ is the only bound better than those implied by general results in [Song1] (see below).

Multicolor general graphs:

$R(C_4, C_4, C_4) = 11$	[BS, see also Clap]
$R(C_5, C_5, C_5) = 17$	[YR1]
$R(C_6, C_6, C_6) = 12$	[YR3]
$R_4(C_4) \geq 18$ and $R_5(C_4) \geq 25$	[Ex9]
$R(C_4, C_4, K_3) = 12$	[Schu]
$R(C_4, K_3, K_3) = 17$	[ER]
$R(K_4 - e, K_4 - e, P_3) = 11$	[Ex6]
$28 \leq R(K_4 - e, K_4 - e, K_4 - e) \leq 30$	[Ex6] [Piw4]
$R(C_4, C_4, C_4, T) = 16$ for $T = P_4$ and $T = K_{1,3}$	[ER]

All colorings on at least 14 vertices for (K_3, K_3, K_3), and all colorings for $(K_4 - e, K_4 - e, P_3)$ were found in [Piw4].

General multicolor results:

- General bounds for $R_k(G)$ [CH3].
- Bounds for $R_k(3)$ [Fre, Chu1, Chu2, ChGri, GrRö, Wan].
- Formulas for $R_k(G)$ for G being P_3, $2K_2$ and $K_{1,3}$ for all k, and for P_4 if k is not divisible by 3 [Ir]. Wallis [Wal] showed $R_6(P_4) = 13$, which already implied $R_{3t}(P_4) = 6t + 1$, for all $t \geq 2$. Independently, the case $R_k(P_4)$ for $k \neq 3^m$ was completed by Lindström in [Lin], and later Bierbrauer proved $R_{3^m}(P_4) = 2 \cdot 3^m + 1$ for all $m \geq 1$.
- $R_k(4) \geq 3 \cdot 5^{k-1} + 1$ and $R_k(5) \geq 4 \cdot 6.48^{k-1} + 1$ for all k, and other general lower bounds on $R_k(n)$ [Song1].
- $R_k(C_4) \leq k^2 + k + 1$ for all $k \geq 1$, and $R_k(C_4) \geq k^2 - k + 2$ for all $k - 1$ a prime power [Ir, Chu2, ChGra]. For small k some improvements on the latter are known: $R_3(C_4) = 11$ [BS], $R_4(C_4) \geq 18$ and $R_5(C_4) \geq 25$ [Ex9].
- Bounds for the bipartite graphs $R_k(K_{s,t})$, in particular for $K_{2,2} = C_4$ [ChGra].
- Formulas for $R(C_n, C_m, C_k)$ and $R(C_n, C_m, C_k, C_l)$ for n sufficiently large [EFRS1].
- Formulas for $R(P_{n_1}, \cdots, P_{n_k})$, except few cases [FS2].
- Monotone paths and cycles [Lef].
- Formulas for $R(S_1, \cdots, S_k)$, where S_i's are arbitrary stars [BuRo1].
- Formulas for $R(S_1, \cdots, S_k, K_n)$, where S_i's are arbitrary stars [Jac].
- Formulas for $R(S_1, \cdots, S_k, nP_2)$, where S_i's are arbitrary stars [CL2].
- Formulas for $R(pP_3, qP_3, rP_3)$ and $R(pP_4, qP_4, rP_4)$ [Scob].

- Cockayne and Lorimer [CL1] found the exact formula for $R(n_1P_2, \cdots, n_kP_2)$, and later Lorimer [Lor] extended it to a more general case of $R(K_m, n_1P_2, \cdots, n_kP_2)$. Still more general cases of the latter, with multiple copies of the complete graph and forests, were studied in [Stahl, LorSe, LorSo].
- If G is connected and $R(K_k, G) = (k-1)(n(G)-1)+1$, in particular if G is any tree, then $R(K_{n_1}, \cdots, K_{n_k}, G) = (R(K_{n_1}, \cdots, K_{n_k})-1)(n(G)-1)+1$ [BE2]. A generalization for connected $G_1, \ldots, G_n$ in place of G appeared in [Jac].
- Study of $R(S, G_1, \cdots, G_k)$ for large sparse S [EFRS1, Bu3].
- Bounds for trees $R_k(T)$ and forests $R_k(F)$ [EG, GRS, BB, GT, Bra1, Bra2].
- See also surveys listed in section 7.

