A Mathematical Bibliography of
Signed and Gain Graphs and Allied Areas

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Colleagues:
HELP!

If you have any suggestions whatever for items to include in this bibliography, or for other changes, please let me hear from you. Thank you.

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Typeset by \TeX
A signed graph is a graph whose edges are labeled by signs. This is a bibliography of signed graphs and related mathematics.

Several kinds of labelled graph have been called “signed” yet are mathematically very different. I distinguish four types:

- **Group-signed graphs**: the edge labels are elements of a 2-element group and are multiplied around a polygon (or along any walk). Among the natural generalizations are larger groups and vertex signs.

- **Sign-colored graphs**, in which the edges are labelled from a two-element set that is acted upon by the sign group: $-$ interchanges labels, $+$ leaves them unchanged. This is the kind of “signed graph” found in knot theory. The natural generalization is to more colors and more general groups—or no group.

- **Weighted graphs**, in which the edge labels are the elements $+1$ and $-1$ of the integers or another additive domain. Weights behave like numbers, not signs; thus I regard work on weighted graphs as outside the scope of the bibliography—except (to some extent) when the author calls the weights “signs”.

- Labelled graphs where the labels have no structure or properties but are called “signs” for any or no reason.

Each of these categories has its own theory or theories, generally very different from the others, so in a logical sense the topic of this bibliography is an accident of terminology. However, narrow logic here leads us astray, for the study of true signed graphs, which arise in numerous areas of pure and applied mathematics, forms the great majority of the literature. Thus I regard as fundamental for the bibliography the notions of balance of a polygon (sign product equals $+$, the sign group identity) and the vertex-edge incidence matrix (whose column for a negative edge has two $+1$’s or two $-1$’s, for a positive edge one $+1$ and one $-1$, the rest being zero); this has led me to include work on gain graphs (where the edge labels are taken from any group) and “consistency” in vertex-signed graphs, and generalizable work on two-graphs (the set of unbalanced triangles of a signed complete graph) and on even and odd polygons and paths in graphs and digraphs.

Nevertheless, it was not always easy to decide what belongs. I have employed the following principles:

- Only works with mathematical content are entered, except for a few representative purely applied papers and surveys. I do try to include:

- Any (mathematical) work in which signed graphs are mentioned by name or signs are put on the edges of graphs, regardless of whether it makes essential use of signs. (However, due to lack of time and in order to maintain “balance” in the bibliography, I have included only a limited selection of items concerning binary clutters and postman theory, two-graphs, signed digraphs in qualitative matrix theory, and knot theory. For clutters, see Cornuejols (20xxa) when it appears; for postman theory, A. Frank (1996a). For two-graphs, see any of the review articles by Seidel. For qualitative matrix theory, see e.g. Maybee and Quirk (1969a) and Brualdi and Shader (1995a). For knot theory there
are uncountable books and surveys.)

- Any work in which the notion of balance of a polygon plays a role. Example: gain graphs. (Exception: purely topological papers concerning ordinary graph embedding.)
- Any work in which ideas of signed graph theory are anticipated, or generalized, or transferred to other domains. Examples: vertex-signed graphs; signed posets and matroids.
- Any mathematical structure that is an example, however disguised, of a signed or gain graph or generalization, and is treated in a way that seems in the spirit of signed graph theory. Examples: even-cycle and bicircular matroids; bidirected graphs; binary clutters (which are equivalent to signed binary matroids); some of the literature on two-graphs and double covering graphs.
- And some works that have suggested ideas of value for signed graph theory or that have promise of doing so in the future.

As for applications, besides works with appropriate mathematical content I include a few (not very carefully) selected representatives of less mathematical papers and surveys, either for their historical importance (e.g., Heider (1946a)) or as entrances to the applied literature (e.g., Taylor (1970a) and Wasserman and Faust (1993a) for psychosociology and Trinajstic (1983a) for chemistry). Particular difficulty is presented by spin glass theory in statistical physics—that is, Ising models and generalizations. Here one usually averages random signs and weights over a probability distribution; the problems and methods are rarely graph-theoretic, the topic is very specialized and hard to annotate properly, but it clearly is related to signed (and gain) graphs and suggests some interesting lines of graph-theoretic research. See Mézard, Parisi, and Virasoro (1987a) and citations in its annotation.

Plainly, judgment is required to apply these criteria. I have employed mine freely, taking account of suggestions from my colleagues. Still I know that the bibliography is far from complete, due to the quantity and even more the enormous range and dispersion of work in the relevant areas. I will continue to add both new and old works to future editions and I heartily welcome further suggestions.

There are certainly many errors, some of them egregious. For these I hand over responsibility to Sloth, Pride, Ambition, Envy, and Confusion. As Diedrich Knickerbocker says:

Should any reader find matter of offense in this [bibliography], I should heartily grieve, though I would on no account question his penetration by telling him he was mistaken, his good nature by telling him he was captious, or his pure conscience by telling him he was startled at a shadow. Surely when so ingenious in finding offense where none was intended, it were a thousand pities he should not be suffered to enjoy the benefit of his discovery.

Corrections, however, will be gratefully accepted by me.
Bibliographical Data. Authors’ names are given usually in only one form, even should the name appear in different (but recognizably similar) forms on different publications. Journal abbreviations follow the style of Mathematical Reviews (MR) with minor ‘improvements’. Reviews and abstracts are cited from MR and its electronic form MathSciNet, from Zentralblatt für Mathematik (Zbl.) and its electronic version (For early volumes, “Zbl. VVV, PPP” denotes printed volume and page; the electronic item number is “(e VVV.PPPNN)”.), and occasionally from Chemical Abstracts (CA) or Computing Reviews (CR). A review marked (q.v.) has significance, possibly an insight, a criticism, or a viewpoint orthogonal to mine.

Some—not all—of the most fundamental works are marked with a ††; some almost as fundamental have a †. This is a personal selection.

Annotations. I try to describe the relevant content in a consistent terminology and notation, in the language of signed graphs despite occasional clumsiness (hoping that this will suggest generalizations), and sometimes with my [bracketed] editorial comments. I sometimes try also to explain idiosyncratic terminology, in order to make it easier to read the original item. Several of the annotations incorporate open problems (of widely varying degrees of importance and difficulty).

I use these standard symbols:

- $\Gamma$ is a graph (undirected), possibly allowing loops and multiple edges. It is normally finite unless otherwise indicated.
- $\Sigma$ is a signed graph. Its vertex and edge sets are $V$ and $E$; its order is $n = |V|$. $E_+$, $E_-$ are the sets of positive and negative edges and $\Sigma_+$, $\Sigma_-$ are the corresponding spanning subgraphs (unsigned).
- $[\Sigma]$ is the switching class of $\Sigma$.
- $A(\ )$ is the adjacency matrix.
- $\Phi$ is a gain graph.
- $\Omega$ is a biased graph.
- $l(\ )$ is the frustration index (= line index of imbalance).
- $G(\ )$ is the bias matroid of a signed, gain, or biased graph.
- $L(\ ), L_0(\ )$ are the lift and extended lift matroids.

Some standard terminology (much more will be found in the Glossary (Zaslavsky 1998c)):

- polygon, circle: The graph of a simple closed path, or its edge set.
- cycle: In a digraph, a coherently directed polygon, i.e., “dicycle”. More generally: in an oriented signed, gain, or biased graph, a matroid circuit (usually, of the bias matroid) oriented to have no source or sink.

Acknowledgement. I cannot name all the people who have contributed advice and criticism, but many of the annotations have benefited from suggestions by the authors or others and a number of items have been brought to my notice by helpful correspondents. I am very grateful to you all. Thanks also to the people who maintain the invaluable MR and Zbl. indices (and a special thank-you for creating our very own MSC classification: 05C22). However, I insist on my total responsibility for the final form of all entries, including such things as my restatement of results in signed or gain graphic language and, of course, all the praise and criticism (but not errors; see above) that they contain.
### Subject Classification Codes

A code in *lower case* means the topic appears implicitly but not explicitly. A suffix *w* on S, SG, SD, VS denotes signs used as weights, i.e., treated as the numbers $+1$ and $-1$, added, and (usually) the sum compared to 0. A suffix *c* on SG, SD, VS denotes signs used as colors (often written as the numbers $+1$ and $-1$), usually permuted by the sign group. In a string of codes a colon precedes subtopics. A code may be refined through being suffixed by a parenthesised code, as S(M) denoting signed matroids (while S: M would indicate matroids of signed objects; thus S(M): M means matroids of signed matroids).

<table>
<thead>
<tr>
<th>Code</th>
<th>Description</th>
</tr>
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<tbody>
<tr>
<td>A</td>
<td>Adjacency matrix, eigenvalues.</td>
</tr>
<tr>
<td>Alg</td>
<td>Algorithms.</td>
</tr>
<tr>
<td>Appl</td>
<td>Applications other than (Chem), (Phys), (PsS) (partial coverage).</td>
</tr>
<tr>
<td>Aut</td>
<td>Automorphisms, symmetries, group actions.</td>
</tr>
<tr>
<td>B</td>
<td>Balance (mathematical), co-balance.</td>
</tr>
<tr>
<td>Bic</td>
<td>Bicircular matroids.</td>
</tr>
<tr>
<td>Chem</td>
<td>Applications to chemistry (partial coverage).</td>
</tr>
<tr>
<td>Cl</td>
<td>Clusterability.</td>
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<tr>
<td>Col</td>
<td>Vertex coloring.</td>
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<tr>
<td>Cov</td>
<td>Covering graphs, double coverings.</td>
</tr>
<tr>
<td>D</td>
<td>Duality (graphs, matroids, or matrices).</td>
</tr>
<tr>
<td>E</td>
<td>Enumeration of types of signed graphs, etc.</td>
</tr>
<tr>
<td>EC</td>
<td>Even-cycle matroids.</td>
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<tr>
<td>ECol</td>
<td>Edge coloring.</td>
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<tr>
<td>Exp</td>
<td>Expository.</td>
</tr>
<tr>
<td>Exr</td>
<td>Interesting exercises (in an expository work).</td>
</tr>
<tr>
<td>Fr</td>
<td>Frustration (imbalance); esp. frustration index (line index of imbalance).</td>
</tr>
<tr>
<td>G</td>
<td>Connections with geometry, including toric varieties, complex complement, etc.</td>
</tr>
<tr>
<td>GD</td>
<td>Digraphs with gains (or voltages).</td>
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<tr>
<td>Gen</td>
<td>Generalization.</td>
</tr>
<tr>
<td>GG</td>
<td>Gain graphs, voltage graphs, biased graphs; includes Dowling lattices.</td>
</tr>
<tr>
<td>GN</td>
<td>Generalized or gain networks. (Multiplicative real gains.)</td>
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<tr>
<td>Hyp</td>
<td>Hypergraphs with signs or gains.</td>
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<tr>
<td>I</td>
<td>Incidence matrix, Kirchhoff matrix.</td>
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<tr>
<td>K</td>
<td>Signed complete graphs.</td>
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<tr>
<td>Knot</td>
<td>Connections with knot theory (sparse coverage if signs are purely notational).</td>
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<tr>
<td>LG</td>
<td>Line graphs.</td>
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<tr>
<td>M</td>
<td>Matroids and geometric lattices, chain-groups, flows.</td>
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<tr>
<td>N</td>
<td>Numerical and algebraic invariants of signed graphs, etc.</td>
</tr>
<tr>
<td>O</td>
<td>Orientations, bidirected graphs.</td>
</tr>
<tr>
<td>OG</td>
<td>Ordered gains.</td>
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<tr>
<td>P</td>
<td>All-negative or antibalanced signed graphs; parity-biased graphs.</td>
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<tr>
<td>p</td>
<td>Includes problems on even or odd length of paths or polygons (partial coverage).</td>
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<tr>
<td>Phys</td>
<td>Applications in physics (partial coverage).</td>
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<tr>
<td>PsS</td>
<td>Psychological, sociological, and anthropological applications (partial coverage).</td>
</tr>
<tr>
<td>QM</td>
<td>Qualitative (sign) matrices: sign stability, sign solvability, etc. (sparse coverage).</td>
</tr>
<tr>
<td>Rand</td>
<td>Random signs or gains, signed or gain graphs.</td>
</tr>
<tr>
<td>Ref</td>
<td>Many references.</td>
</tr>
<tr>
<td>S</td>
<td>Signed objects other than graphs and hypergraphs: mathematical properties.</td>
</tr>
<tr>
<td>SD</td>
<td>Signed digraphs: mathematical properties.</td>
</tr>
<tr>
<td>SG</td>
<td>Signed graphs: mathematical properties.</td>
</tr>
</tbody>
</table>
**Sol** Sign solvability, sign nonsingularity (partial coverage).
**Sta** Sign stability (partial coverage).
**Str** Structure theory.
**Sw** Switching of signs or gains.
  - **T** Topology applied to graphs; surface embeddings. (Not applications to topology.)
  - **TG** Two-graphs, graph (Seidel) switching (partial coverage).
  - **VS** Vertex-signed graphs ("marked graphs"); signed vertices and edges.
**WD** Weighted digraphs.
**WG** Weighted graphs.
**X** Extremal problems.
Robert P. Abelson
See also M.J. Rosenberg.


§II: “Mathematical models of social structure.” Part B: “The balance principle.” Reviews basic notions of balance and clusterability in signed (di)graphs and measures of degree of balance or clustering. Notes that signed $K_n$ is balanced iff $I + A = vv^T$, $v = \pm 1$-vector. Proposes: degree of balance $= \lambda_1/n$, where $\lambda_1$ = largest eigenvalue of $I + A(\Sigma)$ and $n$ = order of the (di)graph. [Cf. Phillips (1967a).] Part C, 3: “Clusterability revisited.”

(SG, SD: B, Cl, Fr, A)

Robert P. Abelson and Milton J. Rosenberg

Basic formalism: the “structure matrix”, an adjacency matrix $R(\Sigma)$ with entries $o, p, n$ [corresponding to 0, +1, −1] for nonadjacency and positive and negative adjacency and $a$ for simultaneous positive and negative adjacency. Defines addition and multiplication of these symbols (p. 8) so as to decide balance of $\Sigma$ via $\text{per}(I + R(\Sigma))$. [See Harary, Norman, and Cartwright (1965a) for more on this matrix.] Analyzes switching, treated as Hadamard product of $R(\Sigma)$ with “passive $T$-matrices” [essentially, matrices obtained by switching the square all-1's matrix]. Thm. 11: Switching preserves balance. Proposes (p. 12) “complexity” [frustration index] $l(\Sigma)$ as measure of imbalance. [Cf. Harary (1959b).] Thm. 12: Switching preserves frustration index. Thm. 14: $\max l(\Sigma)$, over all $\Sigma$ of order $n$, equals $[(n - 1)^2]/4$. (Proof omitted. [Proved by Petersdorf (1966a) and Tomescu (1973a) for signed $K_n$'s and hence for all signed simple graphs of order $n$.])

(PsS)(SG: A, B, sw, Fr)

B. Devadas Acharya
See also M.K. Gill.

1973a On the product of $p$-balanced and $l$-balanced graphs. Graph Theory Newsletter 2, No. 3 (Jan., 1973), Results Announced No. 1. (SG, VS: B)


(SG, SD: B, A, Ref)(PsS: Exp, Ref)


Begins an attack on the problem of characterizing by forbidden induced subgraphs the simple graphs that switch to forests. Among them are $K_5$ and $C_n$, $n \geq 7$. Problem. Find any others that may exist. [Forests that switch to forests are characterized by Hage and Harju (1998a).]


Find the fewest colors to color the edges so that in each polygon the number of edges of some color is even. [Possibly, inspired by §2 of Acharya and Acharya (1983a).]


Converts a vertex-signed graph $(\Gamma, \mu)$ into a signed graph $\Sigma$ such that $(\Gamma, \mu)$ is consistent iff every polygon in $\Sigma$ is all-negative or has an even number of all-negative components. [See S.B. Rao (1984a) and Hoede (1992a) for the definitive results on consistency.]


Notably: nicely characterizes consistent vertex-signed graphs in which the subgraph induced by negative vertices is connected. [Subsumed by S.B. Rao (1984a).]


Includes an exposition of Sampathkumar and Nanjundaswamy (1973a).


Expounds the procedure of Katai and Iwai (1978a). Proposes a generalization to those $\Sigma$ that have a certain kind of polygon basis. Construct a “dual” graph whose vertex set is a polygon basis supplemented by the sum of basic polygons. A “dual” vertex has sign as in $\Sigma$. Let $T = \text{set of negative “dual” vertices. A } T\text{-join in the “dual”, if one exists, yields a negation set for } \Sigma$. A minimum $T$-join need not yield a minimum negation set. Indeed the procedure is unlikely to yield a minimum negation set (hence the frustration index $l(\Sigma)$) for all signed graphs, since it can be performed in polynomial time while $l(\Sigma)$ is NP-complete. Questions. To which signed graphs is the procedure applicable? For which ones does a minimum $T$-join yield a minimum negation set? Do the latter include all those that forbid an interesting subdivision or minor (cf. Gerards and Schrijver (1986a), Gerards (1988a, 1989a))?]

B. Devadas Acharya and Mukti Acharya [M.K. Gill]

The first half (most of §1) was improved and published as (1986a).
The second half (§§2–3) appears to be unpublished. Given; a graph \( \Gamma \), a
vertex signing \( \mu \), and a covering \( F \) of \( E(\Gamma) \) by cliques of size \( \leq 3 \). Define
a signed graph \( S \) by; \( V(S) = F \) and \( QQ' \in E(S) \) when at least half the
elements of \( Q \) or \( Q' \) lie in \( Q \cap Q' \); sign \( QQ' \) negative iff there exist vertices
\( v \in Q \setminus Q' \), and \( w \in Q' \setminus Q \) such that \( \mu(v) \neq \mu(w) \). Suppose there is no
edge \( QQ' \) in which \( |Q| = 3 \), \( |Q'| = 2 \), and the two members of \( Q \setminus Q' \) have
differing sign. [This seems a very restrictive supposition.] Main result (Thm.
7): \( S \) is balanced. The definitions, but not the theorem, are generalized
to multiple vertex signs \( \mu \), general clique covers, and clique adjacency rules
that differ slightly from that of the theorem. (GG, VS, SG: B)


Four criteria for balance in an arbitrary gain graph. [See also Harary, Lind-
strom, and Zetterstrom (1982a).] (GG: B, sw)


1984a Quasicospectral graphs and digraphs. In: \textit{Proceedings of the National Symposium

A signed graph, or digraph, is “cycle-balanced” if every polygon, or ev-
ery cycle, is positive. Graphs, or digraphs, are “quasicospectral” if they
have cospectral signings, “strictly quasicospectral” if they are quasicospec-
tral but not cospectral, “strongly cospectral” if they are cospectral and have
icospectral cycle-unbalanced signings. There exist arbitrarily large sets of
strictly quasicospectral digraphs, which moreover can be assumed strongly
connected, weakly but not strongly connected, etc. There exist 2 unbalanced
strictly quasicospectral signed graphs; existence of larger sets is not unsolved.
There exist arbitrarily large sets of nonisomorphic, strongly cospectral con-
nected graphs; also, weakly connected digraphs, which moreover can be taken
to be strongly connected, unilaterally connected, etc. Proofs, based on ideas
of A.J. Schwenk, are sketched. (SD, SG: A)

Mukti Acharya [Mukhtiar Kaur Gill]

See also B.D. Acharya and M.K. Gill.

1988a Switching invariant three-path signed graphs. In: M.N. Gopalan and G.A. Pat-
wardhan, eds., \textit{Optimization, Design of Experiments and Graph Theory} (Bombay,
Zbl. 744.05054.

L. Adler and S. Cosares

1991a A strongly polynomial algorithm for a special class of linear programs. \textit{Oper. Res.}

The class is that of the transshipment problem with gains. Along the way, a
time bound on the uncapacitated, demands-only flows-with-gains problem.
(GN: I(D), Alg)

S.N. Afriat

1963a The system of inequalities \( a_{rs} > X_r - X_s \). \textit{Proc. Cambridge Philos. Soc.} 59

See also Roy (1959a). (GG: OG, Sw, b)

**A.A. Ageev, A.V. Kostochka, and Z. Szigeti**


A Seymour graph satisfies with equality a general inequality between $T$-join size and $T$-cut packing. Thm.: A graph is not a Seymour graph iff it has a conservative ±1-weighting such that there are two polygons with total weight 0 whose union is an antibalanced subdivision of $-K_n$ or $-Pr_3$ (the triangular prism). (SGw: Str, B, P)


Virtually identical to (1995a). (SGw: Str, B, P)

**J.K. Aggarwal**

See M. Malek-Zavarei.

**Ron Aharoni, Rachel Manber, and Bronislaw Wajnryb**


When do all perfect matchings in a signed bipartite graph have the same sign product? Solved. (sg: b, Alg)(qm: Sol)

**R. Aharoni, R. Meshulam, and B. Wajnryb**


Given an edge weighting $w : E \to K$ where $K$ is a finite abelian group. Main topic: perfect matchings $M$ such that $\sum_{e \in M} w(e) = 0$ [I’ll call them 0-weight matchings]. (Also, in §2, $= c$ where $c$ is a constant.) Generalizes and extends Aharoni, Manber, and Wajnryb (1990a). Continued by Kahn and Meshulam (1998a). (WG)

Prop. 4.1 concerns vertex-disjoint polygons whose total sign product is + in certain signed digraphs. (SD)

**Ravindra K. Ahuja, Thomas L. Magnanti, and James B. Orlin**


§12.6: “Nonbipartite cardinality matching problem”. Nicely expounds theory of blossoms and flowers (Edmonds (1965a), etc.). Historical notes and references at end of chapter. (p: o, Alg: Exp, Ref)

§5.5: “Detecting negative cycles”; §12.7, subsection “Shortest paths in directed networks”. Weighted arcs with negative weights allowed. Techniques for detecting negative cycles and, if none exist, finding a shortest path. (WD: OG, Alg: Exp)

Ch. 16: “Generalized flows”. Sect. 15.5: “Good augmented forests and linear programming bases”, Thm. 15.8, makes clear the connection between flows with gains and the bias matroid of the underlying gain graph. Some terminology: “breakeven cycle” = balanced polygon; “good augmented forest” = basis of the bias matroid, assuming the gain graph is connected and unbalanced. (GN: M(Bases), Alg: Exp, Ref)
Martin Aigner

In §VII.1, pp. 333–334 and Exerc. 13–15 treat the Dowling lattices of $GF(q)^x$ and higher-weight analogs. (GG, GG(Gen): M: N, Str)

M. Aigner [Martin Aigner]

J. Akiyama, D. Avis, V. Chvátal, and H. Era

Bounds for $D(\Gamma)$, the largest frustration index $l(\Gamma, \sigma)$ over all signings of a fixed graph $\Gamma$ (not necessarily simple) of order $n$ and size $m = |E|$. Main Thm.: $\frac{1}{2}m - \sqrt{mn} \leq D(\Gamma) \leq \frac{1}{2}m$. Thm. 4: $D(K_{t,t}) \leq \frac{1}{2}t^2 - c_0t^{3/2}$, where $c_0$ can be taken $= \pi/480$. Probabilistic methods are used. Thus, Thm. 2: Given $\Gamma$, $\text{Prob}(l(\Gamma, \sigma) > \frac{1}{2}m - \sqrt{mn}) \geq 1 - \left(\frac{2}{e}\right)^n$. Moreover, let $n_b(\Sigma)$ be the largest order of a balanced subgraph of $\Sigma$. Thm. 5: $\text{Prob}(n_b(K_n, \sigma) \geq k) \leq \binom{n}{k}/2^k$. (The problem of evaluating $n - n_b$ was raised by Harary; see (1959b.) Finally, Thm. 1: If $\Sigma$ has vertex-disjoint balanced induced subgraphs with $m'$ edges, then $l(\Sigma) \leq \frac{1}{2}(m - m')$. [See Poljak and Turzik (1982a), Solé and Zaslavsky (1994a) for more on $D(\Gamma)$; Brown and Spencer (1971a), Gordon and Witsenhausen (1972a) for $D(K_{t,t})$; Harary, Lindström, and Zetterström (1982a) for a result similar to Thm. 1.] (SG: Fr, Rand)

S. Alexander and P. Pincus

Kazutoshi Ando and Satoru Fujishige

Kazutoshi Ando, Satoru Fujishige, and Takeshi Naitoh
A balanced bisubmodular system corresponds to a bidirected graph that is balanced. The “flows” are arbitrary capacity-constrained functions, not satisfying conservation at a vertex. (sg: O, B)

Kazutoshi Ando, Satoru Fujishige, and Toshio Nemoto

Thomas Andreae
Partially anticipates the “count” matroids of graphs (see Whiteley (1996a)).