6. Hypergraph Numbers

The only known value of a classical Ramsey number for hypergraphs:

$R(4,4;3) = 13$	[MR1]
more than 200000 critical colorings	

Other hypergraph cases:

$R(K_4-t, K_4-t; 3) = 7$	[Ea4]
$R(K_4-t, K_4; 3) = 8$	[Sob, Ex1, MR1]
$14 \le R(K_4-t, K_5; 3)$	[Ex1]
$32 \le R(4,5;3)$	[Ex7]
$63 \le R(5,5;3)$	[Ea1]
$34 \le R(5,5;4)$	[Ex10]
$56 \le R(4,4,4;3)$	[Ex7]
$13 \le R(K_4-t, K_4-t, K_4-t; 3) \le 17$	[Ex1] [Ea1]

The computer evaluation of $R(4,4;3)$ in [MR1] consisted of an improvement of the upper bound from 15 to 13, which followed an extensive theoretical study of this number in [Gi2, Is1, Sid1]. Exoo in [Ex1] announced the bounds $R(4,5;3) \ge 30$ and $R(5,5;4) \ge 27$ without presenting the constructions. The best published bound of $R(4,5;3) \ge 24$ was obtained by Isbell [Is2]. Shastri in [Sha] shows a weak bound $R(5,5;4) \ge 19$, nevertheless his lemmas and those in [Ka3, Abb, GRS] can be used to derive other lower bounds for higher numbers. Study of lower bounds on $R_m(k;s)$ can be found in [DLR], and on $R(p,q;4)$ in [SYL, Song2]. In [AS] it is shown that for some a, b the numbers $R(m,a,b;3)$ are at least exponential in m.

Theoretical results on hypergraph numbers are gathered in [GrRö, GRS].

7. Cumulative Data and Surveys

Cumulative data for two colors:

[CH1]	$R(G,G)$ for all graphs G without isolates on at most 4 vertices.
[CH2]	$R(G,H)$ for all graphs G and H without isolates on at most 4 vertices.
[Clan]	$R(G,H)$ for all graphs G on at most 4 vertices and H on 5 vertices, except five entries (now all solved).
[He4]	All critical colorings for $R(G,H)$, for isolate-free graphs G and H as in [Clan] above.
[Bu4]	$R(G,G)$ for all graphs G without isolates and with at most 6 edges.
[He1]	$R(G,G)$ for all graphs G without isolates and with at most 7 edges.
[HM2]	$R(G,G)$ for all graphs G on 5 vertices and with 7 or 8 edges.
[He2]	$R(G,H)$ for all graphs G and H on 5 vertices without isolates, except 7 entries (5 still open).
[HoMe]	$R(G,H)$ for $G=K_{1,3}+e$ and $G=K_4-e$ versus all connected graphs H on 6 vertices, except $R(K_4-e,K_6)$. The result $R(K_4-e,K_6)=21$ was claimed by McNamara [McN, unpublished].
[FRS4]	$R(G,T)$ for all connected graphs G on at most 5 vertices and all (except some cases) trees T.
[FRS1]	$R(K_3,G)$ for all connected graphs G on 6 vertices.
[Jin,SchSch]	$R(K_3,G)$ for all connected graphs G on 7 vertices. Some errors in [Jin] were found by [SchSch].
[Brin]	$R(K_3,G)$ for all connected graphs G on at most 8 vertices. The numbers for K_3 versus sets of graphs with fixed number of edges, on at most 8 vertices, were presented in [KM].
[BBH]	$R(K_3,G)$ for all connected graphs G on 9 vertices.
[JR3]	$R(C_4,G)$ for all graphs G on 6 vertices.