St. Antohe and E. Olaru


A “congruence” is an equivalence relation $R$ on $V(\Sigma)$ such that no negative edge is within an equivalence class. The quotient $\Sigma/R$ has the obvious simple underlying graph and signs $\sigma(\overline{xy}) = \sigma(xy)$ [which is ambiguous]. A signed-graph homomorphism is a function $f : V_1 \rightarrow V_2$ that is a sign-preserving homomorphism of underlying graphs. [This is inconsistent, since the sign of edge $f(x)f(y)$ can be ill defined. The defect might perhaps be remedied by allowing multiple edges with different signs or by passing entirely to multigraphs.] The canonical map $\Sigma \rightarrow \Sigma/R$ is such a homomorphism. Composition of homomorphisms is well defined and associative; hence one has a category $\text{Graph}^\text{sign}$. The categorial product is $\prod_{i \in I} \Sigma_i :=$ Cartesian product of the $|\Sigma_i|$ with the component-wise signature $\sigma((\ldots, u_i, \ldots)(\ldots, v_i, \ldots)) := \sigma_i(u_i,v_i)$. Some further elementary properties of signed-graph homomorphisms and congruences are proved. [The paper is hard to interpret due to mathematical ambiguity and grammatical and typographical errors.]

Katsuaki Aoki

See M. Iri.

Julián Aráoz, William H. Cunningham, Jack Edmonds, and Jan Green-Krótki


The “minimum-cost capacitated $b$-matching problem in a bidirected graph $B$” is to minimize $\sum_e c_e x_e$ subject to $0 \leq x \leq u \in \{0, 1, \ldots, \infty\}^E$ and $I(B)x = b \in \mathbb{Z}^V$. The paper proves, by reduction to the ordinary perfect matching problem, Edmonds and Johnson’s (1970a) description of the convex hull of feasible solutions.

Dan Archdeacon


A compilation from various sources and contributors, updated every so often. “The genus sequence of a signed graph”, p. 10: A conjecture due to Širáň (?) on the demigenus range (here called “spectrum” [though unrelated to matrices]) for orientation embedding of $\Sigma$, namely, that the answer to Question 1 under Širáň (1991b) is affirmative.


§2.5 describes orientation embedding (called “signed embedding” [although there are other kinds of signed embedding]) and switching (called “sequence of local switches of sense”) of signed graphs with rotation systems. §5.5,

SG: T: Exp

Dan Archdeacon and Jozef Siran

A “claw” consists of a vertex and three incident half edges. Let $C$ be the set of claws in $\Gamma$ and $T$ the set of theta subgraphs. Fix a rotation of each claw. Call $t \in T$ an “edge” with endpoints $c, c'$ if $t$ contains $c$ and $c'$; sign it $+$ or $-$ according as $t$ can or cannot be embedded in the plane so the rotations of its trivalent vertices equal the ones chosen for $c$ and $c'$. This defines, independently (up to switching) of the choice of rotations, the “signed triple graph” $T^\pm(\Gamma)$. Theorem: $\Gamma$ is planar iff $T^\pm(\Gamma)$ is balanced. (SG, Sw)

Srinivasa R. Arikati and Uri N. Peled

Given a graph with edges weighted from a group. The weight of a path is the product of its edge weights in order (not inverted, as with gains). Problem: to determine whether between two given vertices there is a chordless path of given weight. This is NP-complete in general but for chordal graphs there is a fast algorithm (linear in $(|E| + |V|)$ · (group order)). [Question. What if the edges have gains rather than weights?] (WG: p(Gen): Alg)


Esther M. Arkin and Christos H. Papadimitriou

E.M. Arkin, C.H. Papadimitriou, and M. Yannakakis

Modular poise gains in digraphs (gain $+1$ on each oriented edge). (gg: B)

Christos A. Athanasiadis

See Headley (1997a) for definitions of the Shi arrangements. Here the characteristic polynomials of these and other arrangements are evaluated combinatorially. §3: “The Shi arrangements”. §4: “The Linial arrangement”: this represents $\text{Lat}^b(K_n, \varphi_1)$ (see Stanley (1996a) for notation). §5: “Other interesting hyperplane arrangements”, treats: the arrangement representing $\text{Lat}^b(L \cdot K_n)$ where $L = \{-k, \ldots, k-1, k\}$, which is the semilattice of $k$-composed partitions (see Zaslavsky (20xxh), also Edelman and Reiner (1996a)) and several generalizations, including to arbitrary sign-symmetric gain sets $L$ and to Weyl analogs; also, an antibalanced analog of the $A_n$ Shi arrangement (Thm. 5.4); and more. (sg, gg: G, M, N)

The Shi arrangement of hyperplanes [of type $A_{n-1}$] represents $\text{Lat}^b \Phi$ where $\Phi = (K_n, \varphi_0) \cup (K_n, \varphi_1)$ (see Stanley (1996a) for notation). (\text{gg: G, M, N})


The arrangements considered are the subarrangements of the projectivized Shi arrangements of type $A_{n-1}$ that contain $A_{n-1}$. Thms. 4.1 and 4.2 characterize those that are free or supersolvable. Arrangements representing the extended lift matroid $L_0(\Phi)$ where $\Phi = \bigcup_{i=1}^a (K_n, \varphi_i)$ and $a \geq 1$ ($a = 1$ giving the Shi arrangement), and a mild generalization, are of use in the proof (see Stanley (1996a) for notation). (\text{gg: G, M, N})

20xxa Deformations of Coxeter hyperplane arrangements and their characteristic polynomials. Submitted.

**David Avis**
See J. Akiyama.

**Constantin P. Bachas**

The frustration index decision problem on signed (3-dimensional) cubic lattice graphs is NP-complete. [Proof is incomplete; completed and improved by Green (1987a).] [Cf. Barahona (1982a).] (\text{SG: Fr: Alg})

**G. David Bailey**
20xxa Inductively factored signed-graphic arrangements of hyperplanes. Submitted. (\text{SG: G, M})

**V. Balachandran**

**V. Balachandran and G.L. Thompson**

**Egon Balas**


Linear (thus “fractional”, meaning half-integral) vs. integral programming solutions to maximum matching. The difference of their maxima = $\frac{1}{2}$ (max number of matching-separable vertex-disjoint odd polygons). Also noted (p. 12): (max) fractional matchings in $\Gamma$ correspond to (max) matchings in the
double covering graph of $-\Gamma$. [Question. Does this lead to a definition of maximum matchings in signed graphs?]  

E. Balas and P.L. Ivanescu [P.L. Hammer]

K. Balasubramanian
Here a “signed graph” means, in effect, an acyclically oriented graph $D$ along with the antisymmetric adjacency matrix $A_\pm(D) = A(+D) - A(-D)$, $D^*$ being the converse digraph. [That is, $A_\pm(D) = A(D) - A(D)^t$. The “signed graphs” are just acyclic digraphs with an antisymmetric adjacency matrix and, correspondingly, what we may call the ‘antisymmetric characteristic polynomial’.] Proposes an algorithm for the polynomial. Observes in some examples a relationship between the characteristic polynomial of $\Gamma$ and the antisymmetric characteristic polynomial of an acyclic orientation.

Argues (heuristically) that a certain algorithm is superior to another, in particular for the antisymmetric polynomial defined in (1988a).

Computed for graphs of six different cages of three different orders, in both ordinary and “signed” (see (1988a)) versions. Observes a property of the “signed graph” polynomials [which is due to antisymmetry, as explained by P.W. Fowler (Comment on “Characteristic polynomials of fullerene cages”. Chemical Physics Letters 203 (1993), 611–612)].

The “signed graphs” are as in (1988a). Simplified contents: It is shown by example that the antisymmetric characteristic polynomials of two non-isomorphic acyclic orientations of a graph (see (1988a)) may be equal or unequal. [Much smaller examples are provided by P.W. Fowler (Comment on “Characteristic polynomials of fullerene cages”. Chemical Physics Letters 203 (1993), 611–612).] [Question. Are there examples for which the underlying (di)graphs are nonisomorphic?] [For cospectrality of other kinds of signed graphs, see Acharya, Gill, and Patwardhan (1984a) (signed $K_n$’s).]

R. Balian, J.M. Drouffe, and C. Itzykson
(SG: Phys, Sw, B)

Jørgen Bang-Jensen and Gregory Gutin
A rich source for problems on bidirected graphs. An edge 2-coloration of a graph becomes an all-negative bidirection by taking one color class to con-
sist of introverted edges and the other to consist of extroverted edges. An alternating path becomes a coherent path; an alternating polygon becomes a coherent polygon. [General Problem. Generalize to bidirected graphs the results on edge 2-colored graphs mentioned in this paper. (See esp. §5.) Question. To what digraph properties do they specialize by taking the underlying signed graph to be all positive?] [See e.g. Bánkfalvi and Bánkfalvi (1968a) (q.v.), Bang-Jensen and Gutin (1998a), Das and Rao (1983a), Grossman and Häggkvist (1983a), Mahadev and Peled (1995a), Saad (1996a).]


The longest coherent trail, having degrees bounded by a specified degree vector, in a bidirected all-negative complete multigraph that satisfies an extra hypothesis. Generalization of Das and Rao (1983a) and Saad (1996a), thus ultimately of Thm. 1 of Bánkfalvi and Bánkfalvi (1968a) (q.v.). Also, a polynomial-time algorithm.

M. Bánkfalvi and Zs. Bánkfalvi

Let \( \Sigma \) be a bidirected \(-K_{2n}\) which has a coherent 2-factor. (“Coherent” means that, at each vertex in the 2-factor, one edge is directed inward and the other outward.) Thm. 1: \( B \) has a coherent Hamiltonian polygon iff, for every \( k \in \{2, 3, \ldots, n-2\} \), \( s_k > k^2 \), where \( s_k := \text{the sum of the } k \text{ smallest indegrees and the } k \text{ smallest outdegrees.} \) Thm. 2: The number of \( k \)'s for which \( s_k = k^2 \) equals the smallest number \( p \) of polygons in any coherent 2-factor of \( B \). Moreover, the \( p \) values of \( k \) for which equality holds imply a partition of \( V \) into \( p \) vertex sets, each inducing \( B_i \) consisting of a bipartite [i.e., balanced] subgraph with a coherent Hamiltonian polygon and in one color class only introverted edges, while in the other only extroverted edges. [Problem. Generalize these remarkable results to an arbitrary bidirected complete graph. The all-negative case will be these theorems; the all-positive case will give the smallest number of cycles in a covering by vertex-disjoint cycles of a tournament that has any such covering.] [See Bang-Jensen and Gutin (1997a) for further developments on alternating walks.]

Zs. Bánkfalvi
See M. Bánkfalvi.

C. Bankwitz

Introduces the sign-colored graph of a link diagram. [Further work by numerous writers, e.g., S. Kinoshita et al. and esp. Kauffman (1989a) and successors.]

Francisco Barahona

Given a 2-connected \( \Sigma \) whose underlying graph is toroidal, polynomial-time algorithms are given for calculating the frustration index \( l(\Sigma) \) and the generating function of switchings \( \Sigma^\mu \) by \(|E_-(\Sigma^\mu)|\). The technique is to


The frustration-index problem, that is, minimization of \( |E_-(\Sigma^n)| \) over all switching functions \( \eta : V \to \{\pm 1\} \), for signed planar and toroidal graphs and subgraphs of 3-dimensional grids. Analyzed structurally, in terms of perfect matchings in a modified dual graph, and algorithmically. The last is NP-hard, even when the grid has only 2 levels; the former are polynomial-time solvable even with weighted edges. Also, the problem of minimizing \( |E_-(\Sigma^n)| + \sum_v \eta(v) \) for planar grids (“2-dimensional problem with external magnetic field”), which is NP-hard. (This corresponds to adding an extra vertex, positively adjacent to every vertex.) (SG: Phys, Fr, Fr( Gen): D, Alg)


Section 2: “Spin glasses.” (SG: Phys, Fr: Exp)


Negative cutsets, where signs come from a network with real-valued capacities. Dual in the plane to negative polygons. See \S2. (SG: D: B, Alg)

Francisco Barahona and Adolfo Casari

Francisco Barahona, Martin Grötschel, and Ali Ridha Mahjoub

The polytope \( P_B(\Gamma) \) is the convex hull in \( \mathbb{R}^E \) of incidence vectors of bipartite edge sets. Various types of and techniques for generating facet-defining inequations, thus partially extending the description of \( P_B(\Gamma) \) from the weakly bipartite case (Grötschel and Pulleyblank (1981a)) in which all facets are due to edge and odd-polygon constraints. [Some can be described best via signed graphs; see Poljak and Tuza (1987a).] [A brief expository treatment of the polytope appears in Poljak and Tuza (1995a).] (sg: p: fr: G)

Francisco Barahona and Enzo Maccioni
Discusses a 3-dimensional analog of Barahona, Maynard, Rammal, and Uhry (1982a). (Here there may not always be a combinatorial LP optimum; hence LP may not completely solve the problem.)

**Francisco Barahona and Ali Ridha Mahjoub**


Call \( P_{BS}(\Sigma) \) the convex hull in \( \mathbb{R}^E \) of incidence vectors of negation sets (or “balancing [edge] sets”) in \( \Sigma \). Finding a minimum-weight negation set in \( \Sigma \) corresponds to a maximum cut problem, whence \( P_{BS}(\Sigma) \) is a linear transform of the cut polytope \( P_C(|\Sigma|) \), the convex hull of cuts. Conclusions follow about facet-defining inequalities of \( P_{BS}(\Sigma) \). See §5: “Signed graphs”.


The “balanced induced subgraph polytope” \( P_{BIS}(\Sigma) \) is the convex hull in \( \mathbb{R}^V \) of incidence vectors of vertex sets that induce balanced subgraphs. Conditions are studied under which certain inequalities of form \( \sum_{i \in Y} x_i \leq f(Y) \) define facets of this polytope: in particular, \( f(Y) = \max \) size of balance-inducing subsets of \( Y \), \( f(Y) = 1 \) or 2, \( f(Y) = |Y| - 1 \) when \( Y = V(C) \) for a negative polygon \( C \), etc.


More on \( P_{BIS}(\Sigma) \) (see (1989a)). A balance-inducing vertex set in \( \pm \Gamma = \text{stable set in } \Gamma \). [See Zaslavsky (1982b) for a different correspondence.] Thm. 2.1 is an interesting preparatory result: If \( \Sigma = \Sigma_1 \cup \Sigma_2 \) where \( \Sigma_1 \cap \Sigma_2 \cong \pm K_k \), then \( P_{BIS}(\Sigma) = P_{BIS}(\Sigma_1) \cap P_{BIS}(\Sigma_2) \). The main result is Thm. 2.2: If \( \Sigma \) has a 2-separation into \( \Sigma_1 \) and \( \Sigma_2 \), the polytope is the projection of the intersection of polytopes associated with modifications of \( \Sigma_1 \) and \( \Sigma_2 \). §5: “Compositions of facets”, derives the facets of \( P_{BIS}(\Sigma) \).

F. Barahona, R. Maynard, R. Rammal, and J.P. Uhry


§2: “The frustration model as the Chinese postman’s problem”, describes how to find the frustration index \( l(-\Sigma) = \min_\eta |E_-(\Sigma, \eta)| \) (over all switching functions \( \eta \)) of a signed planar graph by solving a Chinese postman \((T\text{-join})\) problem in the planar dual graph, \( T \) corresponding to the frustrated face boundaries. [This was solved independently by Katai and Iwai (1978a).] The postman problem is solved by linear programming, in which there always is a combinatorial optimum: see §3: “Solution of the frustration problem by duality: rigidity”. Of particular interest are vertex pairs, esp. edges, for which \( \eta(v)\eta(w) \) is the same for every “ground state” (i.e., minimizing \( \eta \)); these are called “rigid”. §5: “Results” (of numerical experiments) has interesting discussion. [Barahona (1981a) generalizes to signed toroidal graphs.]

In the preceding one minimizes \( f_0(\eta) = \sum_{E} \sigma(vw)\eta(v)\eta(w) \). More general problems discussed are (1) allowing positive edge weights (due to variable
bond strengths); (2) minimizing $f_0(\eta) + c\sum_v \eta(v)$, with $c \neq 0$ because of an external magnetic field. Then one cannot expect the LP to have a combinatorial optimum. 

F. Barahona and J.P. Uhry

J. Wesley Barnes
See P.A. Jensen.

Lowell Bassett, John Maybee, and James Quirk

Lemma 3: A square matrix with every diagonal entry negative is sign-
nonsingular i every cycle is negative in the associated signed digraph. Thm. 4: A square matrix with negative diagonal is sign-invertible i all cycles are negative and the sign of any (open) path is determined by its endpoints. And more. (QM: Sol, Sta: sd)

Vladimir Batagelj


M. Behzad and G. Chartrand

[L.] W. Beineke and F. Harary

Lowell W. Beineke and Frank Harary

A digraph with signed vertices is “consistent” (that is, every cycle has positive sign product) iff its vertices have a bipartition so that every arc with a positive tail lies within a set but no arc with a negative tail does so. (The reason is that a strongly connected digraph with vertex signs can be regarded as edge-signed and the bipartition criterion for balance can be applied.) A corollary: the digraphs that have consistent vertex signs are characterized.

VS


A graph with signed vertices is “consistent” if every polygon has positive sign product. Elementary results, but a characterization of consistent vertex-signed graphs is presented as an open problem. For a good solution see Hoede (1992a); Rao (1984a) had found a more complicated solution. (VS)

Jacques Bélair, Sue Ann Campbell, and P. van den Driessche
The signed digraph of a square matrix is “frustrated” if it has a negative cycle. Somewhat simplified: frustration is necessary for there to be oscillation caused by intraneuronal processing delay. (SD: QM, Ref)

A. Bellacicco and V. Tulli

Signed digraphs (“spin graphs”) are defined. The main concepts—“dissimilarity”, “balance”, and “cluster”—do not involve signs. Eigenvalues are mentioned. [This may be an announcement. There are no proofs. It is hard to be sure what is being said.] (SD: A)

Joachim von Below

Here a periodic graph [of dimension m] is defined as a connected graph Γ = Ψ where Ψ is a finite $\mathbb{Z}^m$-gain graph with gains contained in \{0, $b_1$, $b_i - b_j$\}. ($b_1, \ldots, b_m$ are the unit basis vectors of $\mathbb{Z}^m$.) Let us call such a Ψ a small-gain base graph for Γ. Any Φ, where Φ is a finite $\mathbb{Z}^m$-gain graph, has a small-gain base graph Ψ; thus this definition is equivalent to that of Collatz (1978a). The “index” $I(\Gamma)$, analogous to the largest eigenvalue of a finite graph, is the spectral radius of $A(\Gamma, N)$ for any small-gain base graph of Γ. The paper contains basic theory and the lower bound $L_m = \inf \{I(\Gamma) : \Gamma$ is $m$-dimensional\}, where $1 = L_1$, $\sqrt{9/2} = L_2 \leq L_3 \leq \ldots$ (GG(Cov): A)

Edward A. Bender and E. Rodney Canfield

§3: “Self-dual signed graphs.” gives the number of $n$-vertex graphs that are signed, vertex-signed, or both; connected or not; self-isomorphic by reversing edge and/or vertex signs or not, for all $n \leq 12$. Some of this appeared in Harary, Palmer, Robinson, and Schwenk (1977a). (SG, VS: E)

Riccardo Benedetti

§8, “Spin manifolds”, hints at a use for decorated signed graphs in the structure theory of spin 3-manifolds. (sg: Appl: Exp)

Curtis Bennett and Bruce E. Sagan

To illustrate the generalization, most of the article calculates the chromatic polynomial of $\pm K_n^{(k)}$ (called $DB_{n,k}$; this has half edges at $k$ vertices), builds an “atom decision tree” for $k = 0$, and describes and counts the bases of $G(\pm K_n^{(k)})$ (called $D_n$) that contain no broken circuits. (SG: M, N, col)
M.K. Bennett, Kenneth P. Bogart, and Joseph E. Bonin

Moussa Benoumhani


C. Benzaken
See also P.L. Hammer.

C. Benzaken, S.C. Boyd, P.L. Hammer, and B. Simeone

Cl. Benzaken, P.L. Hammer, and B. Simeone

C. Benzaken, P.L. Hammer, and D. de Werra

Claude Berge and A. Ghouila-Houri


Joseph Berger, Bernard P. Cohen, J. Laurie Snell, and Morris Zelditch, Jr.
See Ch. 2: “Explicational models.”

Abraham Berman and B. David Saunders

Gora Bhaumik
See P.A. Jensen.

V.N. Bhave
See E. Sampathkumar.

I. Bieche, R. Maynard, R. Rammal, and J.P. Uhry

Dan Bienstock

Given a graph. Problem 1: Is there an odd hole on a particular vertex? Problem 2: Is there an odd induced path joining two specified vertices? Problem 3: Is every pair of vertices joined by an odd-length induced path?

All three problems are NP-complete. [Obviously, one can replace the graph by a signed graph and “odd length” by “negative” and the problems remain NP-complete.]

Norman Biggs

Ch. 19: “The covering graph construction.” Especially see Exercise 19A: “Double coverings.” These define what we might call the canonical covering graphs of gain graphs.


As in (1974a), but Exercise 19A has become Additional Result 19a.

Robert E. Bixby
They study lattices $\Pi_{n,k,h}$ (for $0 < h \leq k \leq n$) consisting of all spanning subgraphs of $\pm K_n^\circ$ that have at most one nontrivial component $K$, for which $K$ is complete and $|V(K)| \geq k$ if $K$ is balanced, $K$ is induced and $|V(K)| \geq h$ if $K$ is unbalanced (also a generalization). Characteristic polynomial, homotopy and homology of the order complex, cohomology of the real complement.

Anders Björner, Michel Las Vergnas, Bernd Sturmfels, Neil White, and Günter M. Ziegler


The adjacency graph of bases of an oriented matroid is signed, using circuit signatures, to make the “signed basis graph”. See §3.5, “Basis orientations and chirotopes”, pp. 132–3.

Andreas Blass


Treats the generalized Dowling lattices of Hanlon (1991a) as congruence lattices of certain quasi-varieties, in order to calculate characteristic polynomials and generalizations.

Andreas Blass and Frank Harary


The theorem that deletion index = negation index of a signed graph (Harary (1959b)) is shown to be a special case of a very general phenomenon involving hereditary classes of “partial choice functions”. Another special case: deletion index = alteration index of a gain graph [an immediate corollary of Harary, Lindstöm, and Zetterström (1982a), Thm. 2]. (SG, GG: B, Fr)

Andreas Blass and Bruce Sagan


§3: “Non-crossing $B_n$ and $D_n$”. Lattices of noncrossing signed partial partitions. Atoms of the lattices are defined as edge fibers of the signed covering graph of $\pm K_n^\circ$, thus corresponding to edges of $\pm K_n^\circ$. [The “half edges” are perhaps best regarded as negative loops.] The lattices studied, called $NCB_n, NCD_n, NCBD_n(S)$, consist of the noncrossing members of the Dowling and near-Dowling lattices of the sign group, i.e., $\text{Lat} G(\pm K_n^\circ)$ for $T = [n], \emptyset, [n]\setminus S$, respectively. (SG, N, cov)


Signed-graph chromatic polynomials are recast geometrically by observing that the number of $k$-colorings equals the number of points of $\{-k, -k+1, \ldots, k-1, k\}^n$ that lie in none of the edge hyperplanes of the signed graph. The interesting part is that this generalizes to subspace arrangements of signed graphs and, somewhat ad hoc, to the hyperplane arrangements of the exceptional root systems. [See also Zaslavsky (20xxi). For applications see articles of Sagan and Zhang.] (SG, Gen: M(Gen), G: col, N)
T.B. Boffey

Kenneth P. Bogart
See M.K. Bennett, J.E. Bonin, and J.R. Weeks.

Ethan D. Bolker

Bela Bollobás
A rich source of problems: find interesting generalizations to signed graphs of questions involving even or odd polygons, or bipartite graphs or subgraphs. (p: X)

§3.2, Thm. 2.2, is Lovász’s (1965a) characterization of the graphs having no two vertex-disjoint polygons. (GG: Polygons)

§6.6, Problem 47, is the theorem on all-negative vertex elimination number from Bollobás, Erdős, Simonovits, and Szemerédi (1978a). (p: Fr)

B. Bollobás, P. Erdös, M. Simonovits, and E. Szemerédi
Thm. 9 asymptotically estimates upper bounds on frustration index and vertex elimination number for all-negative signed graphs with fixed negative girth. [Sharpened by Komlós (1997a).] (p: Fr)

J.A. Bondy and L. Lovász
If $\Gamma$ is $k$-connected [and not bipartite], then any $k$ $[k-1]$ vertices lie on an even [odd] polygon. [Problem. Generalize to signed graphs, this being the all-negative case.] (sg: b)

J.A. Bondy and M. Simonovits
If a graph has enough edges, it has even polygons of all moderately small lengths. [Problem 1. Generalize to positive polygons in signed graphs, this being the antibalanced (all-negative) case. For instance, Problem 2. If an unbalanced signed simple graph has positive girth $\geq l$ (i.e., no balanced polygon of length $<l$), what is its maximum size? Are the extremal examples antibalanced? Balanced?] (p: b(Polygons), X)

Joseph E. Bonin
See also M.K. Bennett.