Chvátal and Harary [CH1, CH2] formulated several simple but very useful observations how to discover values of some numbers. All five missing entries in the tables of Clancy [Clan] have been solved. Out of 7 open cases in [He2] 2 have been solved, the bounds for 2 were improved, and the status of the other 3 did not change. Section 4 of this survey under "Mixed special cases" lists 4 of them (labeled [He2], 1 solved, 3 open). $R(4,5)=R(G_{19},G_{23})=25$ is the second solved case. The other 2 open entries are K_5 versus K_5 (see section 2) and K_5 versus K_5-e (see section 3).

Cumulative data for three colors:

[YR2]	$R_3(G)$ for all graphs G with at most 4 edges and no isolates.

[YR1] $R_3(G)$ for all graphs G with 5 edges and no isolates, except $K_4 - e$. The case of $R_3(K_4 - e)$ remains open (see section 5).

[YY] $R_3(G)$ for all graphs G with 6 edges and no isolates, except 10 cases.

[AKM] $R(F, G, H)$ for most triples of isolate-free graphs with at most 4 vertices.

Surveys:

[Bu1] A general survey of results in Ramsey graph theory by S.A. Burr (1974)

[Par5] A general survey of results in Ramsey graph theory by T.D. Parsons (1978)

[Ha2] Summary of progress by Frank Harary (1981)

[ChGri] A general survey of bounds and values by F.R.K. Chung and C.M. Grinstead (1983)

[JGT] Special volume of *the Journal of Graph Theory* (1983)

[Rob] Nice textbook-type review of Ramsey graph theory for newcomers (1984)

[Bu6] What can we hope to accomplish in generalized Ramsey Theory ? (1987)

[GrRö] Survey of asymptotic problems by R.L. Graham and V. Rödl (1987)

[GRS] An excellent book by R.L. Graham, B.L. Rothschild and J.H. Spencer, second edition (1990)

[FRS7] Survey of graph goodness results, i.e. conditions for the formula $R(G, H) = (\chi(G) - 1)(n(H) - 1) + s(G)$ (1991)

[Neš] A chapter in *Handbook of Combinatorics* (1996)

[Caro] Survey of zero-sum Ramsey theory (1996)

The surveys by S.A. Burr [Bu1] and T.D. Parsons [Par5] contain extensive chapters on general exact results in graph Ramsey theory. F. Harary presented the state of the theory in 1981 in [Ha2], where he also gathered many references including seven to other survey papers. A decade ago, Chung and Grinstead in their survey paper [ChGri] gave less data than in this note, but included a broad discussion of different methods used in Ramsey computations in the classical case. S.A. Burr, one of the most experienced researchers in Ramsey graph theory, formulated in [Bu6] seven conjectures on Ramsey numbers for sufficiently large and sparse graphs, and reviewed the evidence for them found in the literature. Recently three of them have been refuted in [Bra3].

For newer extensive presentations see [GRS, GrRö, FRS7, Neš], though these focus on asymptotic theory not on the numbers themselves. Finally, this compilation could not pretend to be complete without mentioning a special volume of the Journal of Graph Theory [JGT] dedicated entirely to Ramsey theory. Besides a number of research papers, it includes historical notes and presents to us Frank P. Ramsey (1903-1930) as a person.

8. Concluding Remarks

This compilation does not include information on numerous variations of Ramsey numbers, nor related topics, like size Ramsey numbers, zero-sum Ramsey numbers, irredundant Ramsey numbers, local Ramsey numbers, connected Ramsey numbers, chromatic Ramsey numbers, avoiding sets of graphs in some colors, coloring graphs other than complete, or the so called Ramsey multiplicities. Interested reader can find such information in the surveys listed in section 7 here.

The author apologizes for any omissions or other errors in reporting results belonging to the scope of this work. Suggestions for any kind of corrections and/or additions will be greatly appreciated.

Acknowledgement

I would like to thank Brendan McKay, Geoffrey Exoo and Heiko Harborth for their help in gathering data for earlier versions of this survey.

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We mark the papers containing results obtained with the help of computer algorithms with stars. We identify two categories of such papers: marked with * involving some use of computers, where the results are easily verifiable with some computations, and those marked with **, where cpu intensive algorithms have to be implemented to replicate or verify the results. The first category contains mostly constructions done by algorithms, while the second mostly nonexistence results or claims of complete enumerations of special kinds of graphs.

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