A weight- \( k \) higher Dowling geometry of rank \( n \), \( Q_{n,k}(\text{GF}(q)^\times) \), is the union of all coordinate \( k \)-flats of \( \text{PG}(n-1,q) \): i.e., all flats spanned by \( k \) elements of a fixed basis. If \( k > 2 \), the automorphism groups are those of \( \text{PG}(n-1,q) \) for \( q > 2 \) and are symmetric groups if \( q = 2 \).  


See definition in (1993a). For \( k > 2 \) the only nontrivial modular flats are the projective coordinate \( k \)-flats and their subflats. This gives some information about the characteristic polynomials [which, however, are still only partially known]. [Kung (1996a), §6, has further results.]


The automorphisms of a Dowling geometry of a nontrivial group are the compositions of a coordinate permutation, switching, and a group automorphism. A similar result holds, with two exceptions, if some or all coordinate points are deleted.


*Problem 6.1.* If a finite matroid embeds in the Dowling geometry of a group, does it embed in the Dowling geometry of some finite group? [The answer may be “no” (Squier and Zaslavsky, unwritten and possibly unrecoverable).]

**Joseph E. Bonin and Kenneth P. Bogart**


**Joseph E. Bonin and Joseph P.S. Kung**


**Joseph E. Bonin and William P. Miller**

20xxa Characterizing geometries by numerical invariants. Submitted

Dowling geometries are characterized amongst all simple matroids by numerical properties of large flats of ranks \( \leq 7 \) (Thm. 3.4); amongst all matroids by their Tutte polynomials.

**Joseph E. Bonin and Hongxun Qin**

20xxa Size functions of subgeometry-closed classes of representable combinatorial geometries. Submitted

Extremal matroid theory. The Dowling geometry \( Q_3(\text{GF}(3)^\times) \) appears as an exceptional extremal matroid in Thm. 2.10. The extremal subset of \( \text{PG}(n-1,q) \) not containing the higher-weight Dowling geometry \( Q_{m,m-1}(\text{GF}(q)^\times) \) (see Bonin 1993a) is found in Thm. 2.14.

**C. Paul Bonnington and Charles H.C. Little**

Signed-graph imbedding: see §2.3, §2.6 (esp. Thm. 2.4), pp. 44–48 (for the colorful 3-gem approach to crosscaps), §3.3, and Ch. 4 (esp. Thms. 4.5, 4.6).

E. Boros, Y. Crama, and P.L. Hammer


(\text{sg: T, b})

Endre Boros and Peter L. Hammer

Includes finding a minimum-weight deletion set (as in Boros, Crama, and Hammer (1991a)).

(\text{sg: G, Alg})

André Bouchet

(\text{sg: O, Appl})


Introduces nowhere-zero flows on signed graphs. A connected, coloop-free signed graph has a nowhere-zero integral flow with maximum weight \leq 216. The value 216 cannot be replaced by 5, but Bouchet conjectures that it can be replaced by 6. [See Khelladi (1987a) for some progress on this.] A topological application is outlined. [The bidirection is inessential; it is a device to keep track of the flow.]

(\text{sg: O, Flows, Appl})

Jean-Marie Bourjolly

[See Sewell (1996a).]

(\text{sg: O, GG: Alg})

J.-M. Bourjolly, P.L. Hammer, and B. Simeone

(\text{p: o})

Jean-Marie Bourjolly and William R. Pulleyblank

[It is hard to escape the feeling that we are dealing with all-negative signed graphs and that something here will generalize to other signed graphs. Especially see Theorem 5.1. Consult the references for related work.]

(\text{P; Ref})

John Paul Boyd

(\text{SG: B})
S.C. Boyd  
See C. Benzaken.

A.J. Bray, M.A. Moore, and P. Reed  
(Phys: SG: Fr)

Floor Brouwer and Peter Nijkamp  
(QM, SD: Sol, Sta: Exp)

Edward M. Brown and Robert Messer  
Their “signed graph” we might call a type of Eulerian partially bidirected graph. That is, some edge ends are oriented (hence “partially bidirected”), and every vertex has even degree and at each vertex equally many edge ends point in and out (“Eulerian”). More specially, at each vertex all or none of the edge ends are oriented.  
(sg: o: gen: Appl)

Gerald G. Brown and Richard D. McBride  
(GN: M(bases))

Kenneth S. Brown and Persi Diaconis  
The real hyperplane arrangement representing $-K_n$ is studied in §3D. It leads to a random walk on threshold graphs.  
(p: G)

Thomas A. Brown  
See also F.S. Roberts.

T.A. Brown, F.S. Roberts, and J. Spencer  
(SDw)

Thomas A. Brown and Joel H. Spencer  
Asymptotic estimates of $l(K_{r,s})$, the maximum frustration index of signatures of $K_{r,s}$. Improved by Gordon and Witsenhausen (1972a). Also, exact values stated for $r \leq 4$ [extended by Sole and Zaslavsky (1994a)].  
(sg: Fr)

William G. Brown, ed.  
See esp.: §208: “Signed graphs (+ or – on each edge), balance” (undirected and directed), Vol. 1, pp. 569–571.  
(SG, SD)

Richard A. Brualdi and Herbert J. Ryser  
See §7.5.  
(QM: Sol, SD, b)(Exp, Ref)
Richard A. Brualdi and Bryan L. Shader  
Innumerable results and references on signed digraphs are contained herein.  
(QM, SD: Sol, Sta)(Exp, Ref, Alg)

Michael Brundage  
A concise expository survey. Ch. 1: “Even cycles in directed graphs”. Ch. 2: “$L$-matrices and sign-solvability”, esp. sect. “Signed digraphs”. Ch. 3: “Beyond”, esp. sect. “Balanced labellings” (vertices labelled from $\{0, +1, -1\}$ so that from each vertex labelled $\epsilon \neq 0$ there is an arc to a vertex labelled $-\epsilon$) and sect. “Pfaffian orientations”.  
(SD, P: Polygons, Sol, Alg, VS: Exp, Ref)

Fred Buckley, Lynne L. Doty, and Frank Harary  
“Signed invertible graph” [i.e., sign-invertible graph] = graph $\Gamma$ such that $A(\Gamma)^{-1} = A(\Sigma)$ for some signed graph $\Sigma$. Finds two classes of such graphs.  
Characterizes sign-invertible trees. [Cf. Godsil (1985a) and, for a different notion, Greenberg, Lundgren, and Maybee (1984b).]  
(SG: A)

James R. Burns and Wayland H. Winstead  
§IV: “The computation of contradictory redundancy.” Summarized in modified notation: In a signed graph, define $w_{ij}^\epsilon(r) =$ number of walks of length $r$ and sign $\epsilon$ from $v_i$ to $v_j$. Define an adjacency matrix $A$ by $a_{ij} = w_{ij}^+(1) + w_{ij}^-(1)\theta$, where $\theta$ is an indeterminate whose square is 1. Then $w_{ij}^+(r) + w_{ij}^-(r)\theta = (A^r)_{ij}$ for all $r \geq 1$. [We should regard this computation as taking place in the group ring of the sign group. The generalization to arbitrary gain graphs and digraphs is obvious.] Other sections also discuss signed digraphs [but have little mathematical content].  
(SD, gd: A, Paths)

F.C. Bussemaker, P.J. Cameron, J.J. Seidel, and S.V. Tsaranov  
(SG: Sw)

F.C. Bussemaker, D.M. Cvetković, and J.J. Seidel  
The 187 simple graphs with eigenvalues $\geq -2$ that are not (negatives of) reduced line graphs of signed graphs are found, with computer aid. By Cameron, Goethals, Seidel, and Shult (1976a), all are represented by root systems $E_d$, $d = 6, 7, 8$. Most interesting is Thm. 2: each such graph is Seidel-switching equivalent to a line graph of a graph. [Problem. Explain this within signed graph theory.]  
(LG: p: A)

Announces the results of (1976a).

F.C. Bussemaker, R.A. Mathon, and J.J. Seidel


“The most important tables from” (1979a).

Leishen Cai and Baruch Schieber

By the negative-subdivision trick (subdividing each positive edge into two negative ones), the algorithm will find the intersection of all negative polygons of a signed graph.

Peter J. Cameron
See also F.C. Bussemaker.


The first step towards (1977b), Thm. 3.1.


Introducing the cohomological theory of two-graphs. A two-graph $\tau$ is a 2-coboundary in the complex of GF(2)-cochains on $E(K_n)$. [The 1-cochains are the signed complete graphs, equivalently the graphs that are their negative subgraphs. Cf. D.E. Taylor (1977a).] Write $Z_i$, $Z^i$, $B^i$ for the $i$-cycle, $i$-cocycle, and $i$-coboundary spaces. Switching a signed complete graph means adding a 1-cocycle to it; a switching class of signed complete graphs is viewed as a coset of $Z^1$ and is equivalent to a two-graph.

Take a group $G$ of automorphisms of $\tau$. Special cohomology elements $\gamma \in H^1(G, B^1)$ and $\beta \in H^2(G, B^0)$ (where $B^0 = \{0, V(K_n)\}$, the reduced 0-coboundary group) are defined. Thm. 3.1: $\gamma = 0$ iff $G$ fixes a graph in $\tau$. Thm. 5.1: $\beta = 0$ iff $G$ can be realized as an automorphism group of the canonical double covering graph of $\tau$ (viewing $\tau$ as a switching class of signed complete graphs). Conditions are explored for the vanishing of $\gamma$ (related to Harries and Liebeck (1978a)) and $\beta$.

$Z^1$ is the annihilator of $Z_1 =$ the space of even-degree simple graphs; the theorems of Mallows and Sloane (1975a) follow immediately. More generally: Lemma 8.2: $Z^i$ is the annihilator of $Z_i$. Thm. 8.3. The numbers of isomorphism types of $i$-cycles and $i$-cocycles are equal, for $i = 1, \ldots, n-2$.

§8 concludes with discussion of possible generalizations, e.g., to oriented two-graphs (replacing GF(2) by GF(3)$^*$) and double coverings of complete digraphs (Thms. 8.6, 8.7). [A full ternary analog is developed in Cheng and Wells (1986a).]

Exposition of parts of (1977b) with a simplified proof of the connection between $\beta$ and $\gamma$.

(TG: Aut, E, G, Exp)


[For generalized line graphs see Zaslavsky (1984c).] If two generalized line graphs are isomorphic, their underlying graphs and cocktail-party attachments are isomorphic, with small exceptions related to exceptional isomorphisms and automorphisms of root systems. The proof, along the lines of Cameron, Goethals, Seidel, and Shult (1976a), employs the canonical vector representation of the underlying signed graph.

(sg: LG: Aut, G)


Let $T$ be a tree. Construction 1 (simplifying Seidel and Tsaranov (1990a)): Take all triples of edges such that none separates the other two. This defines a two-graph on $E(T)$ [whose underlying signed complete graph is described by Tsaranov (1992a)]. Construction 2: Choose $X \subseteq V(T)$. Take all triples of end vertices of $T$ whose minimal connecting subtree has its trivalent vertex in $X$. The two-graphs $(V, T)$ that arise from these constructions are characterized by forbidden substructures, namely, the two-graphs of (1) $C_5$ and $C_6$; (2) $C_5^\perp$. Also, trees that yield identical two-graphs are characterized.

(TG)


Counting two-graphs of the types constructed in (1994a).

(TG: E)

P.J. Cameron, J.M. Goethals, J.J. Seidel, and E.E. Shult


The essential idea is that graphs with least eigenvalue $\leq -2$ are represented by the angles of root systems. It follows that line graphs are so represented. [Similarly, signed graphs with largest eigenvalue $\leq 2$ are represented by the inner products of root systems, as in Vijayakumar et al. These include the line graphs of signed graphs as in Zaslavsky (1984c), since simply signed graphs are represented by $B_n$ or $C_n$ with a few exceptions. The representation of ordinary graphs by all-negative signed graphs is motivated in Zaslavsky (1984c).]

(LG: sg: A, G, Sw)

P.J. Cameron, J.J. Seidel, and S.V. Tsaranov


A generalized Coxeter group $\text{Cox}(\Sigma)$ and a Tsaranov group $\text{Ts}(\Sigma)$ are defined via Coxeter relations and an extra relation for each negative polygon in $\Sigma$. They generalize Coxeter groups of tree Coxeter graphs and the Tsaranov groups of a two-graph ($|\Sigma| = K_n$; see Seidel and Tsaranov (1990a)). A new operation of “local switching” is introduced, which changes the edge set of $\Sigma$ but preserves the associated groups.
§2, “Signed graphs”, proves some well-known properties of switching and reviews interesting data from Bussemaker, Cameron, Seidel, and Tsaranov (1991a). §3, “Root lattices and Weyl groups”: The “intersection matrix” $2I + A(\Sigma)$ is a hyperbolic Gram matrix of a basis of $\mathbb{R}^n$ whose vectors form only angles $\pi/2, \pi/3, 2\pi/3$. To these vectors are associated the lattice $L(\Sigma)$ of their integral linear combinations and the Weyl group $W(\Sigma)$ generated by reflecting along the vectors. $W$ is finite iff $2I + A(\Sigma)$ is positive definite (Thm. 3.1). Problem 3.6. Determine which $\Sigma$ have this property. §4 introduces local switching to partially solve Problem 4.1: Which signed graphs generate the same lattice? Results and some experimental data are reported. All-negative signed graphs play a special role. §6, “Coxeter groups”: The relationship between the Coxeter and Weyl groups of $\Sigma$. $\text{Cox}(\Sigma)$ is $\text{Cox}(|\Sigma|)$ with additional relations for antinegative (i.e., negative in $-\Sigma$) induced polygons. §7: “Signed complete graphs”. §8: “Tsaranov groups” of signed $K_n$’s. Dictionary: \(\Gamma(f)\) = \(\Sigma = (\Gamma, \sigma)\). “Fundamental signing” = all-negative signing, giving the antibalanced switching class. “The balance” of a cycle (i.e., polygon) = its sign $\sigma(C)$; “the parity” = $\sigma(-C)$ where $-C = C$ with all signs negated. “Even” = positive and “odd” = negative (referring to “parity”). “The balance” of $\Sigma$ = the partition of all polygons into positive and negative classes $C^+$ and $C^-$; this is the bias on $|\Sigma|$ due to the signing and should not be confused with the customary meaning of “balance”, i.e., all polygons are positive.

[A more natural definition of the intersection matrix would be $2I - A$. Then signs would be negative to those in the paper. The need for “parity” would be obviated, ordinary graphs would correspond to all-positive signings (and those would be “fundamental”), and the extra Coxeter relations would pertain to negative induced polygons.]

P.J. Cameron and Albert L. Wells, Jr.

Sue Ann Campbell
See J. Bélair.

E. Rodney Canfield
See E.A. Bender.

D.-S. Cao
See R. Simion.

Dorwin Cartwright
See also T.C. Gleason; Harary, Norman, and Cartwright (1965a, etc.)

Dorwin Cartwright and Terry C. Gleason

Dorwin Cartwright and Frank Harary
Expounds Harary (1953a, 1955a) with sociological discussion. Proposes to measure imbalance by the proportion of balanced polygons (the “degree of balance”) or polygons of length \( k \) (“degree of \( k \)-balance”).

\((\text{PsS, SG: B, Fr})\)


\((\text{SG: CI})\)


\((\text{SD, B})\)


\((\text{SD: Cl})\)


\((\text{SG, SD, VS: B, Fr, Cl, A: Exp})\)

Adolfo Casari
See F. Barahona.

Paul A. Catlin

Thm. 2: If \( \Gamma \) is 4-chromatic, \([-\Gamma]\) contains a subdivision of \([-K_4]\) (an “odd-\(K_4\)”). \(\text{[Question.}\) Can this possibly be a signed-graph theorem? For instance, should it be interpreted as concerning the 0-free (signed) chromatic number of \(-\Gamma\)?] (\(\text{p: col}\))

\((\text{SD, SG, GG: A, I})\)


Connects a problem on common covectors of two subspaces of \( \mathbb{R}^m \), and more generally of a pair of oriented matroids, to the problem of sign-solvability of a matrix and the even-cycle problem for signed digraphs. \((\text{Sol, sd: P, Alg})\)


Possible generalizations to oriented matroids of sign-nonsingularity of a matrix. \((\text{Sol, SD: P})\)

Seth Chaiken

\((\text{SD, SG, GG: A, I})\)


Determining whether a gain graph with real multiplicative gains has a balanced polygon, i.e., is not contrabalanced, is NP-hard. So is determining whether a real matrix is projectively equivalent to the incidence matrix of a contrabalanced real gain graph. \((\text{GN, Bic: I, Alg})\)

Vijaya Chandru, Collette R. Coullard, and Donald K. Wagner

Determining whether a gain graph with real multiplicative gains has a balanced polygon, i.e., is not contrabalanced, is NP-hard. So is determining whether a real matrix is projectively equivalent to the incidence matrix of a contrabalanced real gain graph. \((\text{GN, Bic: I, Alg})\)
Chung-Chien Chang and Cheng-Ching Yu
Modified the method of Iri, Aoki, O’Shima, and Matsuyama (1979a) of constructing the diagnostic signed digraph, e.g., by considering transient and steady-state situations. (SD: Appl, Ref)

Gerard J. Chang

A. Charnes, M. Kirby, and W. Raike

A. Charnes and W.M. Raike

Gary Chartrand
See also M. Behzad.

Gary Chartrand, Heather Gavlas, Frank Harary, and Michelle Schultz
Net degree sequences (i.e., \( d^+ - d^- \); called “signed degree sequences”) of signed simple graphs. A Havel–Hakimi-type reduction formula, but with an indeterminate length parameter [improved in Yan, Lih, Kuo, and Chang (1997a)]; a determinate specialization to complete graphs. A necessary condition for a sequence to be a net degree sequence. Examples: paths, stars, double stars. [Continued in Yan, Lih, Kuo, and Chang (1997a).]
[This is a special case of weighted degree sequences of \( K_n \) with integer edge weights chosen from a fixed interval of integers. In this case the interval is \([-1,+1]\). There is a theory of such sequences; however, it seems not to yield the exact results obtained here.] (SGw: N)
[One can interpret net degrees as the net indegrees (\( d_{in} - d_{out} \)) of certain bidirected graphs. Change the positive (negative) edges to extroverted (resp., introverted). Then we have the net indegree sequence of an oriented \(-\Gamma\).]

Problem 1. Generalize this paper and Yan, Lih, Kuo, and Chang (1997a) to all bidirected (simple, or simply signed) graphs, especially \( K_n \’s \). Problem 2. Find an Erdős–Gallai-type characterization of net degree sequences of signed simple graphs. Problem 3. Characterize the separated signed degree sequences of signed simple graphs, where the separated signed degree is \((d^+(v), d^-(v))\). Problem 4. Generalize Problem 3 to edge \( k \)-colorings of \( K_n \).] (SG: O: N)

Gary Chartrand, Frank Harary, Hector Hevia, and Kathleen A. McKeon
What is the smallest order of an edge-disjoint union of two (isomorphism types of) simple graphs, \( \Gamma \) and \( \Gamma' \)? Bounds, constructions, and special cases. (The union is called a signed graph with \( \Gamma \) and \( \Gamma' \) as its positive
and negative subgraphs.) Thm. 13: If $\Gamma'$ is bipartite (i.e., the union is balanced) with color classes $V'_1$ and $V'_2$, the minimum order $= \min(|V'_1|, |V'_2|) + \max(|V'_1|, |V'_2|)$.

Guy Chaty


Clarifies the structure of “free cyclic” digraphs and shows they include strong “upper” digraphs (see Harary, Lundgren, and Maybee (1985a)). (SD: Str)

P.D. Chawathe and G.R. Vijayakumar


Jianer Chen, Jonathan L. Gross, and Robert G. Rieper


Ying Cheng


This article studies what are described as $\mathbb{Z}_4$-gain graphs $\Phi$ with underlying simple graph $\Gamma$. [However, see below.] They are regarded as digraphs $D$, the gains being determined by $D$ as follows: $\varphi(u,v) = 1$ or 2 if $(u,v)$ is an arc, 2 or 3 if $(v,u)$ is an arc. [N.B. $\Gamma$ is not uniquely determined by $D$.] Cheng’s “switching” is gain-graph switching but only by switching functions $\eta : V \rightarrow \{0, 2\};$ I will call this “semiswitching”. His “isomorphisms” are vertex permutations that are automorphisms of $\Gamma$; I will call them “$\Gamma$-isomorphisms”. The objects of study are equivalence classes under semiswitching (semiswitching classes) or semiswitching and $\Gamma$-isomorphism (semiswitching $\Gamma$-isomorphism classes). Prop. 3.1 concerns adjacency of vertex orbits of a $\Gamma$-isomorphism that preserves a semiswitching class (call it a $\Gamma$-automorphism of the class). Thm. 4.3 gives the number of semiswitching $\Gamma$-isomorphism classes. Thm. 5.2 characterizes those $\Gamma$-automorphisms of a semiswitching class that fix an element of the class; Thm. 5.3 characterizes the $\Gamma$-isomorphisms $g$ that fix an element of every $g$-invariant semiswitching class.

[Likely the right viewpoint, as is hinted in §6, is that the edge labels are not $\mathbb{Z}_4$-gains but weights from the set $\{\pm 1, \pm 2, \ldots, \pm k\}$ with $k = 2$. Then semiswitching is ordinary signed switching, and so forth. However, I forbear to reinterpret everything here.]

In §6, $\mathbb{Z}_4$ is replaced by $\mathbb{Z}_{2k}$ [but this should be $\{\pm 1,\pm 2,\ldots,\pm k\}$]; semiswitching functions take values $0, k$ only. Generalizations of Sects. 3, 4 are sketched and are applied to find the number of $H$-equivalent matrices of given size with entries $\pm 1, \pm 1, \ldots, \pm k$. ($H$-[or Hadamard] equivalence means permuting rows and columns and scaling by $-1$.)

Ying Cheng and Albert L. Wells, Jr.


A two-digraph is a switching class of $\mathbb{Z}_2$-gain graphs based on $K_n$. (sg, wg, GG: Sw, Aut, E)
HH Hyeong-ah Choi, Kazuo Nakajima, and Chong S. Rim

Vertex biparticity (the fewest vertices to delete to get a bipartite graph) is compared to edge biparticity (for cubic graphs) and studied algorithmically.

(p: Fr)

Debashish Chowdhury

Includes brief survey of how physicists look upon frustration. See esp. §1.3, “An elementary introduction to frustration”, where the signed square lattice graph illustrates balance vs. imbalance; Ch. 20, “Frustration, gauge invariance, defects and SG [spin glasses]”, discussing planar duality (see e.g. Barahona (1982a), “gauge theories”, where gains are in the orthogonal or unitary group (and switching is called “gauge transformation” by physicists), and functions of interest to physicists; Addendum to Ch. 10, pp. 378–379, mentioning results on when the proportion of negative bonds is fixed and on gauge theories.

(Phys: SG, GG, VS, Fr: Exp, Ref)

San Yan Chu
See S.-L. Lee.
V. Chvátal
See J. Akiyama.

F.W. Clarke, A.D. Thomas, and D.A. Waller

Bernard P. Cohen
See J. Berger.

Edith Cohen and Nimrod Megiddo


Given: a gain graph $\Phi$ with gains in $\mathbb{Z}^d$ (a “static graph”). Found: algorithms for (1) connected components and (2) bipartiteness of the covering graph $\hat{\Phi}$ (the “periodic graph”) and, (3) given costs on the edges of $\Phi$, for a minimum-average-cost spanning tree in the covering graph. Many references to related work. (GG( Cov): Alg, Ref)


Preliminary version of (1994a), differing only slightly. (GN: Alg)(sg: O: Alg)


Looking for a closed walk (“cycle”) with gain 0 in a gain digraph with (additive) gains in $\mathbb{Q}^d$. [Cf. Kodialam and Orlin (1991a).] (GD: B: Alg)


Maximize the fraction of demand satisfied by a flow on a network with gains. Positive real gains in §3. Bidirected networks with positive gains in §4; these are more general than networks with arbitrary non-zero real gains. (GN: Alg)(sg: O: Alg)


Charles J. Colbourn and Derek G. Corneil

Deciding switching equivalence of graphs is polynomial-time equivalent to graph isomorphism. (TG: Alg)
L. Collatz
Zbl. 402.05054.
Introducing periodic graphs: these are connected canonical covering graphs \( \Gamma = \Phi \) of finite \( \mathbb{Z}^d \)-gain graphs \( \Phi \). The “spectrum” of \( \Gamma \) is the set of all eigenvalues of \( A(||\Phi||) \) for all possible \( \Phi \). The spectrum, while infinite, is contained in the interval \([-r,r]\) where \( r \) is the largest eigenvalue of each \( A(||\Phi||) \) [the “index” of von Below (1994a)]. The inspiration is tilings.

(GG(Cov): A)

Barry E. Collins and Bertram H. Raven
“Graph theory and structural balance,” pp. 106–109. (PsS: SG: Exp, Ref)

Ph. Combe and H. Nencka
\( \Sigma \) is balanced iff a fundamental system of polygons is balanced [as is well known; see i.a. Popescu (1979a), Zaslavsky (1981b)]. An algorithm [incredibly complicated, compared to the obvious method of tracing a spanning tree] to determine all vertex signings of \( \Sigma \) that switch it to all positive. Has several physics references.

(SG: B, Fr, Alg, Ref)

F.G. Commoner

(SD: B)

Michele Conforti, Gérard Cornuélols, Ajai Kapoor, and Kristina Vušković

(SG: B)

The structure of graphs that are signable to be “without odd holes”: that is, so that each triangle is negative and each chordless polygon of length greater than 3 is positive. Proof based on Truemper (1982a).

(SG: B, Str)

\( \Gamma \) is “universally signable” if it can be signed so as to make every triangle negative and the holes independently positive or negative at will. Such graphs are characterized by a decomposition theorem which leads to a polynomial-time recognition algorithm.

(SG: B, Str)

1999a Even and odd holes in cap-free graphs. J. Graph Theory 30 (1999), 289–308.

(SG: B)

20xxa Triangle-free graphs that are signable without even holes. Submitted

(SG: B)
Michele Conforti, Gérard Cornuéjols, and Kristina Vušković

Michele Conforti and Ajai Kapoor

A new proof of Truemper’s theorem on prescribed hole signs; discussion of applications. (SG: B)

Derek G. Corneil

Gérard Cornuéjols
See also M. Conforti.


The topic is linear optimization over a clutter, esp. a “binary clutter”, which is the class of negative circuits of a signed binary matroid. The class $C_-(\Sigma)$ is an important example (see Seymour 1977a), as is its blocker $bC_-(\Sigma)$ which is the class of minimal balancing edge sets; hence the frustration index $l(\Sigma) =$ minimum size of a member of the blocker.

Ch. 5: “Graphs without odd-$K_5$ minors”, i.e., signed graphs without $-K_5$ as a minor. Some esp. interesting results: Thm. 5.0.7 (special case of Seymour (1977a), Main Thm.): The clutter of negative polygons of $\Sigma$ has the “Max-Flow Min-Cut Property” (Seymour’s “Mengerian” property) iff $\Sigma$ has no $-K_4$ minor. Conjecture 5.1.11 is Seymour’s (1981a) beautiful conjecture (his “weak MFMC” is here called “ideal”). §5.2 reports the partial result of Guenin (1998b). (See also §8.4.)

Def. 6.2.6 defines a signed graph “$G(A)$” of a $0, \pm 1$-matrix $A$, whose transposed incidence matrix is a submatrix of $A$. §6.3.3: “Perfect $0, \pm 1$-matrices, bidirected graphs and conjectures of Johnson and Padberg” (1982a), associates a bidirected graph with a system of 2-variable pseudoboolean inequalities; reports on Sewell (1997a) (q.v.).

§8.4: “On ideal binary clutters”, reports on Cornuéjols and Guenin (20xxa), Guenin (1998a), and Novick and Sebő (1995a) (qq.v.).

(S(M), SG: M, G, I(Gen), O: Exp, Ref, Exr)

Gérard Cornuéjols and Bertrand Guenin
20xxa On ideal binary clutters and a conjecture of Seymour. In preparation

A partial proof of Seymour’s (1981a) conjecture. Main Thm.: A binary clutter is ideal if it has as a minor none of the circuit clutter of $F_7$, $C_-(K_5)$ or its blocker, or $C_-(K_4)$ or its blocker. Important are the lift and extended lift matroids, $L(M, \sigma)$ and $L_0(M, \sigma)$, defined as in signed graph theory. [See Cornuéjols (20xxa), §8.4.] (S(M), SG: M, G)

S. Cosares
See L. Adler.
Collette R. Coullard
See also V. Chandru.

Collette R. Coullard, John G. del Greco, and Donald K. Wagner

§4: §4.1 describes 4 fairly simple types of “legitimate” graph operation that preserve the bicircular matroid. Thm. 4.11 is a converse: if \( \Gamma_1 \) and \( \Gamma_2 \) have the same connected bicircular matroid, then either they are related by a sequence of legitimate operations, or they belong to a small class of exceptions, all having order \( \leq 4 \), whose bicircular matroid isomorphisms are also described. This completes the isomorphism theorem of Wagner (1985a).

§5: If finitely many graphs are related by a sequence of legitimate operations (so their bicircular matroids are isomorphic), then they have contr-balanced real gains whose incidence matrices are row equivalent. These results are also found by a different approach in Shull *et al.* (1989a, 20xxa).


Yves Crama
See also E. Boros.


Balance and switching are used to study pseudo-Boolean functions. (Sects. 2.2 and 4.) (SG: B, Sw)

Yves Crama and Peter L. Hammer

“Adjoint” = unoriented positive part of the line graph of a bidirected graph. “Quadratic graph” = graph that is an adjoint. Recognition of adjoints of bidirected simple graphs is NP-complete. (sg: O: LG: Alg)

Yves Crama, Peter L. Hammer, and Toshihide Ibaraki

§7: Signed hypergraphs, with a surprising generalization of balance. (S(Hyp): B)

Y. Crama, M. Loebl, and S. Poljak

William H. Cunningham
See J. Aráoz.

Dragoš M. Cvetković
See also F.C. Bussemaker and M. Doob.


Pp. 128–130 discuss switching-equivalent graphs. Some of the theory is invariant, hence applicable to two-graphs. [Question. How can this be generalized to signed graphs and their switching classes?]

(TG: A)

**Dragos M. Cvetković, Michael Doob, Ivan Gutman, and Aleksandar Torgašev**


Signed graphs on pp. 44-45. All-negative signatures are implicated in the infinite-graph eigenvalue theorem of Torgašev (1982a), Thm. 6.29 of this book.

(SG, p: A: Exp, Ref)

**Dragoš M. Cvetković, Michael Doob, and Horst Sachs**


§4.6: Signed digraphs with multiple edges are employed to analyze the characteristic polynomial of a digraph. (Signed) switching, too. Pp. 187–188: Exercises involving Seidel switching and the Seidel adjacency matrix. Thm. 6.11 (Doob (1973a)): The even-cycle matroid determines the eigenvaluicity of $-2$. §7.3: “Equiangular lines and two-graphs.”

(SD, p, TG: Sw, A, G: Exp, Exr, Ref)


Appendices update the second, slightly corrected edn. of (1980a), beyond the updating in Cvetković, Doob, Gutman, and Torgašev (1988a). App. B.3, p. 381: mentions work of Vijayakumar (q.v.). P. 422: Pseudo-inverse graphs $(A(\Gamma))^{-1} = A(\Sigma)$ for some balanced $\Sigma$; $|\Sigma|$ is the “pseudo-inverse” of $\Gamma$.

(SD, p, TG: A, Sw, G, B: Exp, Exr, Ref)

**Dragoš Cvetković, Michael Doob, and Slobodan Simić**


(SG: LG, A(LG), Aut(LG))

**Dragoš M. Cvetković and Slobodan K. Simić**


**E. Damiani, O. D’Antona, and F. Regonati**


Dowling lattices are an example.

(gg: M: N)

**George B. Dantzig**


Prabir Das and S.B. Rao

Given an all-negative bidirected $K_n$ and a positive integer $f_i = 2g_i$ for each vertex $v_i$. There is a connected subgraph having in-degree and out-degree $= g_i$ at $v_i$ iff there is a $g$-factor of introverted and one of extroverted edges and the degrees satisfy a complicated degree condition. Generalizes Thm. 1 of Báncfalvi and Báncfalvi (1968a). [See Bang-Jensen and Gutin (1997a) for how to convert an edge 2-coloring to an orientation of an all-negative graph and for further developments on alternating walks.] (p: o)

James A. Davis


James A. Davis and Samuel Leinhardt

Analysis of a sociological theory incorporating structural balance in relation to both randomly generated and observational data. (PsS: SG)

A.C. Day, R.B. Mallion, and M.J. Rigby

A clumsy but intriguing way of representing some signed (or more generally, $\mathbb{Z}_n$-weighted) graphs: via 2-page (or, $n$-page) looseleaf book embedding (all vertices are on the spine and each edge is in a single page), with an edge in page $k$ weighted by the “sheet parity index” $\alpha_k = (-1)^k$ (or, $e^{2\pi ik/n}$). (Described in the [unnecessary] terminology of an $n$-sheeted Riemann surface.) [A $\mathbb{Z}_n$-weighted) graph has such a representation iff the subgraph of edges with each weight is outerplanar.]

A variation to get switching classes of signed polygons: replace $\alpha_k$ by the “connectivity parity index” $\sigma_k^\alpha$ where $\sigma_k = \text{number of edges in page } k$. [The variation is valid only for polygons.] [Questions vaguely suggested by these procedures: Which signed graphs can be switched so that the edges of each sign form an outerplanar graph? Also, the same for gain graphs. And there are many similar questions: for instance, the same ones with “outerplanar” replaced by “planar.”]

Anne Delandtsheer

John G. del Greco
See also C.R. Coullard.
How to decide, given a matroid $M$ and a biased graph $\Omega$, whether $M = G(\Omega)$. (GG: M)

B. Derrida, Y. Pomeau, G. Toulouse, and J. Vannimenus
1979a Fully frustrated simple cubic lattices and the overblocking effect. J. Physique 40 (1979), 617–626. (SG: Phys, Fr)

Michel Marie Deza and Monique Laurent
A main object of interest is the cut polytope, which is the bipartite subgraph polytope (see Barahona, Grötschel, and Mahjoub (1985a)) of $K_n$, i.e. the balanced subgraph polytope (Poljak and Turzik (1987a)) of $-K_n$. §4.5, “An application to statistical physics”, briefly discusses the spin glass application. §26.3, “The switching operation”, discusses graph switching and its generalization to sets. §30.3, “Circulant inequalities”, mentions Poljak and Turzik (1987a, 1992a). No explicit mention of signed graphs. (p: fr: G: Exp)

Persi Diaconis
See K.S. Brown.

V. Di Giorgio

Yvo M.I. Dirkx and M.R. Rao

Michael Doob
See also D.M. Cvetković.
A readable, tutorial introduction to (1973a) (without matroids). (ec: LG, I, A(LG))

Along with Simões-Pereira (1973a), introduces to the literature the even-cycle matroid \(G(–\Gamma)\) [previously invented by Tutte, unpublished]. The multiplicity of \(-2\) as an eigenvalue (in characteristic 0) equals the number of independent even polygons \(n - \text{rk} G(–\Gamma)\). In characteristic \(p\) there is a similar theorem, but the pertinent matroid is \(G(\Gamma)\) if \(p = 2\) and, when \(p \mid n\), the matroid has rank 1 greater than otherwise [a fact that mystifies me].

(EC: LG, I, A(LG))


Michael Doob and Dragos Cvetković


Patrick Doreian, Roman Kapuscinski, David Krackhardt, and Janusz Szczyzypula


They propose indices for clusterability that generalize the frustration index. Fix \(k \geq 2\) and \(\alpha \in [0, 1]\). For a partition \(\pi\) of \(V\) into \(k\) parts, they define \(P(\pi) := \alpha n_+ + (1 - \alpha)n_-\), where \(n_+ := |E_+\langle \pi \rangle| = \text{number of positive edges between parts and } n_- := |E_-\langle \pi \rangle| = \text{number of negative edges within parts. The first proposed measure is } \min \{P(\pi)\}, \text{minimized over } k\text{-partitions. [Call this } P_{k,\alpha}.\text{] A second suggestion is the \textit{negation-minimal index of generalized imbalance} [i.e., of clusterability], smallest number of edges whose negation (equivalently, deletion) makes } \Sigma \text{ clusterable; it } = \min_{k \geq 2} 2P_{k,\frac{1}{2}}. \text{[Note that } P(\pi) \text{ effectively generalizes the Potts Hamiltonian as given by Welsh (1993a).}\

\textbf{Question.} \text{Does } P(\pi) \text{ fit into an interesting generalized Potts model?} \text{[} P(\pi) \text{ also resembles the Potts Hamiltonian in} \]
Fischer and Hertz (1991a) (q.v. for a related research question).]

They employ a local optimization algorithm to evaluate $P_{k,\alpha}$ and find an optimal partition: random descent from partition to neighboring partition, where $\pi$ and $\pi'$ are neighbors if they differ by transfer of one vertex or exchange of two vertices between two parts. This was found to work well if repeated many times. [A minimizing partition into at most $k$ parts is equivalent to a ground state of the $k$-spin Potts model in the form given by Welsh (1993a), but not quite of that in Fischer and Hertz (1991a).]

Terminology: $P(\pi)$ is called the “criterion function” [more explicitly, one might call it the ‘clusterability (adjusted by $\alpha$)’ of $\pi$]; clusterability is “$k$-balance” or “generalized balance”. The partition’s parts are “plus-sets”. Signed digraphs are employed in the notation but direction is ignored.

(SD: sg: B, Cl: Fr(Gen), Alg, PsS)


Similar to (1996a). Some lesser theoretical detail; some new examples. The $k$-clusterability index $P_{k,\alpha}$ (see (1996a)) is compared for different values of $k$, seeking the minimum. [But for which value(s) of $\alpha$ is not stated.] Interesting observation: optimal values of $k$ were small. It is said that positive edges between parts are far more acceptable socially than negative edges within parts [thus, in the criterion function $\alpha$ should be rather near 1].

(SD: sg: B, Cl: Fr(Gen), Alg, PsS)

W. Dörfler


(SG: Cov, LG)(SD, S(Hyp): Cov)


Lynne L. Doty

See F. Buckley.

Peter Doubilet


Peter Doubilet, Gian-Carlo Rota, and Richard Stanley


Section 5.3: Brief gain-graphic treatment of Dowling lattices. (GG: M)
T.A. Dowling
Pp. 221–223: The first intimations of Dowling lattices/geometries/matroids,
as in (1973a, 1973b), and their higher-weight relatives (see Bonin 1993a).

1973a A \(q\)-analog of the partition lattice. Ch. 11 in: J. N. Srivastava et al.,
Linear-algebraic progenitor of (1973b). Treats the Dowling lattice of group
\(GF(q)^\times\) as naturally embedded in \(PG^{n-1}(q)\). Interesting is p. 105, Remark:
One might generalize some results to any ambient (simple) matroid.

47 #8369. Zbl. 264.05022.
Introduces the Dowling lattices of a group, treated as lattices of group-
labelled partial partitions. Equivalent to the bias matroid of complete \(\mathcal{G}\)-gain
graph \(\mathcal{G}K_n^\bullet\). [The gain-graphic approach was known to Dowling (1973a, p.
109) but first published in Doubilet, Rota, and Stanley (1972a).] Isomor-
phism, vector representation, Whitney numbers and characteristic polyno-
mial. [The first and still fundamental paper.]

Pauline van den Driessche
See van den Driessche (under ‘V’).

J.M. Drouffe
See R. Balian.

Richard A. Duke, Paul Erdős, and Vojtěch Rödl
776.05057.
All graphs are simple. This is one of four related papers that prove extremal
results concerning subgraphs of \(-\Gamma\) within which every two edges belong to
a balanced polygon of length at most \(2k\), for all or particular \(k\). Typical
theorem: Let \(F_l(n,m) = \) the largest number \(m' = m'(n,m)\) such that every
\(-\Gamma\) with \(|V| = n\) and \(|E| \geq m\) has a subgraph \(\Sigma'\) with \(|E'| = m'\) in which
every two edges belong to a balanced polygon of length at most \(l\). For \(m = m(n) \geq n^{3/2}\),
there is a constant \(c_3 > 0\) such that \(F_l(n,m) \leq c_3m^2n^{-2}\) for
all \(l\). (§2, (2).) [Problem. Extend these extremal results in an interesting way
to arbitrary signed simple graphs, or to simply signed graphs (no repeated
edges with the same sign). (Merely allowing positive edges in addition to
negative ones just makes the problem easier. Something more is required.])

Arne Dür
1986a Möbius Functions, Incidence Algebras and Power Series Representations. Lecture
592.05006.
Dowling lattices are an example of a categorial approach to incidence-algebra
techniques in Ch. IV, §7. Computed are the characteristic polynomial and
second kind of Whitney numbers. Binomial concavity, hence unimodality of
the latter [cf. Stonesifer (1975a)] is proved by showing that a suitable generating polynomial has only distinct, negative roots [cf. Benoumhani (1999a)].

Paul H. Edelman and Victor Reiner

Characterizes all $\Sigma \supseteq +K_n$ whose bias matroid $G(\Sigma)$ is supersolvable, free, or inductively free. Essentially, iff the negative links form a threshold graph. [Continued in Bailey (20xxa). Generalized in part to arbitrary gain groups in Zaslavsky (20xxh).]


Paul H. Edelman and Michael Saks

Given $\Gamma$ and abelian group $\mathfrak{A}$. Vertex and edge labelings $\lambda : V \to \mathfrak{A}$ and $\eta : E \to \mathfrak{A}$ are “compatible” if $\lambda(v) = \sum_{e \in \delta(v)} \eta(e)$ for every vertex $v$, the sum taken over all edges incident with $v$. $\lambda$ is “admissible” if it is compatible with some $\eta$. Admissible vertex labellings are characterized (differently for bipartite and nonbipartite graphs) and the number of edge labelings compatible with a given vertex labelling is computed. [Dual in a sense to Gimbel (1988a).]


Followed up by much work, e.g., Witzgall and Zahn (1965a); see Ahuja, Magnanti, and Orlin (1993a) for some references.


Alludes to the polyhedron of Edmonds and Johnson (1970a).

Richard Ehrenborg and Margaret A. Readdy

An abstract additive approach to the characteristic polynomial, applied in particular to “divisor Dowling arrangements” of hyperplanes and certain
interpolating arrangements. Let $\Phi = G_1 K_1 \cup \cdots \cup G_n K_n$, where $V(K_i) = \{v_1, \ldots, v_i\}$ and $G_1 \supseteq \cdots \supseteq G_n$ is a chain of subgroups of a gain group $G = G_1$. When $G$ is finite cyclic, the complex hyperplane representation of $\Phi^*$ is a “divisor Dowling arrangement”. Its polynomial equals the chromatic polynomial of $\Phi^*$, which is easily computed via gain-graph coloring without the restriction to cyclic gain group. The same appears to be true for the other arrangements treated herein. 


The Dowling lattice is that of a finite cyclic group $Z_k$. Thm. 4.9 is a recursive formula for its flag $h$-vector (in the form of the ab-index). Thm. 5.2 is a similar formula for the $c,2d$-index of the face lattices of the real root system arrangements $A_n$ and $B_n$, whose intersection lattices are the Dowling lattices of $Z_1$ and $Z_2$. §6 presents a combinatorial description of the face lattice of $B_n$ [which it is interesting to compare with that in Zaslavsky (1991b)].

A. Ehrenfeucht, T. Harju, and G. Rozenberg


The “heierarchical structure” of a switching class of skew gain graphs based on $K_n$.


Given a gain graph $(K_n, \varphi, \mathcal{G})$, a word $w$ in the oriented edges of $K_n$ has a gain $\varphi(w)$; call this $\psi_w(\varphi)$. A “free invariant” is a $\psi_w$ that is an invariant of switching classes. Thm.: There is a number $d = d(K_n, \mathcal{G})$ such that the group of free invariants is generated by $\psi_w$ with $w = z_1^d \cdots z_i^d u_1 \cdots u_l$ where $w_i$ are triangular cycles (directed!) and $u_i$ are commutators. [The whole paper applies mutatis mutandis to arbitrary graphs, the triangular cycles being replaced by any set of cycles containing a fundamental system.] Dictionary: “Inversive 2-structure” = gain graph based on $K_n$.

Andrzej Ehrenfeucht and Grzegorz Rozenberg


They prove that a complicated definition of “reversible dynamic labeled 2-structure” $G$ amounts to a complete graph with a set, closed under switching, of twisted gains in a gain group $\Delta$. The twist is a gain-group automorphism $\alpha$ such that $\lambda(e; x, y) = [\alpha \lambda(e; y, x)]^{-1}$, $\lambda$ being the gain function. Dictionary: their “domain” $D = \text{vertex set}$, “labeling function” $\lambda$ (or equivalently, $g$) = gain function, “alphabet” = gain group, “involution” $\delta = \alpha \circ \alpha$ inversion, “$\delta$-selector” $\delta = \text{switching function}$, “transformation induced by $\delta$” = switching by $\delta$; a “single axiom” d.l. 2-structure consists of a single switching class.

Further, they investigate “clans” of $G$. Given $g$ (i.e., $\lambda$), deleting identity-gain edges leaves isolated vertices (“horizons”) and forms connected components, any union of which is a “clan” of $g$. A clan of $G$ is any clan of any $g \in G$.


Combinations and decompositions of complete graphs with twisted gains.

Kurt Eisemann

Joyce Elam, Fred Glover, and Darwin Klingman

David P. Ellerman

M.N. Ellingham

Main theorem ($\S$2) characterizes, given two signings of $K_n$ (where $n$ may be infinite) and a vertex set $S$, when switching $S$ makes the signings isomorphic. [Problem 1. Generalize to other underlying graphs. Problem 2. Prove an analog for bidirected $K_n$’s.] A corollary ($\S$3) characterizes when vertices $u, v$ of $\Sigma = (K_n, \sigma)$ satisfy $\Sigma^\{u\} \cong \Sigma^\{v\}$ and discusses when in addition no automorphism of $\Sigma$ moves $u$ to $v$. All is done in terms of Seidel (graph) switching (here called “vertex-switching”) of unsigned simple graphs.

(k: sw, TG)


Deepens the folded-cube theory of Ellingham and Royle (1992a). Nicely generalizing Stanley (1985a), the number of subgraphs of a signed $K_n$ that are isomorphic to a fixed signed $K_m$ is reconstructible from the $s$-vertex switching deck if the Krawtchouk polynomial $K_n^s(x)$ has no even zeros between 0 and $m$. (Closely related to Krasikov and Roditty (1992a), Theorems 5
Remark 4: balance equations (Krasikov and Roditty (1987a)) and Krawtchouk polynomials both reflect properties of folded cubes. All is done in terms of Seidel switching of unsigned simple graphs. It seems clear that the folded cube appears because it corresponds to the effect of switchings on signatures of $K_n$ (or any connected graph), since switching by $X$ and $X^c$ have the same effect. For the bidirected case (Problem 2 under Stanley (1985a)), the unfolded cube should play a similar role. Question. When treating a general underlying graph $\Gamma$, will a polynomial influenced by $\text{Aut}\, \Gamma$ replace the Krawtchouk polynomial?

M.N. Ellingham and Gordon F. Royle


Reconstruction of induced subgraph numbers of a signed $K_n$ from the $s$-vertex switching deck, dependent on linear transformation and thence Krawtchouk polynomials as in Stanley (1985a). The role of those polynomials is further developed. Done in terms of Seidel switching of unsigned simple graphs, with the advantage of reconstructing arbitrary subgraph numbers as well. A gap is noted in Krasikov and Roditty (1987a), proof of Lemma 2.5. [Methods and results are closely related to Krasikov (1988a) and Krasikov and Roditty (1987a, 1992a)].

Gernot M. Engel and Hans Schneider


R.C. Entringer


See Erdős, Rubin, and Taylor (1980a).

H. Era

See J. Akiyama.

Pál Erdős [sometimes, Paul Erdős]

See also B. Bollobás and R.A. Duke.


P. 119 mentions the theorem of Duke, Erdős, and Rödl (1991a) on even polygons.

Pp. 120–121 mention (amongst similar problems) a theorem of Erdős and Hajnal (source not stated): Every all-negative signed graph with chromatic number $N_1$ contains every finite bipartite graph [i.e., every finite, balanced,
all-negative signed graph]. [Problem. Find generalizations to signed graphs. For instance: Conjecture. Every signed graph with chromatic number \( r_1 \), that does not become antibalanced upon deletion of any finite vertex set, contains every finite, balanced signed graph up to switching equivalence.]

[The MR review: “this is one of the best collections of problems that Erdös has published.”]

P. Erdős, R.J. Faudree, A. Gyárfás, and R.H. Schelp

A large, nonbipartite, 2-connected graph with large minimum degree contains a polygon of given odd length or is one of a single type of exceptional graph. [Question. Can this be generalized to negative polygons in unbalanced signed graphs?]

P. Erdős, E. Győri, and M. Simonovits

Assume \(|\Sigma|\) simple of order \( n \) and \( \not\exists \) a fixed graph \( \Delta \). Results on frustration index \( l(\Sigma) \) of antibalanced \( \Sigma \) if \( \Delta \) is 3-chromatic, esp. \( C_3 \). Thm.: If \(|E| > n^2/5 - o(n^2)\), then \( l(\Sigma) < n^2/25 - o(n^2) \). Conjecture (Erdős): For \( \Delta = C_3 \) the hypothesis on \(|E|\) is unnecessary. [Question 1(a). Is the answer different when \( \Sigma \) need not be antibalanced? Question 2(a). Exclude a fixed signed graph whose signed chromatic number = 1. Question 3(a). In particular, exclude \(-K_3\). Question 4(a). Exclude \(-K_1\). Question 5(a). Exclude an unbalanced \( C_1 \). Questions 1–5(b). Even if \( l(\Sigma) \) cannot be estimated, is there always an extremal graph that is antibalanced—as when no graph is excluded, by Petersdorf (1966a)?]

Paul Erdős, Arthur L. Rubin, and Herbert Taylor

Rubin’s block theorem (Thm. R, p. 136): a block graph, not complete or an odd polygon, contains an induced even polygon with at most one chord. [See also Entringer (1985a).] [Question. Does this generalize to signed graphs, Rubin’s block theorem being the antibalanced case? Rubin’s 2-choosability theorem, p. 132, is also tantalizingly reminiscent of antibalanced graphs, but in reverse.]

Cloyd L. Ezell

Arthur M. Farley and Andrzej Proskurowski
1981a Computing the line index of balance of signed outerplanar graphs. Proc. Twelfth

Calculating frustration index is NP-complete, since it is more general than max-cut. However, for signed outerplanar graphs with bounded size of bounded faces, it is solvable in linear time. [It is quickly solvable for signed planar graphs. See Katai and Iwai (1978a), Barahona (1981a, 1982a), and more.]

*SG: Fr*

**M. Farzan**


A “double cover of a graph” means the double cover of a signing of a simple graph.

*sg: Cov, Aut*

**R.J. Faudree**

See P. Erdős.

**Katherine Faust**

See S. Wasserman.

**N.T. Feather**


*PsS: B: Exp* (WD: B)

**Lori Fern, Gary Gordon, Jason Leasure, and Sharon Pronchik**

20xxa Matroid automorphisms and symmetry groups. Submitted.

Consider a subgroup $W$ of the hyperoctahedral group $O_n$ that is generated by reflections. Let $M(W)$ be the vector matroid of the vectors corresponding to reflections in $W$. The possible direct factors of any automorphism group of $M(W)$ are $S_k$, $O_k$, and $O_k^+$. The proof is strictly combinatorial, via signed graphs.

*SG: Aut, G*

**Miroslav Fiedler**


*SG: G*


Miroslav Fiedler and Vlastimil Ptak


Joseph Fiksel

Steven D. Fischer

§1.2: “Signed posets”. Definition of signed poset: a positively closed subset of the root system $B_n$ whose intersection with its negative is empty. (Following Reiner (1990).) Equivalent to a partial ordering of $\pm [n]$ in which negation is a self-duality and each dual pair of elements is comparable. [This is really a special type of signed poset. The latter restriction does not hold in general.] Relevant contents: Ch. 2: “Cohen-Macaulay signed posets”, §2.2: “EL-labelings of posets and signed posets”, and shellability. Ch. 3: “Euler characteristics”, and a fixed-point theorem. §5.1: “The homology of the signed posets $S_H$” (a particular example). App. A: “Open problems”, several concerning signed posets.

[Partially summarized by Hanlon (1996a).] (S: sg, o, G, N)

K.H. Fischer and J.A. Hertz

§2.5, “Frustration”, discusses the spin glass Ising model (essentially, signed graphs) in square and cubical lattices, including the “Mattis model” (a switching of all positive signs), as well as a vector analog, the “XY” model (planar spins) and (p. 46) even a general gain-graph model with switching-invariant Hamiltonian. From the point of view of physics (mainly theoretical physics).

(Phys: SG: Fr, Sw: Exp, Ref)

§3.7: “The Potts glass”. The Hamiltonian (without edge weights) is $H = -\frac{1}{2} \sum \sigma(e_{ij})(k\delta(s_i,s_j) - 1)$. It is not clear that the authors intend to permit negative edges. If they are allowed, $H$ is rather like Doreian and Mrvar’s (1996a) $P(\pi)$. Question. Is there a worthwhile generalized signed and
weighted Potts model with Hamiltonian that specializes both to this form of $H$ and to $P$? [Also cf. Welsh (1993a) on the Ashkin–Teller–Potts model.] (Phys: sg, cl: Exp)

P.C. Fishburn and N.J.A. Sloane

The maximum frustration index of a signed $K_t$; $t$, which equals the covering radius of the Gale–Berlekamp code, is evaluated for $t \leq 10$, thereby extending results of Brown and Spencer (1971a). See Table 1. (sg: Fr)

Claude Flament

Signed graphs are treated on pp. 126–129. (SG: B, PsS: Exp)


Ch. III: “Processus d’équilibriation.” (SG: K: B, Alg)


C.M. Fortuin and P.W. Kasteleyn

Most of the paper recasts classical physical and other models (percolation, ferromagnetic Ising, Potts, graph coloring, linear resistance) in a common form that is generalized in §7, “Random cluster model”. The “cluster (generating) polynomial” $Z(\Gamma; p, \kappa)$, where $p \in \mathbb{R}^E$ and $\kappa \in \mathbb{R}$, is a 1-variable specialization of the general parametrized dichromatic polynomial. In the notation of Zaslavsky (1992b) it equals $Q_{\Gamma}(q, p; \kappa, 1)$, where $q_e = 1 - p_e$. Thus it partially anticipates the general polynomials of Przytycka and Przytycki (1988a), Traldi (1989a), and Zaslavsky (1992b) that were based on Kauffman’s (1989a) sign-colored Tutte polynomial. A spanning-tree expansion is given only for the resistance model. A feature [that seems not to have been taken up by subsequent workers] is the differentiation relation (7.7) connecting $\partial \ln Z / \partial q_e$ with [I think!] the expectation that the endpoints of $e$ are disconnected in a subgraph. [Grimmett (1994a) summarizes subsequent work in the probabilistic direction.] (sgc: Gen: N, Phys)

J.-C. Fournier

Aviezri S. Fraenkel and Peter L. Hammer

András Frank

A “conservative $\pm 1$-weighting” of $G$ is an edge labelling by $+1$’s and $-1$’s so that in every polygon the sum of edge weights is nonnegative. It is a tool in several theorems. [Related: Ageev, Kostochka, and Szegedi (1995a), Sebő (1990a).] (SGw: Str, Alg: Exp, Ref)

Howard Frank and Ivan T. Frisch


Ove Frank and Frank Harary

The model: an edge is present with probability $\alpha$ and positive with probability $p$. The expected value is computed for two kinds of measures of imbalance: the number of balanced triangles (whose variance is also given), and the number of induced subgraphs of order 3 having specified numbers of positive and negative edges. (SG: Rand, Fr)

Ivan T. Frisch
See H. Frank.

Toshio Fujisawa

Satoru Fujishige
See K. Ando.

David Gale
See also A. J. Hoffman.

David Gale and A.J. Hoffman

Marianne L. Gardner [Marianne Lepp]
See R. Shull.

Michael Gargano and Louis V. Quintas
Characterizes balance in abelian gain graphs. [See Harary, Lindström, and Zetterström (1982a).] Very simple results on existence, for a given graph, of balanced nowhere-zero gains from a given abelian group. [Elementary, if one notes that such gains exist iff the graph is $|G|$-colorable, $G$ being the gain group]. Comparison with the approach of Sampathkumar and Bhave (1973a). Dictionary: “Symmetric $G$-weighted digraph” = gain graph with gains in the (abelian) group $G$. “Weight” = gain. “Non-trivial” (of the gain function) = nowhere zero.

Michael L. Gargano, John W. Kennedy, and Louis V. Quintas  

An abelian gain graph $\Phi$ is cobalanced (here called “cut-balanced”) if the sum of gains on the edges of each coherently oriented cutset is 0. [This generalizes Kabell (1985a).] Given $\Phi$ with $||\Phi||$ embedded in a surface, the surface dual graph is given gains by a right-rotation rule, thus forming a surface dual $\Phi^*$ of $\Phi$. [This appears to require that the surface be orientable. Note that cobalance generalizes to nonabelian gains on orientably embedded graphs, since the order of multiplication for the gain product on a cutset is given by the embedding.] Thm. 3.2: For a plane embedding of $\Phi$, $\Phi$ is cobalanced if $\Phi^*$ is balanced. Thm. 3.4 restates as criteria for cobalance of $\Phi$ the standard criteria for balance of $\Phi^*$, as in Gargano and Quintas (1985a). More interesting are “well-balanced” graphs, which are both balanced and cobalanced. Problem. Characterize them. Dictionary (also see Gargano and Quintas 1985a): Balance is called “cycle balance”.

Gilles Gastou and Ellis L. Johnson  

§10 introduces the co-postman and “odd circuit” problems, treated more thoroughly in Johnson and Mosterts (1987a) (q.v). “Odd” edges and circuits are precisely negative edges and polygons in an edge signing. The “odd circuit matrix” represents $L(\Sigma)$ (p. 30). The “odd circuit problem” is to find a shortest negative polygon; a simple algorithm uses the signed covering graph (pp. 30–31). The “Fulkerson property” may be related to planarity and $K_5$ minors [which suggests comparison with Barahona (1990a), §5].

Heather Gavlas  
See G. Chartrand.

Joseph Genin and John S. Maybee  

A.M.H. Gerards  

If an antibalanced, unbalanced signed graph has no homomorphism into its shortest negative polygon, then it contains a subdivision of $-K_4$ or of a loose $\pm C_3$ (here called an “odd $K_4$” and an “odd $K_2^3$”). (A loose $\pm C_n$ consists of $n$ negative digons in circular order, each adjacent pair joined either at a common vertex or by a link.) [Question. Do the theorem and proof carry
over to any unbalanced signed graph?] Other results about antibalanced signed graphs are corollaries. Several interesting results about signed graphs are lemmas. 


Let \(\Sigma\) be antibalanced and without isolated vertices and contain no subdivision of \(-K_4\). Then max. stable set size = min. cost of a cover by edges and negative polygons. Also, min. vertex-cover size = max. profit of a packing of edges and negative polygons. Also, weighted analogs. *Question. Do the theorem and proof extend to any \(\Sigma\)?*  


The proof of Lemma 3 uses a signed graph.


(Very incomplete annotation.) Thm.: Given \(\Sigma\), the set \(\{x \in \mathbb{R}^n : d_1 \leq x \leq d_2, b_1 \leq I(\Sigma)^T x \leq b_2\}\) has Chvatal rank \(\leq 1\) for all integral vectors \(d_1, d_2, b_1, b_2\), iff \(\Sigma\) contains no subdivided \(-K_4\).  


Signed graphs used to prove Tutte's theorem. The signed-graph matroid employed is the extended lift matroid ("extended even cycle matroid"). The main theorem (Thm. 2): Let \(\Sigma\) be a signed graph with no \(-K_4\), \(\pm K_3\), \(-Pr_3\), or \(\Sigma_4\) link minor; then \(\Sigma\) can be converted by Whitney 2-isomorphism operations ("breaking" = splitting a component in two at a cut vertex, "glueing" = reverse, "switching" = twisting across a vertex 2-separation) to a signed graph that has a balancing vertex ("blocknode"). Here \(\Sigma_4\) consists of \(+K_4\) with a 2-edge matching doubled by negative edges and one other edge made negative.

More translation: His "\(\Sigma\)" is our \(E_\cdot\). "Even, odd" = positive, negative (for edges and polygons). "Bipartite" = balanced; "almost bipartite" = has a balancing vertex.

**A.M.H. Gerards and M. Laurent**


Thm. 5.1: The collection of negative polygons of \(\Sigma\) is box \(\frac{1}{d}\)-integral for some/any integer \(d \geq 2\) iff it does not contain \(-K_4\) as a link minor.  


**A.M.H. Gerards, L. Lovász, A. Schrijver, P.D. Seymour, and K. Truemper**

published—though one hopes not! See Seymour (1995a) for description of two main theorems.]  

A.M.H. Gerards and A. Schrijver  
Essential, major theorems. The (extended) lift matroid of a signed graph is one of the objects studied. Some of this material is published in Gerards (1990a). This paper is in the process of becoming Gerards, Lovász, et al. (1990a).  

A subsidiary result: If \( -\Gamma \) contains no subdivided \( -K_4 \), then \( \Gamma \) is \( t \)-perfect.  

A.M.H. Gerards and F.B. Shepherd  
Extension of Gerards (1989a). An “odd-\( K_4 \) is a graph whose all-negative signing is a subdivided \( -K_4 \). A “bad-\( K_4 \)” is an odd-\( K_4 \) which does not consist of exactly two undivided \( K_4 \) edges that are nonadjacent while the other edges are replaced by even paths. Thm. 1: A graph that contains no bad-\( K_4 \) as a subgraph is \( t \)-perfect. Thm. 2 characterizes the graphs that are subdivisions of 3-connected graphs and contain an odd-\( K_4 \) but no bad-\( K_4 \). [The fact that ‘badness’ is not strictly a parity property weighs against the possibility that Gerards (1989a) extends well to signed graphs.]  

Anna Maria Ghirlanda  
See L. Muracchini.  

A. Ghouila-Houri  
See C. Berge.  

Rick Giles  
MR 82m:05075a,b,c. Zbl. 468.90053, 468.90054, 468.90055.  

Mukhtiar Kaur Gill [Mukti Acharya]  
See also B.D. Acharya.  


Most of the results herein have been published separately. See Gill (1981a, 1981b), Gill and Patwardhan (1981a, 1983a, 1986a).  (SG, SD: B, LG, A)

M.K. Gill and B.D. Acharya
(SG: A)

(SG: B, G)

M.K. Gill and G.A. Patwardhan
(SG: LG)

(SG: LG)

(SG, Sw)

John Gimbel

The topic is “induced” edge labellings, that is, $w(e_{uv}) = f(u)f(v)$ for some $f : V \rightarrow \mathbb{A}$. The number of $f$ that induce a given induced labelling, the number of induced labellings, and a characterization of induced labellings. All involve the 2-torsion subgroup of $\mathbb{A}$, unless $\Gamma$ is bipartite. The inspiration is dualizing magic graphs. [Somewhat dual to Edelman and Saks (1979a).]

(T: VS(1): E)

Terry C. Gleason
See also D. Cartwright.

Terry C. Gleason and Dorwin Cartwright
(SG: Cl, A)

Fred Glover
See also J. Elam.

F. Glover, J. Hultz, D. Klingman, and J. Stutz
(GN: Alg, M( bases): Exp, Ref)

Fred Glover and D. Klingman
(GN: B, I)

(GN: M( bases), g)

Fred Glover, Darwin Klingman, and Nancy V. Phillips
Textbook. See especially Ch. 5: “Generalized networks.” (GN: Alg: Exp)

F. Glover, D. Klingman, and J. Stutz
1973a Extensions of the augmented predecessor index method to generalized network problems. Transportation Sci. 7 (1973), 377–384. (GN: M( bases), m)

C.D. Godsil

If $T$ is a tree with a perfect matching, then $A(T)^{-1} = A(\Sigma)$ where $\Sigma$ is balanced and $|\Sigma| \geq \Gamma$. Question. When does $|\Sigma| = \Gamma$? [Solved by Simion and Cao (1989a).] [Cf. Buckley, Doty, and Harary (1984a) and, for a different notion, Greenberg, Lundgren, and Maybee (1984b).] (sg: A, B)

J.M. Goethals
See also P.J. Cameron.

Jay R. Goldman and Louis H. Kauffman

The parametrized Tutte polynomial [as in Zaslavsky (1992b) et al.] of an $\mathbb{R}^*$-weighted graph is used to define a two-terminal “conductance”. Interpreting weights as crossing signs in a planar link diagram with two blocked regions yields invariants of tunnel links. [Also see Kauffman (1997a).] (SGw: Gen: N, Knot, Phys)

Richard Z. Goldstein and Edward C. Turner

Harry F. Gollub

Martin Charles Golumbic

Further results on chordal bipartite graphs. Their properties imply standard properties of ordinary chordal graphs. [See (1980a) for more.] (The “only if” portion of Thm. 4 is false, according to (1980a), p. 267.) (sg: b, cov)


§12.3: “Perfect elimination bipartite graphs,” and §12.4: “Chordal bipartite graphs,” expound perfect elimination and chordality for bipartite graphs from Golumbic and Goss (1978a) and Golumbic (1979a). In particular, Cor. 12.11: A bipartite graph is chordal bipartite iff every induced subgraph has perfect edge elimination scheme. [Problem. Guided by these results, find a signed-graph generalization of chordality that corresponds to supersolvability and perfect vertex elimination (cf. Zaslavsky (20xxh)).] (sg: b, cov)

Martin Charles Golumbic and Clinton F. Goss
A perfect edge elimination scheme is a bipartite analog of a perfect vertex elimination scheme. A chordal bipartite graph is a bipartite graph in which every polygon longer than 4 edges has a chord. Analogs of properties of chordal graphs, e.g., Dirac’s separator theorem, are proved. In particular, a chordal bipartite graph has a perfect edge elimination scheme. [See Golumbic (1980a) for more.]

Gary Gordon
See also L. Fern.

An explicit bijection between the regions of the real hyperplane arrangement corresponding to $\pm K_n^\circ$ and the set of “good signed [complete] mixed graphs” $G_a$ of order $n$. The latter are a notational variant of the acyclic orientations $\tau$ of $\pm K_n^\circ$ [and are therefore in bijective correspondence with the regions, by Zaslavsky (1991b), Thm. 4.4]; the dictionary is: a directed edge in $G_a$ is an oriented positive edge in $\tau$, while a positive or negative undirected edge in $G_a$ is an introverted or extroverted negative edge of $\tau$. The main result, Thm. 1, is an interesting and significant explicit description of the acyclic orientations of $\pm K_n^\circ$. Namely, one orders the vertices and directs all positive edges upward; then one steps inward randomly from both ends of the ordered vertex set, one vertex at a time, at each new vertex orienting all previously unoriented negative edges to be introverted if the vertex was approached from below, extroverted if from above in the vertex ordering. [This clearly guarantees acyclicity.] [Problem. Generalize to arbitrary signed graphs.]

Lemma 2, “a standard exercise”, is that an orientation of $\pm K_n^\circ$ (with the loops replaced by half edges) is acyclic iff the magnitudes of its net degrees are a permutation of \{1, 3, ..., 2n − 1\}. [Similarly, an orientation of $\pm K_n^\circ$ is acyclic iff its net degree vector is a signed permutation of \{2, 4, ..., 2n\} (Zaslavsky (1991b), p. 369, but possibly known beforehand in other terminology). Both follow easily from Zaslavsky (1991b), Cor. 5.3: an acyclic orientation has a vertex that is a source or sink.]

20xxa The answer is $2^n \cdot n!$ What’s the question? Amer. Math. Monthly 106, No. 7 (August–September, 1999), 636–645.

§5 presents the signed-graph question: an appealing presentation of material from (1997a).

Y. Gordon and H.S. Witsenhausen

Asymptotic estimates of $l(K_{r,s})$, the maximum frustration index of signatures of $K_{r,s}$, improving the bounds of Brown and Spencer (1971a). (sg: Fr)

Clinton F. Goss
See M.C. Golumbic.

R.L. Graham and N.J.A. Sloane

See Example b, p. 396 (the Gale–Berlekamp code). (sg: Fr)
Ante Graovac, Ivan Gutman, and Nenad Trinajstić

§2.7. “Extension of graph-theoretical considerations to Möbius systems.”

(SG: A, Chem)

A. Graovac and N. Trinajstić

(SG: A, Chem)


The “Möbius graph” (i.e., signed graph of a suitably twisted ring hydrocarbon) is introduced with examples of the adjacency matrix and characteristic polynomial.

(Chem: SG: A)

John G. del Greco
See del Greco (under ‘D’).

F. Green

Proves polynomial time for the reduction employed in Bachas (1984a) and improves the theorem to: the frustration index decision problem on signed (3-dimensional) cubic lattice graphs with 9 layers is NP-complete. [Cf. Barahona (1982a).]

(SG: Fr: Alg)

Jan Green-Krótki
See J. Araoz.

Harvey J. Greenberg, J. Richard Lundgren, and John S. Maybee

From a matrix $B$, with row set $R$ and column set $C$, form the “signed bipartite graph” $BG^+$ with vertex set $R \cup C$ and an edge $r,c_k$ signed $\text{sgn} b_{rk}$ whenever $b_{rk} \neq 0$. The “signed row graph” $RG^+$ is the two-step signed graph of $BG^+$ on vertex set $R$: that is, $r_i,r_j$ is an edge if $\text{dist}_{BG^+}(r_i,r_j) = 2$ and its sign is the sign of any shortest $r_i,r_j$-path. If some edge has ill-defined sign, $RG^+$ is undefined. The “signed column graph” $CG^+$ is similar. The paper develops simple criteria for existence and balance of these graphs and the connection to matrix properties. It examines simple special forms of $B$.

(QM: SG, B, Appl)


Application of (1983a, 1984b). “Netform” = incidence matrix of a positive real gain graph (neglecting a minor technicality). Thm. 1: $B$ is a netform iff $RG^+(B)$ exists and is all negative. (Then $CG^+(B)$ also exists.) Thm. 2: If the row set partitions so that all negative elements are in some rows and all positives are in the other rows, then $RG^+(B)$ is all negative and balanced. Thm. 3: If $\Sigma$ is all negative and balanced, then $B$ exists as in Thm. 2 with $RG^+(B) = \Sigma$. [Equivalent to theorem of Hoffman and Gale (1956a).] $B$ is an “inverse” of $\Sigma$. Thm. 4 concerns “inverting” $-\Gamma$ in a minimal way. Then $B$ will be (essentially) the incidence matrix of $+\Gamma$. 

See (1983a). “Inversion” means, given a signed graph $\Sigma_R$, or $\Sigma_R$ and $\Sigma_C$, finding a matrix $B$ such that $\Sigma_R = RG^+(B)$, or $\Sigma_R = RG^+(B)$ and $\Sigma_C = CG^+(B)$. The elementary solution is in terms of coverings of $\Sigma_R$ by balanced cliques. It may be desirable to minimize the size of the balanced clique cover; this difficult problem is not tackled.

Harvey J. Greenberg and John S. Maybee, eds.


Several articles relevant to signed (di)graphs.

Curtis Greene and Thomas Zaslavsky


§9: “Acyclic orientations of signed graphs.” Continuation of Zaslavsky (1991b), counting acyclic orientations with specified unique source; also, with edge $e$ having specified orientation and with no termini except at the ends of $e$. The proof is geometric.

G. Grimmett


Reviews Fortuin and Kasteleyn (1972a) and subsequent developments esp. in multidimensional lattices. The viewpoint is mainly probabilistic and asymptotic. §3.7, “Historical observations,” reports Kasteley’s account of the origin of the model.

Richard C. Grinold


Objective: to find the maximum output for given input. Basic solutions correspond to bases of $G(\Phi')$, $\Phi'$ being the underlying gain graph $\Phi$ together with an unbalanced loop adjoined to the sink. Onaga (1967a) also treats this problem.

Heinz Gröflin and Thomas M. Liebling


Jonathan L. Gross

See also J. Chen.


Jonathan L. Gross and Thomas W. Tucker


1979a Fast computations in voltage graph theory. In: Allan Gewirtz and Louis V. Quintas, eds., Second International Conference on Combinatorial Mathematics (New


Ch. 2: “Voltage graphs and covering spaces.” Ch. 4: “Imbedded voltage graphs and current graphs.”

§3.2.2: “Orientability.” §3.2.3: “Rotation systems.” §4.4.5: “Nonorientable current graphs,” discusses how to deduce, from the signs on a current graph, the signs of the “derived” graph of the dual voltage graph. [The same rule gives the signs on the surface dual of any orientation-embedded signed graph.]

(The sign group here is $\mathbb{Z}_2$.)

Jerrold W. Grossman and Roland Häggkvist


They prove the special case in which $B$ is all negative of the following generalization, which is an immediate consequence of their result. [Theorem. If $B$ is a bidirected graph such that for each vertex $v$ there is a block of $B$ in which $v$ is neither a source nor a sink, then $B$ contains a coherent polygon. (“Coherent” means that at each vertex, one edge is directed inward and the other outward.)]

Martin Grötschel

See also F. Barahona.

M. Grötschel, M. Jünger, and G. Reinelt


§2, “The spin glass model”: finding the weighted frustration index in a weighted signed graph $(\Sigma, w)$, or finding a ground state in the corresponding Ising model, is equivalent to the weighted max-cut problem in $(-\Sigma, w)$. This article concerns finding the exact weighted frustration index. §3, “Complexity”, describes previous results on NP-completeness and polynomial-time solvability. §4, “Exact methods”, discusses previous solution methods. §5, “Polyhedral combinatorics”, shows that finding weighted frustration index is a linear program on the cut polytope; also expounds related work. The remainder of the paper concerns a specific cutting-plane method suggested by the polyhedral combinatorics.

M. Grötschel and W.R. Pulleyblank


Includes a polynomial-time algorithm, which they attribute to “Waterloo folklore”, for shortest (more generally, min-weight) even or odd path, hence (in an obvious way) odd or even polygon. [Attributed by Thomassen (1985a) to Edmonds (unpublished). Adapts to signed graphs by the negative subdivision trick: Subdivide each positive edge of $\Sigma$ into two negative edges, each with half the weight. The min-weight algorithm applied to the subdivision finds a min-weight (e.g., a shortest) negative polygon of $\Sigma$.] [This paper is very easy to understand. It is one of the best written I know.] [Weakly bipartite graphs are certain signed graphs. Further work: Barahona, Grötschel,

**Bertrand Guenin**

See also G. Cornuéjols.


Outline of (20xxa). (SG: G)

20xxa A characterization of weakly bipartite graphs. Submitted

$\Sigma$ is “weakly bipartite” (Grötschel and Pulleyblank 1981a) if its clutter of negative polygons is ideal (i.e., has the “weak MFMC” property of Seymour (1977a)). Thm.: $\Sigma$ is weakly bipartite iff it has no $-K_5$ minor. This proves part of Seymour’s conjecture (1981a) (see Cornuéjols 20xxa). (SG: G)

**Gregory Gutin**

See also J. Bang-Jensen.

**Gregory Gutin, Benjamin Sudakov, and Anders Yeo**


Existence of a coherent polygon with alternating colors in a digraph with an edge 2-coloring is NP-complete. However, if the minimum in- and out-degrees of both colors are sufficiently large, such a cycle exists. [This problem generalizes the undirected, edge-2-colored alternating-polygon problem, which is a special case of the existence of a bidirected coherent polygon—see Bang-Jensen and Gutin (1997a). *Question.* Is this alternating cycle problem also signed-graphic?] (p: o: Polygons: Gen)

**Ivan Gutman**

See also D.M. Cvetković, A. Graovac and S.-L. Lee.


Points out an ambiguity in the definitions of Lee, Lucchese, and Chu (1987a) in the case of multiple eigenvalues. [See Lee and Gutman (1989a) for the repair.] (VS, SGw)

**Ivan Gutman, Shyi-Long Lee, Yeung-Long Luo, and Yeong-Nan Yeh**


How to compute the balanced signing of $\Gamma$ that corresponds to eigenvalue $\lambda_i$ (see Lee, Lucchese, and Chu (1987a)), without computing the eigenvector $X_i$. Theorem: If $v_r, v_s$ are adjacent, then $X_{ir}X_{is} = \sum P f(P; \lambda_i)$, where $f(P; \lambda) := \varphi(G - V(P); \lambda)/\varphi'(G; \lambda)$, $\varphi(G; \lambda)$ is the characteristic polynomial, and the sum is over all paths connecting $v_r$ and $v_s$. Hence $\sigma_i(v_r, v_s) = \text{sgn} (X_{ir}X_{is})$ is determined. [An interesting theorem. *Questions.*}
Does it generalize if one replaces $\Gamma$ by a signed graph, this being the balanced (all-positive) case? In such a generalization, if any, how will $\sigma$ enter in—by restricting the sum to positive paths, perhaps? What about graphs with real gains, or weights?

Ivan Gutman, Shyi-Long Lee, Jeng-Horng Sheu, and Chiuping Li

Points out some difficulties with the method of Lee and Li (1994a).

Ivan Gutman, Shyi-Long Lee, and Yeong-Nan Yeh

A connected graph $\Gamma$ has $n$ eigenvalues and $n$ corresponding balanced signings (see Lee, Lucchese, and Chu (1987a)). Let $S_1 \geq S_2 \geq \cdots \geq S_n$ be the net signs of these signings and $m = |E|$. The net signs satisfy analogs of properties of eigenvalues. (A) If $\Delta \subset \Gamma$, then $S_1(\Delta) < S_1$. (B) $S_1 = m \geq S_2 + 2$. (C, D) For bipartite $\Gamma$, $S_n = -m$. Otherwise, $S_n \geq -m + 2$. From (B, C, D) we have $|S_i| \leq m - 2$ for all $i \neq 1$ and, if $\Gamma$ is bipartite, $i \neq n$. (E, F) If $\Gamma$ is bipartite, then $S_i = -S_{n+1-i}$ and at least $a-b$ net signs equal 0, where $a \geq b$ are the numbers of vertices in the two color classes. The analogy is imperfect, since $S_1 + S_2 + \cdots + S_n \geq 0$, while equality holds for eigenvalues.

Questions. Some of these conclusions require $\Gamma$ to be bipartite. Does that mean that they will generalize to an arbitrary balanced signed graph $\Sigma$ in place of the bipartite $\Gamma$, the eigenvectors being those of $\Sigma$? Will the other results generalize with $\Gamma$ replaced by any signed graph? How about real gains, or weights?

A. Gyárfás
See P. Erdős.

Ervin Győri
See also P. Erdős.

Ervin Győri, Alexandr V. Kostochka, and Tomasz Łuczak

Given all-negative $\Sigma$ and positive $\rho$, suppose every odd polygon has length $\geq n/\rho$. Then $\Sigma$ has frustration index $\leq 200\rho^2(\ln(10\rho))^2$ (best possible up to a constant factor) and vertex deletion number $\leq 15\rho\ln(10\rho)$ (best possible up to a logarithmic factor). The proof is based on an interesting, refining lemma. [Problem. Generalize to arbitrary $\Sigma$.]

Jurriaan Hage and Tero Harju

Classifies the switching-equivalent pairs of forests. Thm. 2.2: In a Seidel switching class of graphs there is at most one isomorphism type of tree; and there is at most one tree, with exceptions that are completely classified. Thms. 3.1 and 4.1: In a switching class that contains a disconnected forest there are at most 3 forests (not necessarily isomorphic); the cases in which there are 2 or 3 forests are completely classified. (Almost all are trees plus isolated vertices.) [Question. Regarding these results as concerning the negative subgraphs of switchings of signed complete graphs, to what extent
do they generalize to switchings of arbitrary signed simple graphs? [B.D. Acharya (1981a) asked which simple graphs switch to forests, with partial results.]

20xxa The size of switching classes with skew gains. Submitted.

Introducing “skew gain graphs”, which generalize gain graphs (see Zaslavsky (1989a)) to incorporate dynamic labelled 2-structures (see Ehrenfeucht and Rozenberg). Inversion is replaced by a gain-group antiautomorphism $\delta$ of period at most 2. Thus $\varphi(e^{-1}) = \delta(\varphi(e))$, while in switching by $\tau$, one defines $\varphi^\tau(e;v,w) = \delta(\tau(v))\varphi(e;v,w)\tau(w)$. The authors find the size of a switching class $[\varphi]_T$ in terms of the centralizers and/or $\delta$-centralizers of various parts of the image of $\varphi_T$, that is, $\varphi$ switched to be the identity on a spanning tree $T$. The exact formulas depend on whether $\Gamma$ is complete, or bipartite, or general, and on the choice of $T$ (the case where $T \cong K_{1,n-1}$ being simplest).

Per Hage and Frank Harary

Signed graphs are treated in Ch. 3 and 6, marked graphs in Ch. 6.

Roland Häggkvist
See J.W. Grossman.

J. Hammann
See E. Vincent.

Peter L. Hammer
See also E. Balas, C. Benzaken, E. Boros, J.-M. Bourjolly, Y. Crama, and A. Fraenkel.


P.L. Hammer, C. Benzaken, and B. Simeone

P.L. Hammer, T. Ibaraki, and U. Peled


See description of Thm. 8.5.2 in Mahadev and Peled (1995a). (p: o)

P.L. Hammer and N.V.R. Mahadev


P.L. Hammer, N.V.R. Mahadev, and U.N. Peled

A restricted line graph with signed edges is a proof tool. (SG, LG)

Peter L. Hammer and Sang Nguyen


Computes the Möbius functions of posets obtained from $\text{Lat} G(\pm K_n^\circ)$ by discarding those flats with unbalanced vertex set in a given lower-hereditary list. Examples include $\text{Lat} G(\pm K_n^{(k)})$, the exponent denoting the addition of $k$ negative loops. Generalized and superseded by Hanlon and Zaslavsky (1998a). (gg: M: Aut)


The lattices are based on a rank, $n$, a group, and a meet sublattice of the lattice of subgroups of the group. The Dowling lattices are a special case. (gg: M: Gen: N)


Partial summary of Fischer (1993a). (S)

Phil Hanlon and Thomas Zaslavsky

Computes the characteristic polynomials (Thm. 4.1) and hence the Möbius functions (Cor. 4.4) of posets obtained from $\text{Lat} G(\Omega)$, $\Omega$ a biased graph, by discarding those flats with unbalanced vertex set in a given lower-hereditary list. Examples include $\text{Lat} G(\mathcal{S}K_n^{(k)})$ where $\mathcal{S}$ is a finite group, the exponent denoting the addition of $k$ unbalanced loops. The interval structure, existence of a rank function, covering pairs, and other properties of these posets are investigated. There are many open problems. (GG: M, Gen: N, Str, Col)
Pierre Hansen


§1: Algorithm 1 labels vertices of a signed graph to detect imbalance and a negative polygon if one exists. [It is equivalent to switching a maximal forest to all positive and looking for negative edges.] §2: Algorithm 2 is the unweighted case of the algorithm of (1984a). Path balance in a signed digraph is discussed. §3: The frustration index of a signed graph is bounded below by the negative-polygon packing number, which can be crudely bounded by Alg. 1.


Improves the characterization by Maybee (1981a) of sign-solvable digraphs with an eye to more effective algorithmic recognition. Thm. 2.2. A signed digraph \( D \) is sign solvable iff its positive subdigraph is acyclic and each strongly connected component has a vertex that is the terminus of no negative, simple directed path. §3: “An algorithm for sign solvability” in time \( O(|V| \cdot |E|) \).


Algorithm to find shortest walks of each sign from vertex \( x_1 \) to each other vertex, in a signed digraph with positive integral (?) weights (i.e., lengths) on the edges. Applied to digraphs with signed vertices and edges; \( N \)-balance in signed graphs; sign solvability. The problem for (simple) paths is discussed [which is solvable by any min-weight parity path algorithm; see the notes on Grötschel and Pulleyblank (1981a)].

Pierre Hansen and Bruno Simeone


Three types of relatively easily maximizable pseudo-Boolean function (“unimodular” and two others) are defined. For quadratic pseudo-Boolean functions \( f \), the three types coincide; \( f \) is unimodular iff an associated signed graph is balanced (Thm. 3). Thus one can quickly recognize unimodular quadratic functions, although not unimodular functions in general. If the graph is a tree, the function can be maximized in linear time.

Frank Harary

See also L.W. Beineke, A. Blass, F. Buckley, D. Cartwright, G. Chartrand, O. Frank, and P. Hage.

[The birth of signed graph theory. Although Thm. 3 was anticipated by König (1936a) (Thm. X.11, for finite and infinite graphs) without the terminology of signs, here is the first recognition of the crucial fact that labelling edges by elements of a group—specifically, the sign group—can lead to a general theory.] The main theorem (Thm. 3) characterizes balanced signings as those for which there is a bipartition of the vertex set such that an edge is positive if it lies within a part [I call this a Harary bipartition]. Thm. 2: A signing of a simple [or a loop-free] graph is balanced if, for each pair of vertices, every path joining them has the same sign. Discussion of the number of nonisomorphic signed graphs with specific numbers of vertices and positive and negative edges.


$\Sigma$ is (locally) balanced at a vertex $v$ if every polygon on $v$ is positive; then Thm. 3': $\Sigma$ is balanced at $v$ iff every block containing $v$ is balanced. $\Sigma$ is $N$-balanced if every polygon of length $\leq N$ is positive; Thm. 2 concerns characterizing $N$-balance. Lemma 3: For each polygon basis, $\Sigma$ is balanced iff every polygon in the basis is positive. [For finite graphs this strengthens König (1936a) Thm. 13.]


"Antithetical duality" (pp. 260–261) introduces antibalance. Remarks on signed and vertex-signed graphs are scattered about the succeeding pages.


Section 6: “Balanced signed graphs”.


See pp. 400–401.


Proposes to measure imbalance by (i) $\beta(\Sigma)$, the proportion of balanced polygons ("degree of balance"), (ii) the frustration index ("line index") [cf. Abelson and Rosenberg (1958a)], i.e., the smallest number of edges whose deletion or equivalently (Thm. 7) negation results in balance, and (iii) the vertex elimination number: the smallest number of vertices whose deletion results in balance ("point index"). Thm. 4 is an upper bound on the minimum $\beta$ of unbalanced blocks with given cyclomatic number. Thm. 5 is a lower bound on the maximum. *Conjecture*. These bounds are best possible. Thm. 6 (contributed by J. Riordan) is an asymptotic evaluation of $\beta(−K_n)$.


1970a Graph theory as a structural model in the social sciences. In: Bernard Harris, ed.,


See remarks of Bixby (p. 111).


Reconstruction from the multiset of vertex-deleted subgraphs. $\Sigma_+$ is reconstructible if $\Sigma$ is connected and balanced and not all positive or all negative.

F. Harary and G. Gupta


§3.9, “Signed graphs”, mentions that deletion index = frustration index (Harary (1959b)).

Frank Harary and Jerald A. Kabell


Frank Harary and Helene J. Kommel


Frank Harary and Bernt Lindström


Thm. 1: The number of balanced signings of matroid $M$ is $\leq 2^{k(M)}$, with equality iff $M$ is binary. Thm. 3: Minimal deletion and negation sets coincide for all signings of $M$ iff $M$ is binary. Thm. 5: For connected binary $M$, a signing is balanced iff every circuit containing a fixed point is balanced.
Frank Harary, Bernt Lindström, and Hans-Olov Zetterström

Implicitly characterizes balance and balancing sets in a gain graph $\Phi$ by switching (proof of Thm. 1). [For balance, see also Acharya and Acharya (1986a), Zaslavsky (1977a) and (1989a), Lemma 5.3. For abelian gains, see also Gargano and Quintas (1985a). In retrospect we can see that the characterization of balanced gains is as the 1-coboundaries with values in a group, which for abelian groups is essentially classical.] Thm. 1: The number of balanced gain functions.

Thm. 2: Any minimal deletion set is an alteration set.

Thm. 3: $l(\Phi) \leq m(1 - |\mathcal{G}|^{-1})$. Thm. 4: $l(\Sigma) \leq \frac{1}{2}(m - \frac{a-1}{2})$, with strict inequality if not all degrees are even. [Compare with Akiyama, Avis, Chvátal, and Era (1981a), Thm. 1.]

Frank Harary, J. Richard Lundgren, and John S. Maybee

Which digraphs $D$ can be signed so that every cycle is negative? Three types of example. Type 1: The vertices can be numbered 1, 2, \ldots, $n$ so that the downward arcs are just (2, 1), (3, 2), \ldots, (n, n – 1). (Strong “upper” digraphs; Thm. 2.) Type 2: No cycle is covered by the remaining cycles (“free cyclic” digraphs). This type includes arc-minimal strong digraphs. Type 3: A symmetric digraph, if and only if the underlying graph $\Gamma$ is bipartite and no two points on a common polygon and in the same color class are joined by a path outside the cycle (Thm. 10; proved by signing $\Gamma$ via Zaslavsky (1981b)). [Further work in Chaty (1988a).]

Frank Harary, Robert Z. Norman, and Dorwin Cartwright

In Ch. 10, “Acyclic digraphs”: “Gradable digraphs”, pp. 275–280. That means a digraph whose vertices can be labelled by integers so that $f(w) = f(v) + 1$ for every arc $(v, w)$. [Equivalently, the Hasse diagram of a graded poset.] [Characterized by Topp and Ulatowski (1987a).]


“Limited balance”, pp. 352–355. Harary (1955a); also: Adjacency matrix (nonsymmetric) $A(D, \sigma)$ of a signed digraph: entries are 0, $\pm 1$. The “valency matrix” is the $R(\Sigma)$ of Abelson and Rosenberg (1958a) Thm. 13.8: Entries of $R(\Sigma)^k$ show the existence of (undirected) walks of length $k$ of each sign between pairs of vertices. [The symbols might be treated as 0, $a_+$, $a_-$, $a_+ + a_-$ in the group ring $R$ of the sign group. Then $R(\Sigma)$ is equivalent to the $R$-valued adjacency matrix $A_R(\Sigma)$. Thm. 13.8 follows upon substituting in $A_R^k$: $0 \leftrightarrow o$, $ma_+ \leftrightarrow p$, $ma_- \leftrightarrow n$, $ma_+ + m'a_- \leftrightarrow a$, where $m, m'$ are positive integers. $A_R^k$ itself provides an exact count of walks of each sign. Obviously, $A_R$ and walk-counting generalize to gain graphs.]

“Cycle-balance and path-balance”, pp. 355–358: here directions of arcs are
taken into account. E.g., Thm. 13.11: Every cycle is positive if each strong component is balanced as an undirected graph.

\[(SG: \text{B, Fr, A: Exp, Exr})(SD: \text{B, Exr})\]


\[(GD: \text{b, Exr})(SG: \text{B, Fr, A: Exp, Exr})(SD: \text{B, Exr})\]

Frank Harary and Edgar M. Palmer

\[(SG: \text{B: E})(SD: \text{B, Exr})\]


Four exercises and a remark concern signed graphs, balanced signed graphs, and signed trees. Russian transl.: Kharari and Palmer (1977a).

\[(SG: \text{E, B})(SD: \text{B, Exr})\]


Russian translation of (1973a).

\[(SG: \text{E, B})(GD: \text{b, Exr})\]

Frank Harary, Edgar M. Palmer, Robert W. Robinson, and Allen J. Schwenk

\[(SG, VS: \text{E})(SD: \text{B, Exr})\]

Frank Harary and Michael Plantholt

\[(SG: \text{LG, B})(SD: \text{B, Exr})\]

Frank Harary and Geert Prins

\[(SG: \text{E})(SD: \text{B, Exr})\]

Frank Harary and Robert W. Robinson

\[(SG, VS: \text{E})(SD: \text{B, Exr})\]

Frank Harary and Bruce Sagan

A signed poset is a (finite) partially ordered set \(P\) whose Möbius function takes on only values in \(\{0, \pm 1\}\). \(S(P)\) is the signed graph with \(V = P\) and \(E_\epsilon = \{xy : x \leq y \text{ and } \mu(x, y) = \epsilon 1\} \text{ for } \epsilon = +, -\). Some examples are chains, tree posets, and any product of signed posets. Thm. 1 characterizes \(P\) such that \(|S(P)| \cong H(P)\), the Hasse diagram of \(P\). Thm. 3 characterizes posets for which \(S(P)\) is balanced. Thm. 4 gives a sufficient condition for clusterability of \(S(P)\). There are many unanswered questions, most basically Question 1. Which signed graphs have the form \(S(P)\)? [See Zelinka (1988a) for a partial answer.]

\[(SG, S)(SD: \text{B, Exr})\]

Frank Harary and Marcello Truzzi
1979a The graph of the zodiac: On the persistence of the quasi-scientific paradigm of

Katsumi Harashima
See H. Kosako.

Tero Harju
See A. Ehrenfeucht and J. Hage.

David Harries and Hans Liebeck

Given $\Sigma = (K_n, \sigma)$ and an automorphism group $\mathfrak{A}$ of the switching class $[\Sigma]$, is $\mathfrak{A}$ “exposable” on $[\Sigma]$ (does it fix a representative of $[\Sigma]$)? General techniques and a solution for the dihedral group. Done in terms of Seidel switching of unsigned simple graphs. (A further development from Mallows and Sloane (1975a). [Related work in M. Liebeck (1982a) and Cameron (1977a).]

Nora Hartsfield and Gerhard Ringel

“Cascades”: see Youngs (1968b).

Kurt Häsigg


Ch. 5: “Verallgemeinerte Fluss- und Potentialdifferenzen-probleme.”

Refael Hassin

Patrick Headley

The characteristic polynomials of the Shi hyperplane arrangements $S(W)$ of type $W$ for each Weyl group $W$, evaluated computationally. $S(W)$ is obtained by splitting the reflection hyperplanes of $W$ in two in a certain way; thus $S(A_{n-1})$ splits the arrangement representing $\text{Lat} G(K_n)$—more precisely, it represents $\text{Lat}^b \Phi$ where $\Phi = (K_n, \varphi_0) \cup (K_n, \varphi_1)$ (see Stanley (1996a) for notation); that of type $B_n$ splits the arrangement representing $\text{Lat} G(\pm K_n^*)$, and so on. [See also Athanasiadis (1996a).]

Fritz Heider
1946a Attitudes and cognitive organization. J. Psychology 21 (1946), 107–112.

No mathematics, but a formative article. [See Cartwright and Harary (1956a).]

Richard V. Helgason
See J.L. Kennington.

I. Heller

I. Heller and C. B. Tompkins

Robert L. Hemminger and Joseph B. Klerlein

An attempt, intrinsically unsuccessful, to represent the (signed) line graph of a digraph (see Zaslavsky 20xxb) by a digraph. [Continued by Klerlein (1975a).] (sg: LG, o)

Robert L. Hemminger and Bohdan Zelinka

J.A. Hertz
See K.H. Fischer.

Hector Hevia
See G. Chartrand.

Dorit S. Hochbaum
20xxa A framework for half integrality and 2-approximations with applications to feasible cut and minimum satisfiability. Submitted.

Slightly extends Hochbaum and Naor (1994a) and Hochbaum, Megiddo, Naor, and Tamir (1993a). (GN: I(D): Alg)

Dorit S. Hochbaum, Nimrod Megiddo, Joseph (Seffi) Naor, and Arie Tamir

Approximate solution of integer linear programs with real, dually gain-graphic coefficient matrix. [See Sewell (1996a).] (GN: I(D): Alg)

Dorit S. Hochbaum and Joseph (Seffi) Naor

Linear and integer programs with real, dually gain-graphic coefficient matrix: feasibility for linear programs, solution of integer programs when the gains are positive (“monotone inequalities”), and identification of “fat” polytopes (that contain a sphere larger than a unit hypercube). (GN: I(D): Alg, Ref)
Cornelis Hoede


Teil 4: “Kognitive Konsistenz.” (PsS: Gen: Exp)


Characterizes when one can sign the vertices of a graph so every polygon has positive sign product, solving the problem of Beineke and Harary (1978b). [The definitive word.] (VS: B: Str)

Alan J. Hoffman
See also David Gale.


Eigenvalues of signed complete graphs. (k: A)


Abstract of (1977b). (SG: LG)


[A. J. Hoffman and D. Gale]


Alan J. Hoffman and Peter Joffe
Alan J. Hoffman and Francisco Pereira  

Franz Höfting and Egon Wanke  

Given a finite gain digraph \( \Phi \) (the “static graph”) with gains in \( \mathbb{Z}^d \) and a rational cost for each edge, find a minimum-cost walk (“path”) in its canonical covering graph \( \tilde{\Phi} \) with given initial and final vertices. \((\text{GD} (\text{Cov}): \text{Alg})\)


Take a gain digraph \( \Phi \) (the “static graph”) with gains in \( \mathbb{Z}_\alpha = \mathbb{Z}_{\alpha_1} \times \cdots \times \mathbb{Z}_{\alpha_d} \) (where \( \alpha = (\alpha_1, \ldots, \alpha_d) \)) and its canonical covering digraph \( \tilde{\Phi} \) (the “toroidal periodic graph”). Treated algorithmically via integer linear programming and linear Diophantine equations: existence of directed paths (NP-complete, but polynomial-time if \( \Phi \) is strongly connected) and number of strongly connected components of \( \tilde{\Phi} \). \((\text{GD}, \text{GG}(\text{Cov}): \text{Alg}, \text{G}, \text{Ref})\)


Full version of (1993a). The min-cost problem is expressed as an integer linear program. Various conditions under which the problem is NP-hard, even a very restricted version without costs (Thms. 3.3, 3.5), or polynomial-time solvable (e.g.: without costs, when \( \Phi \) is an undirected gain graph: Thm. 3.4; with costs, when \( d \) is fixed: Thm. 4.5). \((\text{GD}, \text{GG}(\text{Cov}): \text{Alg}, \text{G}, \text{Ref})\)


Paul W. Holland and Samuel Leinhardt  
1971a Transitivity in structural models of small groups. *Comparative Group Studies* 2 (1971), 107–124. \((\text{PsS}: \text{SG}: \text{B})\)

Paul W. Holland and Samuel Leinhardt, eds.  

John Hultz  
See also F. Glover.

John Hultz and D. Klingman  

John E. Hunter  
C.A.J. Hurkens

Given: a bidirected graph $B$ (with no loose or half edges or positive loops) and an integer weight $b_e$ on each edge. Wanted: an integral vertex weighting $x$ such that $I(B)^T x \leq b$, where $I(B)$ is the incidence matrix. Such $x$ exists iff (i) every coherent polygonal or handcu walk has nonnegative total weight and (ii) each doubly odd Korach walk (a generalization of a coherent handcuff that has a cutpoint dividing it into two parts, each with odd total weight) has positive total weight. This improves a theorem of Schrijver (1991a) and is best possible. Dictionary: “path” (“cycle”) = coherent (closed) walk.

T. Ibaraki
See also Y. Crama and P.L. Hammer.

T. Ibaraki and U.N. Peled

Yoshiko T. Ikebe and Akihisa Tamura
20xxa Perfect bidirected graphs. Submitted

A transitively closed bidirection of a simple graph is perfect iff its underlying graph is perfect. (See Johnson and Padberg (1982a) for definitions.) [Also proved by Sewell (1996a).]

Masao Iri and Katsuaki Aoki

Masao Iri, Katsuaki Aoki, Eiji O’Shima, and Hisayoshi Matsuyama
1976a [A graphical approach to the problem of locating the system failure.] (In Japanese.) [????] 76 (135) (1976), 63–68. 


The process is modelled by a signed digraph with some nodes $v$ marked by $\mu(v) \in \{+, -, 0\}$. (Marks $+$, $-$ indicate a failure in the process.) Object: to locate the node which is origin of the failure. An oversimplified description of the algorithm: $\mu$ is extended arbitrarily to $V$. Arc $(u, v)$ is discarded if $0 \neq \mu(u)\mu(v) \neq \sigma(u, v)$. If the resulting digraph has a unique initial strongly connected component $S$, the nodes in it are possible origins. Otherwise, this extension provides no information. (I have overlooked: special marks on “controlled” nodes; speedup by stepwise extension and testing of $\mu$.) [This article and/or (1976a) seems to be the origin of a whole literature. See e.g. Chang and Yu (1990a), Kramer and Palowitch (1987a).] 

C. Itzykson
See R. Balian.
P.L. Ivanescu [P.L. Hammer]
See E. Balas and P.L. Hammer.

Sousuke Iwai
See O. Katai.

François Jaeger

(This is not the colored Tutte polynomial of Kauffman (1989a).) Jaeger shows that the Kauffman polynomial, originally defined for link diagrams and here transformed to an invariant of signed plane graphs, depends only on the edge signs and the polygon matroid. It can also be reformulated to be essentially independent of signs. Problem. Define a similar invariant for more general matroids.

François Jaeger, Nathan Linial, Charles Payan, and Michael Tarsi

Let $\mathfrak{A}$ be abelian group. $\Gamma$ is “$\mathfrak{A}$-colorable” if every $\mathfrak{A}$-gain graph on $\Gamma$ has a proper group-coloring (as in Zaslavsky (1991a)). Prop. 4.2. Every simple planar graph is $\mathfrak{A}$-colorable for every abelian group $\mathfrak{A}$ of order $\geq 6$. (For the same reason as the classical 6-Color Theorem.) [Improved by Lai and Zhang (20xxb).]

John C. Jahnke
See J.O. Morrissette.

John J. Jarvis and Anthony M. Jezior

Clark Jeffries

Sufficient (and necessary) conditions for sign stability in terms of negative cycles and a novel color test. Proofs are sketched or (for necessity) absent.

Clark Jeffries, Victor Klee, and Pauline van den Driessche

Paul A. Jensen and J. Wesley Barnes


Sec. 5.5: “Negative cycles.”


Russian translation of (1980a).
P.A. Jensen and Gora Bhaumik

Tommy R. Jensen and Bjarne Toft

R.H. Jeurissen

William S. Jewell

Anthony M. Jezior
See J.J. Jarvis.

Samuel Jezný and Marián Trenkler

Peter Joffe
See A.J. Hoffman.

Eugene C. Johnsen

An elaborate classificatory analysis of “triads” (signed complete directed graphs of 3 vertices) vis-à-vis “macrostructures” (signed complete directed graphs) with reference to structural interactions and implications of triadic numerical restrictions on “dyads” (s.c.d.g. of 2 vertices). Connections to certain models of affect in social psychology. [“Impenetrability! That’s what I say!” “Would you tell me, please,” said Alice, “what that means?”]

(K, SD, SG: B, PsS: Exp)
Charles R. Johnson and John Maybee

In square matrix $A$ let $A[S]$ be the principal submatrix with rows and columns indexed by $S$. Thm. 1: Assume $A[S]$ is sign-nonsingular in standard form and $i, j \notin S$. Then the $(i,j)$ entry of the Schur complement of $A[S]$ has sign determined by the sign pattern of $A$ iff, in the signed digraph of $A$, every path $i \rightarrow j$ via $S$ has the same sign. (QM: SD)

Charles R. Johnson, D.D. Olesky, Michael Tsatsomeros, and P. van den Driessche

Suppose the signed digraph $D$ of an $n \times n$ matrix has longest cycle length $k$ and all cycles of $-D$ are negative. Theorem: If $k = n - 1$, the eigenvalues lie in a domain subtending angle $< 2\pi/k$. This is known for $k = 2$ but false for $k = n - 3$. (QM, SD)

Ellis L. Johnson
See also J. Edmonds and G. Gastou.


§9: “Integer programming in an undirected graph.”

(GN: I, M( bases))(ec: I, M( bases), Alg)


Ellis L. Johnson and Sebastiano Mosterts

Two of the problems: Given a signed graph (edges called “even” and “odd” rather than “positive” and “negative”). The co-postman problem is to find a minimum-cost deletion set (of edges). The “odd circuit” problem is to find a minimum-cost negative polygon. The Chinese postman problem is described in a way that involves cobalance and “switching” around a polygon. (SG: Fr(Gen), I)

Ellis L. Johnson and Manfred W. Padberg

Geometry of the bidirected stable set polytope $P(B)$ (which generalizes the stable set polytope to bidirected graphs), defined as the convex hull of 0,1 solutions of $x_i + x_j \leq 1$, $-x_i - x_j \leq -1$, $x_i \leq x_j$ for extroverted, introverted, and directed edges of $B$. (Thus, undirected graphs correspond to extroverted
bidirected graphs.) It suffices to treat transitively closed bidirections of simple graphs ([unfortunately] called “bigraphs”). [Such a bidirected graph must be balanced.] A “biclique” $(S_+, S_-)$ is the Harary bipartition of a balanced complete subgraph $(S_+, S_-)$ are the source and sink sets of the subgraph). It is “strong” if no external vertex has an edge directed out of every vertex of $S_+$ and an edge directed into every vertex of $S_-$. Strong bicliques generate facet inequalities of the polytope. Call $B$ perfect if these facets (and nonnegativity) determine $P(B)$. $\Gamma$ is “biperfect” if every transitively closed bidirection $B$ of $\Gamma$ is perfect. Conjectures: $\Gamma$ is biperfect if it is perfect. $\Gamma$ is perfect if some transitively closed bidirection is perfect. [Both proved by Sewell (1996a) and independently by Ikebe and Tamura (20xxa). See e.g. Tamura (1997a), Conforti (20xxa) for further work.] (sg: O: I, G, sw)

Leif Kjær Jørgensen

Let $\sigma_{op}(\Gamma)$, or $\sigma_{odd}(\Gamma)$, be the largest $s$ for which $-\Gamma$ contains a subdivision of $-K_s$ (an “odd-path-$K_sS$”), or $[-\Gamma]$ contains an antibalanced subdivision of $K_s$ (an “odd-$K_sS$”). Thm. 4: $\sigma_{op}(\Gamma), \sigma_{odd}(\Gamma) \approx \sqrt{n}$. Thms. 7, 8 (simplified): For $p = 4, 5$ and large enough $n = |V|$, $\sigma_{odd}(\Gamma) \geq p$ or $\Gamma$ is a specific exceptional graph. Conjecture 9. The same holds for all $p \geq 4$. [Problem. Generalize this to signed graphs.] (p: X)

Tadeusz Józefiak and Bruce Sagan

Summarizes the freeness results in (1993a). (sg, gg: G, m, N)


The hyperplane arrangements (over fields with characteristic $\neq 2$) corresponding to certain signed graphs are shown to be “free”. Explicit bases and the exponents are given. The signed graphs are: $+K_{n-1} \subseteq \Sigma_1 \subseteq +K_n$ (known), $\pm K_n \subseteq \Sigma_2 \subseteq \pm K_n$, $\pm K_n \subseteq \Sigma_3 \subseteq \pm K_n$; also, those obtained from $+K_n$ or $K_n$ by adding all negative links in the order of their larger vertex (assuming ordered vertices) (Thms. 4.1, 4.2) or smaller vertex (Thms. 4.4, 4.5); and those obtained from $\pm K_{n-1}$ by adding positive edges ahead of negative ones (Thm. 4.3). [For further developments see Edelman and Reiner (1994a).] Similar theorems hold for complex arrangements when the sign group is replaced by the complex $s$-th roots of unity (§5). The Möbius functions of $\Sigma_2$, known from Hanlon (1988a), are deduced in §6. (sg, gg: G, m, N)

M. Jünger
See M. Grötschel.

Mark Jungerman and Gerhard Ringel

“Cascades”: see Youngs (1968b). (sg: O: Appl)

Jerald A. Kabell
See also F. Harary.

Co-balance means that every cutset has positive sign product. Thm.: $\Sigma$ is cobalanced iff every vertex star has evenly many negative edges. For planar graphs, corollaries of this criterion and Harary’s bipartition theorem result from duality. [The theorem follows easily by looking at the negative subgraph.]

(SG: B(D), B)


(Jeff Kahn and Joseph P.S. Kung)


Announcement of (1982a).


A “variety” is a class closed under deletion, contraction, and direct summation and having for each rank a “universal model”, a single member containing all others. There are two nontrivial types of variety of finite matroids: matroids representable over $\text{GF}(q)$, and gain-graphic matroids with gains in a finite group $\mathfrak{G}$. The universal models of the latter are the Dowling geometries $Q_n(\mathfrak{G})$.

It is incidentally proved that Dowling geometries of non-group quasigroups cannot exist in rank $n \geq 4$.


A geometric lattice of rank $\geq 4$, if not a projective geometry with a few points deleted, is a Dowling lattice.

(Jeff Kahn and Roy Meshulam)


Continues Aharoni, Meshulam, and Wajnryb (1995a) (q.v., for definitions), generalizing its Thm. 1.3 (the case $|K|=2$ of the following). Let $m =$ number of 0-weight matchings, $\delta =$ minimum degree. Thm. 1.1: If $m > 0$ then $m \geq (\delta - k + 1)!$ where $k = |K|$. Conjecture 1.2. $k$ can be reduced. (See the paper for details.) [Question. Is there a generalization to weighted digraphs? One could have two kinds of arcs: some weighted from $K$, and some weighted 0. The perfect matching might be replaced by an alternating Hamilton cycle or a spanning union of disjoint alternating cycles.]

(WG)

Thm. 2.1: Let $D$ be a simple digraph with weights in an abelian group $K$. If all outdegrees are $> k$, where $k = |K|$, then there is a nonempty set of disjoint cycles whose total weight is 0.

(Ajai Kapoor)

See M. Conforti.

(Roman Kapuscinski)

See P. Doreian.
Richard M. Karp, Raymond E. Miller, and Shmuel Winograd

P.W. Kasteleyn
See also C.M. Fortuin.

P.W. Kasteleyn and C.M. Fortuin

A specialization of the parametrized dichromatic polynomial of a graph: \( Q_{\Gamma}(q, p; x, 1) \) where \( q_e = 1 - p_e \). [Essentially, announcing Fortuin and Kasteleyn (1972a).]

Osamu Katai

Osamu Katai and Sousuke Iwai


Louis H. Kauffman
See also J.R. Goldman.


A leisurely development of Kauffman’s combinatorial bracket polynomial of a link diagram and the Jones and other knot polynomials, including the basics of (1989a).

(SGc: Knot: N)


The Tutte polynomial, also called “Kauffman’s bracket of a signed graph” and equivalent to his bracket of a link diagram, is defined by a sum over spanning trees of terms that depend on the signs and activities of the edges and nonedges of the tree. The point is that the deletion-contraction recurrence over an edge has parameters dependent on the color of the edge; also,
the parameters of the two colors are related. The purpose is to develop the bracket of a link diagram combinatorially. §3.2, “Link diagrams”: how link diagrams correspond to signed plane graphs. §4, “A polynomial for signed graphs”, defines the general sign-colored graph polynomial $Q[\Sigma](A, B, d)$ by deletion-contraction, modified multiplication on components, and evaluation on graphs of loops and isthmii. §5, “A spanning tree expansion for $Q[G]$” [$G$ means $\Sigma$], proves $Q[\Sigma]$ exists by producing a spanning-tree expansion, shown independent of the edge ordering by a direct argument. [No dichromatic form of $Q[\Sigma]$ appears; but see successor articles.] §6, “Conclusion”, remarks that $Q[\Sigma]$ is invariant under signed-graphic Reidemeister moves II and III. [This significant work, inspired by Thistlethwaite (1988a), led to independent but related generalizations by Przytycka and Przytycki (1988a), Schwärlzer and Welsh (1993a), Traldi (1989a), and Zaslavsky (1992b) that were partially anticipated by Fortuin and Kasteleyn (1972a). Also see (1997a).]


§2, “A state summation for classical electrical networks”, uses a form of the parametrized dichromatic polynomial $Q_T(B, A; 1, 1)$ [as in Zaslavsky (1992b) et al.], where $A(e), B(e) \in \mathbb{C}^*$, to compute conductances as in Goldman and Kauffman (1993a). (sgc: Gen: N: Exp)

§3: “The bracket polynomial”, discusses the connections with signed graphs and electricity. Problem: Is there a signed graph, not reducible by signed-graphic Reidemeister moves (see (1989a)) to a tree with loops, whose sign-colored dichromatic polynomial is trivial? If not, the Jones polynomial detects the unknot. (SGc: N: Exp)(SGc: N)

John G. Kemeny and J. Laurie Snell


John W. Kennedy
See M.L. Gargano.

Jeff L. Kennington and Richard V. Helgason

Ch. 5: “The simplex method for the generalized network problem.” (GN: M( Bases): Exp)

F. Kharari and È. Palmer [Frank Harary and Edgar M. Palmer]
See F. Harary and E.M. Palmer (1977a).

A. Khelladi

Improves the result of Bouchet (1983a). (SG: M, Flows)

Shin’ichi Kinoshita
See also T. Yajima.
Shin’ichi Kinoshita and Hidetaka Terasaka

Employs the sign-colored graph of a link diagram (Bankwitz 1930a) to form certain combinations of links. (SGc: Knot)

M. Kirby
See A. Charnes.

Scott Kirkpatrick

Victor Klee
See also C. Jeffries.


Along with Simões-Pereira (1972a), invents the bicircular matroid (here, for infinite graphs). (Bic)


When are various forms of stability of a linear differential equation \( \dot{x} = Ax \) determined solely by the sign pattern of \( A \)? A survey of elegant combinatorial criteria. Signed digraphs [alas] play but a minor role. (Sta, SD: Exp, Ref)

Victor Klee, Richard Ladner, and Rachel Manber

Victor Klee and Pauline van den Driessche

Peter Kleinschmidt and Shmuel Onn

In a graded partially ordered set with 0 and 1, assign a sign to each covering pair \( (x, y) \) where \( y \) is covered by 1. This is an “exact signing” if in every upper interval there is just one \( y \) whose coverings are all positive. Then the poset is “signable”. (S: G)


See (1995a) for definition. Signability is a generalization to posets of partitionability of a simplicial complex (Prop. 3.1). Shellable posets, and face lattices of spherical polytopes and oriented matroid polytopes, are signable. A stronger property of a simplicial complex, “total signability”, which applies for instance to simplicial oriented matroid polytopes (Thm. 5.12), implies
the upper bound property (Thm. 4.4). Computational complexity of face counting and of deciding shellability and partitionability are discussed in §6.

(S: G, Alg)

Joseph B. Klerlein
See also R.L. Hemminger.


Continues the topic of Hemminger and Kerlein (1977a).

(SG: LG, o)

Darwin Klingman
See J. Elam, F. Glover, and J. Hultz.

Elizabeth Klipsch
20xxa Some signed graphs that are forbidden link minors for orientation embedding. In preparation.

For each $n \geq 5$, either $-K_n$ or its 1-edge deletion, but not both, is a forbidden link minor. Which one it is, is controlled by Euler’s polyhedral formula, provided $n \geq 7$. [A long version with excruciating detail is available.]

(SG: T, P)

Muralidharan Kodialam and James B. Orlin

Linear programming methods to find the strongly connected components of a periodic digraph from the static graph: i.e., of the covering digraph of a gain digraph $\Phi$ with gains in $\mathbb{Q}^d$ by looking at $\Phi$. Cf. Cohen and Megiddo (1993a), whose goals are similar but algorithms differ.

(GD(Cov): B, Polygons: Alg)

János Komlós

Sharp asymptotic upper bounds on frustration index and vertex elimination number for all-negative signed graphs with fixed negative girth. Improves Bollobás, Erdős, Simonovits, and Szemerédi (1978a). [Problem. Generalize to arbitrary signed graphs or signed simple graphs.]

(P: Fr)

Helene J. Kommel
See F. Harary.

Dénes König

§X.3, “Komposition von Büscheln”, contains Thms. 9–16 of Ch. X. I restate them in terms of a signature on the edge set; König says subgraph or $p$-subgraph (“$p$-Teilgraph”) to mean what we would call the negative edge set of a signature or a balanced signature. Instead of signed switching, König speaks of set summation (“composition”) with a vertex star (“Büschel”). His theorems apply to finite and infinite graphs except where stated otherwise.

Thm. 9: The edgewise product of balanced signatures is balanced. Thm.
10: Every balanced signing of a finite graph is a switching of the all-positive signature. Thm. 11: A signature is balanced if and only if it has a Harary bipartition [see Harary (1953a)]. Thm. 12 (cor. of 11): A graph is bicolourable if and only if every polygon has even length. [König makes this fundamental theorem a corollary of a signed-graph theorem!] Thm. 13: A signature is balanced if (not only if) every polygon of a fundamental system is positive. Thm. 14: A graph with $n$ vertices (a finite number) and $c$ components has $2^{n-c}$ balanced signings. Thm. 16: The set of all vertex switchings except for one in each finite component of $\Gamma$ forms a basis for the space of all finitely generated switchings.


English translation of (1936a). §X.3: “Composition of stars”. The term “Kreis” (circle, meaning polygon) is translated as “cycle”—one of the innumerable meanings of “cycle”.


“Variable-signed graph” = signed simple (di)graph $\Sigma$ with switching function $p$ and switched graph $\Sigma^p$. Known basic properties of switching are established. More interesting: planar duality when $|\Sigma|$ is planar. The planar dual $|\Sigma|^*$ inherits the same edge signs; a dual vertex has sign of the surrounding primal face boundary. Property 9 is in effect the statements:

1. If a signed plane graph has $f$ negative face boundaries, then $l(\Sigma) \geq f/2$.
2. If the negative faces fall into two connected groups with oddly many faces in each, (1) can be improved to $\geq f/2 + 1$. Finally, incidence matrices are studied that are only superficially related to signs. [The paper is hard to interpret due to mathematical imprecision and language difficulty.]

(SG: Sw, fr, D, I)

Alexandr V. Kostochka
See A.A. Ageev and E. Győri.


David Krackhardt
See P. Doreian.


Vertex signs indicate directions of change in vertex variables; signed directed edges describe relations among these directions. Truth tables for a signed edge as a function of endpoint signs. Algorithms for deducing logical rules about states (assignments of vertex signs) from the signed digraph. Has a useful discussion of previous literature, e.g., Iri, Aoki, O’Shima, and Matsuyama (1979a).

(SD, VS: Appl, Alg, Ref)

Following up Stanley (1985a), a signed $K_n$ is reconstructible from its single-vertex switching deck if its negative subgraph is disconnected [therefore also if its positive subgraph is disconnected] or if the minimum degree of its positive or negative subgraph is sufficiently small. All done in terms of Seidel switching of unsigned simple graphs.


Following up Krasikov and Roditty (1987a), $(K_n, \sigma)$ is reconstructible from its $s$-vertex switching deck if $s = \frac{1}{2}n - r$ where $r \in \{0, 2\}$ and $r \equiv n \pmod{4}$, or $r = 1 \equiv n \pmod{2}$; also, if $s = 2$ and the minimum degree of the positive or negative subgraph is sufficiently small. Also, bounds on $|E_-|$ if $(K_n, \sigma)$ is not reconstructible. Negative-subgraph degree sequence: reconstructible when $s = 2$ and $n \geq 10$. Done in terms of Seidel switching of unsigned simple graphs.


If the minimum degrees of its positive and negative subgraphs obey certain bounds, a signed $K_n$ is reconstructible from its $s$-switching deck. The main bound involves the least and greatest even zeros of the Krawtchouk polynomial $K_n^s(x)$. Done in terms of Seidel switching of unsigned simple graphs. [More details in Zbl.]

*Ilia Krasikov and Simon Litsyn*


Among the applications mentioned (pp. 72–73): 2. “Switching reconstruction problem”, i.e., graph-switching reconstruction as in Stanley (1985a) etc. 4. “Sign reconstruction problem”, i.e., reconstructing a signed graph from its $s$-edge negation deck, which is the multiset of signed graphs obtained by separately negating each subset of $s$ edges (here called “switching signs”, but it is not signed-graph switching); this is a new problem.

*Ilia Krasikov and Y. Roditty*


§2: “Reconstruction of graphs from vertex switching”. Corollary 2.3. If a signed $K_n$ is not reconstructible from its $s$-vertex switching deck, a certain linear Diophantine system (the “balance equations”) has a certain kind of solution. For $s = 1$ the balance equations are equivalent to Stanley’s (1985a) theorem; for larger $s$ they may or may not be. All is done in terms of Seidel switching of unsigned simple graphs. [Ellingham and Royle (1992a) note a gap in the proof of Lemma 2.5.]


Main Theorem. Fix $s \geq 4$. If $n$ is large and (for odd $s$) not evenly even, every signed $K_n$ is reconstructible from its $s$-vertex switching deck. Different results hold for $s = 2, 3$. (This is based on and strengthens Stanley (1985a).) Theorems 5 and 6 concern reconstructing subgraph numbers. All done in terms of Seidel switching of unsigned simple graphs.

Based on (1987a) and strengthening Stanley (1985a): Theorem 7. A signed $K_n$ is reconstructible if the Krawtchouk polynomial $K_n^s(x)$ “has one or two even roots [lying] far from $n/2$” (the precise statement is complicated). Numerous other partial results, e.g., a signed $K_n$ is reconstructible if $s = \frac{1}{2}(n - r)$ where $r = 0, 1, 3$, or $2, 4, 5, 6$ with side conditions. All is done in terms of Seidel switching of unsigned simple graphs.

**Jan Kratochvíl, Jaroslav Nešetřil, and Ondřej Zýka**


Is a given graph switching equivalent to a graph with a specified property? (This is Seidel switching of simple graphs.) Depending on the property, this question may be in P or be NP-complete, whether the original property is in P or is NP-complete. Properties: containing a Hamilton path; containing a Hamilton polygon; no induced $P_2$; regularity; etc. Thm. 4.1: Switching isomorphism and graph isomorphism are polynomially equivalent.

**Vijaya Kumar [G.R. Vijayakumar]**

See G.R. Vijayakumar.

**Joseph P.S. Kung**

See also J.E. Bonin and J. Kahn.


Examples include Dowling geometries, Ex. (6.2), and the bias matroids of full group expansions of graphs in certain classes; see pp. 98–99. (GG: M)


The Dowling geometry over the sign group is the largest simple ternary matroid not containing the “Reid matroid”. (sg: M: X)


Dowling geometries used in the proof of Prop. (1.2). (gg: M)


Survey and new results. See: §2.7: “Gain-graphic matroids.” P. 30, fn. 9.

(GG: M: X, Str, Exp, Ref)

Dowling lattices are lower-half Sperner. The proof is given only for partition lattices. (gg: M)


Delete from a Dowling geometry a subset $S$ that contains no whole plane. Found: necessary and sufficient conditions for the characteristic polynomial to factor completely over the integers. When the geometry corresponds to a hyperplane arrangement, many more of the arrangements are not free than are free; however, if $S$ contains no whole line, all are free (so the characteristic polynomial factors completely over $\mathbb{Z}$) while many are not supersolvable. (gg: M: N)

**Joseph P.S. Kung and James G. Oxley**


For $n \geq 4$, the Dowling geometry of rank $n$ over the sign group is the unique largest simple matroid of rank $n$ that is representable over $GF(3)$ and $GF(q)$. (sg: M: X)

**David Kuo**


**Richard Ladner**

See V. Klee.

**George M. Lady and John S. Maybee**


In terms of signed graphs, restates and completes the characterizations of sign-invertible matrices $A$ due to Bassett, Maybee, and Quirk (1968a) and George M. Lady (The structure of qualitatively determinate relationships. *Econometrica* 51 (1983), 197–218. MR 85c:90019. Zbl. 517.15004) and reveals the sign pattern of $A^{-1}$ in terms of path signs in the associated signed digraph. (QM: Sol: SD)

**J.C. Lagarias**

Theorem F: Feasibility of integer linear programs with at most two variables per constraint is $NP$-complete. (GN(I): D: Alg)

Hong-Jian Lai and Xiankun Zhang

20xxa Group colorability of graphs. Submitted

Simple graphs only are considered. The [abelian] “group chromatic number” $\chi_1(\Gamma) = \min m$ such that $\Gamma$ is $\mathfrak{A}$-colorable (as in Jaeger, Linial, Payan, and Tarsi (1992a)) for every abelian $\mathfrak{A}$ of order $\geq m$. Various results, e.g., $\Gamma$ is $\mathbb{Z}_2$-colorable iff it is a forest; analog of Brooks’ Theorem (stronger than the original because $\chi_1(\Gamma) \geq \chi(\Gamma)$; analog of Nordhaus-Gaddum Theorem involving the complementary graph. [Thus $\chi_1(\Gamma)$ seems to resemble ordinary chromatic number more than it does gain-graph coloring.] (GG: Col)

20xxb Coloring a graph with elements in an Abelian group. Submitted

Continues (20xxa). Thm.: If $\Gamma$ is simple and has no $K_5$ minor, then $\chi_1(\Gamma) \leq 5$, improving on Jaeger, Linial, Payan, and Tarsi (1992a). (GG: Col)

Kelvin Lancaster


Zbl. 495.93001 (book).

Comment on Maybee (1981a). (QM: Sol: SD)

Andrea S. LaPaugh and Christos H. Papadimitriou


Fast algorithms for existence of even paths between two given vertices (or any two vertices) of a graph. The corresponding digraph problem is $NP$-complete. [Signed (di)graphs are similar, due to the standard reduction by negative subdivision.] [See also, e.g., works by Thomassen.] (P: Paths: Alg)

Michel Las Vergnas

See A. Björner.

Monique Laurent

See M.M. Deza and A.M.H. Gerards.

Eugene L. Lawler


Ch. 6: “Nonbipartite matching.” §3: Bidirected flows. (sg: O)


Jason Leasure

See L. Fern.

Bruno Leclerc


Jon Lee


See Section 9.
Shyi-Long Lee

See also I. Gutman.


Response to Gutman (1988a). Proposes weighted net sign: divide by number of nonzero vertex signs. The goal is to have the ordering of net signs correlate more closely with that of eigenvalues. (VS, SGw, Chem)


Expounds principally Lee, Lucchese, and Chu (1987a) and Lee and Gutman (1989a). Examples include all connected, simple graphs of order ≤ 4 and some aromatics. (VS, SGw, Exp, Chem)


See Lee, Lucchese, and Chu (1987a). More examples; again, eigenvalue and net-sign orderings are compared. (VS, SGw, Chem)

Shyi-Long Lee and Ivan Gutman


 Supplements Lee, Lucchese, and Chu (1987a) to answer an objection by Gutman (1988a), by treating vertex signs corresponding to multidimensional eigenspaces. (VS, SGw, Chem)

Shyi-Long Lee and Chiuping Li


Varies Lee, Lucchese, and Chu (1987a) by taking net signs of all balanced signings, instead of only those obtained from eigenvectors, for small paths, polygons, and polygons with short tails. The distribution of net sign, over all signings of each graph, is more or less binomial. (VS, SGw, Chem)


Abbreviated presentation of (1994a). (VS, SGw: Exp)

Shyi-Long Lee and Feng-Yin Li


Shyi-Long Lee, Feng-Yin Li, and Friday Lin


**Shyi-Long Lee, Robert R. Lucchese, and San Yan Chu**


Introduces the net sign of a (balanced) signed graph. A graph has vertices signed according to the signs of an eigenvector \(X_i\) of the adjacency matrix, \(\mu(v_r) = \text{sgn}(X_{ir})\), and \(\sigma(v_r,v_s) = \mu(v_r)\mu(v_s)\) [hence \(\Sigma\) is balanced]. Note that a vertex can have ‘sign’ 0. Net sign of a [hydrocarbon] chemical graph is applied to prediction of properties of molecular orbitals. (VS, SGw, Chem)

**Shyi-Long Lee, Yeung-Long Luo, and Yeong-Nan Yeh**


See Lee, Lucchese, and Chu (1987a). Net signs for the Platonic polyhedra (Table I). (VS, SGw, Chem)

**Shyi-Long Lee and Yeong-Nan Yeh**


Follows up Lee, Lucchese, and Chu (1987a) and Lee and Gutman (1989a), calculating net signs of eigenspatially signed hypercube graphs of dimensions up to 6 by means of a general graph-product formula. (VS, SGw, Chem)


See Lee, Lucchese, and Chu (1987a). Net signs and eigenvalues are compared. (VS, SGw, Chem)

**Samuel Leinhardt**

See also J.A. Davis and P.W. Holland.

**Samuel Leinhardt, ed.**


An anthology reprinting some basic papers in structural balance theory. (PsS, SG: B, Cl)

**P.W.H. Lemmens and J.J. Seidel**


**Marianne Lepp [Marianne L. Gardner]**

See R. Shull.

**David W. Lewit**

See E.G. Shrader.

**Chiuping Li**


**Feng-Hin Li**

See S.-L. Lee.

**Hans Liebeck**

See D. Harries.
Martin W. Liebeck
Examines the \( F \text{Aut} ([\Sigma]) \)-module \( FV(\Sigma) \), where \( \Sigma \) is a signed complete graph and \( F \) is a field of characteristic 2. \((\text{TG: Aut})\)

Given an abstract group \( \mathfrak{A} \), which of its permutation representations are exposable on every invariant switching class of signed complete graphs [see Harries and H. Liebeck (1978a) for definitions]? \((k: \text{sw}, \text{TG: Aut})\)

Thomas M. Liebling
See H. Grofli.

Magnhild Lien and William Watkins
20xxa Dual graphs and knot invariants. Submitted
The Kirchhoff (“Laplacian”) matrices of a signed plane graph and its dual have the same invariant factors. The proof is via the signed graphs of knot diagrams. \((\text{SGc: D, I, Knot})\)

Ko-Wei Lih

Friday Lin
See S.-L. Lee.

Bernt Lindström
See F. Harary.

Nathan Linial
See F. Jaeger.

Sóstenes Lins
For Eulerian \( \Sigma \) in projective plane, max. number of edge-disjoint negative polygons = min. number of edges cut by a noncontractible closed curve that avoids the vertices. [Generalized by Schrijver (1989a).] \((\text{SG: T, fr, Alg})\)

See §4. \(\text{(sg: T: b)}\)


J.H. van Lint and J.J. Seidel

Marc J. Lipman and Richard D. Ringeisen
Simon Litsyn
See I. Krasikov.

Charles H.C. Little
See C.P. Bonnington.

M. Loebl
See Y. Crama.

D.O. Logofet and N.B. Ul’yanov

Necessity of Jeffries’ (1974a) sufficient conditions. (Sta)

D.O. Logofet and N.B. Ul’yanov [N.B. Ul’yanov]

English translation of (1982a). (Sta)

M. Loréa

Discovers the “count” matroids of graphs (see Whiteley (1996a)). (Bic: Gen)

Janice R. Lourie

L. Lovász
See also J.A. Bondy and Gerards et al. (1990a).


Characterization of the graphs having no two vertex-disjoint polygons. See Bollobás (1978a) for exposition in English. [Major Problem. Characterize the biased graphs having no two vertex-disjoint unbalanced polygons. This theorem is the contrabalanced case. The sign-biased case was also solved by Lovász; see Seymour (1995a). McQuaig (1993a) might be relevant.] (P: Str)


It is hard to escape the feeling that we are dealing with all-negative signed graphs and their $-K_4$ and $-K_2^*$ minors. [And indeed, see Gerards and Schrijver (1986a) and Gerards et al. (1990a) and the notes on Seymour (1995a).] (GG: Polygons)

L. Lovász and M.D. Plummer

L. Lovász, L. Pyber, D.J.A. Welsh, and G.M. Ziegler
§7: “Knots and the Tutte polynomial”, considers the signed graph of a knot diagram (pp. 2076–77).

Robert R. Lucchese
See S.-L. Lee.

Tomasz Łuczak
See E. Győri.

J. Richard Lundgren
See H.J. Greenberg and F. Harary.

Yeung-Long Luo

Enzo Maccioni
See F. Barahona.

Thomas L. Magnanti
See R.K. Ahuja.

N.V.R. Mahadev
See also P.L. Hammer.

N.V.R. Mahadev and U.N. Peled

§8.3: “Bithreshold graphs” (from Hammer and Mahadev (1985a)), and §8.4: “Strict 2-threshold graphs” (from Hammer, Mahadev, and Peled (1989a)), characterize two types of threshold-like graph. In each, a different signed graph $H$ is defined on $E(\Gamma)$ so that $\Gamma$ is of the specified type iff $H$ is balanced. (The negative part of $H$ is the “conflict graph”, $\Gamma^*$. ) The reason is that one wants $\Gamma$ to decompose into two subgraphs, and the subgraphs, if they exist, must be the two parts of the Harary bipartition of $H$. [Thus one also gets a fast recognition algorithm (though not the fastest possible) for the desired type from the fast recognition of balance.] (SG: B: Appl)

§8.5: “Recognizing threshold dimension 2.” Based on Raschle and Simon (1995a). Given: $\Gamma \subseteq K_n$ such that $\Gamma^*$ is bipartite. Orient $-K_n$ so that $\Gamma$-edges are introverted and the other edges are extroverted. Their “alternating cycle” is a coherent closed walk in this orientation. Let us call it “black” (in a given black-white proper coloring of $\Gamma^*$) if its $\Gamma$-edges are all black. Thm. 8.5.2 (Hammer, Ibaraki, and Peled (1981a)): If there is a black coherent closed walk in $E_0$, then there is a coherent tour (closed trail) of length 6 (which is a pair of joined triangles or a hexagon—their $AP_5$ and $AP_6$).

Thm. 8.5.4: Given that there is no black coherent hexagon, one can recolor quickly so there is no black coherent 6-tour. Thm. 8.5.9: Given that there is no ‘double’ coherent hexagon (the book’s “double $AP_6$”), one can recolor quickly so there is no black coherent hexagon. Thm. 8.5.28: Any 2-coloring of $\Gamma^*$ can be quickly transformed into one with no ‘double’ coherent hexagon. [Question. Can any of this, especially Thm. 8.5.2, be generalized to arbitrary oriented all-negative graphs $B$? Presumably, this would require first defining a conflict graph on the introverted edges of $B$. More remotely, consider generalizing to bidirected complete or arbitrary graphs.] (p: o, Alg)

§9.2.1: “Threshold signed graphs.” In this version it’s not clear where the signs are! (and their role is trivial). Real weights are assigned to the vertices and an edge receives the sign of the weight product of its endpoints.
Ali Ridha Mahjoub
See F. Barahona.

J.M. Maillard
See J. Vannimenus.

M. Malek-Zavarei and J.K. Aggarwal

R.B. Mallion
See A.C. Day.

C.L. Mallows and N.J.A. Sloane

Thm. 1: For all \( n \), the number of unlabelled two-graphs of order \( n \) [i.e., switching isomorphism classes of signed \( K_n \)'s] equals the number of unlabelled even-degree simple graphs on \( n \) vertices. The key to the proof is that a permutation fixing a switching class fixes a signing in the class. (Seidel (1974a) proved the odd case, where the fixing property is simple.) Thm. 2: The same for the labelled case. [More in Cameron (1977b), Cameron and Wells (1986a), Cheng and Wells (1984a, 1986a).]

To prove the fixing property they find the conditions under which a given permutation \( \pi \) of \( V(K_n) \) and switching set \( C \) fix some signed \( K_n \). [More in Harries and Liebeck (1978a), M. Liebeck (1982a), and Cameron (1977b).]

(TG: Aut, E)

Rachel Manber
See also R. Aharoni and V. Klee.


Rachel Manber and Jia-Yu Shao

Dănăț Marcu
I cannot vouch for the authenticity of these articles. See MR 97a:05095 and Zbl. 701.51004. Also see MR 92a:51002, 92b:51026, 92h:11026, 97k:05050; and Marcu (1981b).


See Harary, Norman, and Cartwright (1965a) for the definition. (GD: b)


See Harary, Norman, and Cartwright (1965a) for the definition. The tournaments of order 3 are not gradable, whence the titular theorem. (GD: b)


§1, “Preliminary considerations”, appears to be an edited, unacknowledged transcription of portions of Harary, Norman, and Cartwright (1965a) (or
possibly (1968a), pp. 341–345. Wording and notation have been modified, a trivial corollary has been added, and some errors have been introduced; but the mathematics is otherwise the same down to details of proofs. §2, “Results”, is largely a list of the corollaries resulting from setting all signs negative. The exception is Thm. 2.5, for which I am not aware of a source; however, it is simple and well known. (sg(SD): B)


Matroidal families of (multi)graphs (see Simões-Pereira (1973a)) correspond to functions on all isomorphism types of graphs that are similar to matroid rank functions, e.g., submodular. This provides insight into matroidal families, e.g., it immediately shows there are infinitely many. (Bic, EC: Gen)

Harry Markowitz

Also see RAND Corporation Paper P-602, 1954. (GN: m(bases))

Clifford W. Marshall

“Consistency of choice” discusses signed graphs, pp. 262–266. (SG: B, A: Exp)

J.H. Mason

§§2.5–2.6: “The lattice approach” and “Generalized coordinates”, pp. 172–174, propose a purely matroidal and more general formulation of Dowling’s construction of his lattices. (gg(Gen): M)


Dowling matroids are an example in §1. (gg: M)

R.A. Mathon

Hisayoshi Matsuyama
See M. Iri.

Laurence R. Matthews

Thorough study of bicircular matroids, introduced by Klee (1971a) and Simões-Pereira (1972a). (Bic)


Inverts poise, modular poise, and antidirection matroids of a digraph. (M: G)


Laurence R. Matthews and James G. Oxley


Jean François Maurras


John S. Maybee


Survey and simple proofs. (QM: sd, gg, Sta)(Exp)


For comments, see Lancaster (1981a). (QM: Sol: SD)


Signed (di)graphs play a role in characterizations. See e.g. §7. See also Roberts (1989a), §4. (QM, SD)

John S. Maybee and Stuart J. Maybee


A linear-time algorithm to determine balance or antibalence of the undirected signed graph of a signed digraph. The algorithm of Harary and Kabell (1980a) appears to be different. (SG: B, P: Alg)
John Maybee and James Quirk
An important early survey with new results.
(QM, SD: Sol, Sta, b; Exp(in part), Ref)

John S. Maybee and Daniel J. Richman
Square matrix $A$ is a GM-matrix if, for every positive and negative cycle $P$ and $N$ in its signed digraph, $V(P) \supseteq V(N)$. Classification of irreducible GM-matrices; connections with the property that each $p \times p$ principal minor has sign $(-1)^p$; some conclusions about the inverse. (SD: QM)

John S. Maybee and Gerry M. Weiner
An $L$-function is a nonlinear generalization of a qualitative linear function. Signed digraphs play a small role. (QM, SD)

Stuart J. Maybee
See J.S. Maybee.

W. Mayeda and M.E. Van Valkenburg

R. Maynard
See F. Barahona and I. Bieche.

Richard D. McBride
See G.G. Brown.

H. Gilman McCann
See E.C. Johnsen.

William McCuaig
Characterizes the digraphs with no two disjoint cycles as well as those with no two arc-disjoint cycles. [Since cycles do not form a linear subclass of polygons, this is not a biased-graphic theorem, but it might be of use in studying biased graphs that have no two disjoint balanced polygons. See Lovász (1965a).] (Str)

†20xxa Pólya’s permanent problem. Submitted.
Question 1. Does a given digraph $D$ have an even cycle? Question 2. Can a given digraph $D$ be signed so that every cycle is negative? (These problems are easily seen to be equivalent.) The main theorem (the “Even Dicycle Thm.”) is a structural characterization of digraphs that have a signing in which every cycle is negative. (These were previously characterized by forbidden minors in Seymour and Thomassen (1987a).)
The main theorem is proved also in Robertson, Seymour, and Thomas (20xxa). (SD: p: Str)( SG)

20xxb When all dicycles have the same length. Submitted.
Uses the main theorem of (20xxa) and Robertson, Seymour, and Thomas (20xxa) to prove: a digraph has an edge weighting in which all cycles have equal nonzero total weight if and only if it does not contain a “double dicycle”: a symmetric digraph whose underlying simple graph is a circle. There is also a structural description of such digraphs.

William McCuaig, Neil Robertson, P.D. Seymour, and Robin Thomas
20xxa Permanents, Pfaffian orientations, and even directed circuits. Extended abstract.
In: *Proceedings of the 1997 Symposium on the Theory of Computing*
Extended abstract of McCuaig (20xxa) and Robertson, Seymour, and Thomas (20xxa).

W.D. McCuaig and M. Rosenfeld
In a 3-connected graph, almost any two edges are in an even and an odd polygon. [By the negative-subdivision trick this generalizes to signed graphs.]

T.A. McKee


Kathleen A. McKeon
See G. Chartrand.

Nimrod Megiddo
See E. Cohen and D. Hochbaum.

Roy Meshulam
See R. Aharoni and J. Kahn.

Robert Messer
See E.M. Brown.

Marc M{é}zard, Giorgio Parisi, and Miguel Angel Virasoro
Focuses on the Sherrington-Kirkpatrick model, i.e., underlying complete graph, emphasising the Parisi-type model (see articles reprinted herein), which posits numerous metastable states, separated by energy barriers of greatly varying heights and subdividing as temperature decreases. Essentially heuristic (as noted in MR): that is, the ideas awaited [and still largely await] mathematical justification.

Many original articles on Ising and vector models (both of which are based on weighted signed graphs) are reprinted herein, though few are of general signed-graphic interest.

[See also, i.a., Toulouse (1977a, etc.), Chowdhury (1986a), Stern (1989a), Fischer and Hertz (1991a), Vincent, Hammann, and Ocio (1992a) for physics, Barahona (1982a, etc.), Grötschel, Jünger, and Reinelt (1987a) for mathematics.]
Ch. 0, “Introduction”, briefly compares, in the obvious way, balance in social psychology with frustration in spin glasses.  

(Phys, PsS: SG: B: Exp)

Pt. I, “Spin glasses”, Ch. 2, “The TAP approach”: pp. 19–20 describe 1-vertex switching of a weighted signed graph to reduce frustration, not however necessarily producing the frustration index (minimum frustration).

(Phys: SG: Fr, Sw, Alg: Exp)

**Raymond E. Miller**
See R.M. Karp.

**William P. Miller**
See J.E. Bonin.

**Edward Minieka**

(GN: M(indep), B)


(GN: B, Sw, m(indep): Exp)


Russian translation of (1978a).  

(V. Mishra)

**S. Mitra**

Treats only signed $K_n$. Viewed with hindsight, observes that balance holds iff $I + A(\Sigma) = vv^T$ for some vector $v$ of $\pm 1$’s; also, defines switching and observes [I call this the Switching Thm. of Frustration] that frustration index $l(\Sigma) =$ minimum number of negative edges over all switchings. States a simple “algorithm” for computing $l(\Sigma)$ [but without a stopping rule, and the obvious ones are invalid].  

(sg: k: A, sw, Fr)

**Bojan Mohar**

The “overlap matrix” of a signed graph with respect to a rotation system and a spanning tree provides a lower bound on the demigenus that sometimes improves on that from Euler’s formula.  

(SG: T)

**Elliott W. Montroll**


(SG, Phys: Exp)
J.W. Moon and L. Moser

Suck Jung Moon
See H. Kosako.

M.A. Moore
See A.J. Bray.

Michio Morishima

§4: “Alternative expression of the assumptions (1),” can be interpreted with hindsight as proving that, for a signed $K_n$, every triangle is positive iff the signature switches to all positive. (Everything is done with sign-symmetric matrices, not graphs, and switching is not mentioned in any form.) 

Julian O. Morrissette

Proposes that edges have strengths between $-1$ and $+1$ instead of pure signs. The Cartwright-Harary degree of balance (1956a), computed from polygons, is modified to take account of strength. In addition, signed graphs are allowed to have edges of two types, say $U$ and $A$, and only short mixed-type polygons enter into the degree of balance. This is said to be more consistent with the experimental data reported herein. 

Julian O. Morrissette and John C. Jahnke

Reports an experiment; then discusses problems with and alternatives to the Cartwright-Harary (1956a) polygon degree of balance. 

L. Moser
See J.W. Moon.

Sebastiano Mosterts
See E.L. Johnson.

Andrej Mrvar
See P. Doreian.

Luigi Muracchini and Anna Maria Ghirlanda

A partially successful attempt to use unoriented signed graphs to define a line graph of a digraph. [See Zaslavsky (20xxb) for the correct signed-graph approach.] The Harary-Norman line digraph is also discussed. 

Kunio Murasugi

The signature of a sign-colored graph (see 1989a) is an invariant of the sign-colored graphic matroid. 

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*The Electronic Journal of Combinatorics* #DS8

Studies a dichromatic form, $P_\Sigma(x, y, z)$, of Kauffman’s (1989a) Tutte polynomial of a sign-colored graph. The deletion-contraction parameters are $a_\epsilon = 1$, $b_\epsilon = x^\epsilon$ for $\epsilon = \pm 1$; the initial values are such that $P_\Sigma(x, y, z) = y^{-1}Q_\Sigma(a, b; y, z)$ of Zaslavsky (1992b). The polynomial is shown to be, in effect, an invariant of the sign-colored graphic matroid.

Much unusual graph theory is in here. A special focus is the degrees of the polynomial. First Main Thm. 3.1: Formulas for the maximum and minimum combined degrees of $P_\Sigma(x, y, z)$. §7, “Signature of a graph”, studies the signature ($\sigma$ in the paper, $s$ here) of the Kirchhoff matrix $B_\Sigma$ obtained by changing the diagonal of $A(\Sigma)$ so the row sums are 0. Prop. 7.2 is a matrix-tree theorem [entirely different from that of Zaslavsky (1982a)]. The Second Main Thm. 8.1 bounds the signature: $|V| - 2\beta_0(\Sigma_-) + 1 \leq s \leq |V| - 2\beta_0(\Sigma_+) + 1$ ($\beta_0 =$ number of components), with equality characterized. The Kirchhoff matrix is further examined later on. §9, “Dual graphs”: Differing from most studies, here the dual of a sign-colored plane graph is the planar dual with same edge signs [however, negating all colors is a triviality]. §10, “Periodic graphs”: These graphs might be called branched covering graphs of signed gain graphs with finite cyclic gain group. [Thus they generalize the periodic graphs of Collatz (1978a) and others.] §§12–15 concern applications to knot theory.

*SGc: N, I, GG(Cov), D, Knot*


§§1–3 expound results from (1989a) on the dichromatic polynomial and the signature of a sign-colored graph and knot applications. §5 discusses the signed Seifert graph of a link diagram.  

*SGc: N, I, Knot: Exp*


See (1996a).

*SGc: Knot*


*SGc: Knot*

**Kunio Murasugi and Jozef H. Przytycki**


Ch. I, “Index of a graph”. The “index” is the largest number of “independent” edges, where “independent” has a complicated recursive definition (unrelated to matchings), one of whose requirements is that the edges be “singular” (simple, i.e., nonmultiple links). The positive or negative index of a sign-colored graph is similar except that the independent edges must all be positive or negative. [The general notion is that of the index of a graph-subgraph pair. The signs pick out complementary subgraphs.] Thm. 2.4: Each of these indices is additive on blocks of a bipartite graph. The main interest, because of applications to knot theory, is in bipartite plane graphs. Ch. II,
“Link theory”: Pp. 26–27 define the sign-colored Seifert graph of an oriented link diagram and apply the graphical index theory. (SGc: N, D, Knot)

Tadao Murata

Takeshi Naitoh
See K. Ando.

Kazuo Nakajima
See H. Choi.

Daishin Nakamura and Akihisa Tamura

The problem of the title is solvable in polynomial time. See Johnson and Padberg (1982a), Tamura (1997a) for definitions. They reduce to simple graphs, transitively bidirected with no sink or introverted edge (called “canonical” bidirected graphs). (sg: O: G, Sw, Alg)


L. Nanjundaswamy
See E. Sampathkumar.

Joseph (Seffi) Naor
See D. Hochbaum.

C.St.J.A. Nash-Williams


Roman Nedela and Martin Škoviera

By “canonical double covering” of $\Gamma$ they mean the signed covering graph $\Sigma$ of $\Sigma = -\Gamma$, but without reversing orientation at the negative covering vertex [as one would do in a signed covering graph (cf. e.g. Zaslavsky 1992a)], because orientable embeddings of $\Gamma$ are being lifted to orientable embeddings of $\Sigma$. [Thus these can be thought of as not signed graphs but rather voltage (i.e., gain) graphs with 2-element voltage group.] Instead of reversal they twist the negative-vertex rotations by taking a suitable power. In some cases this allows classifying the orientable, regular embeddings of $\Sigma$.

(P: Cov, T, Aut)

Main topic: the theory of twisting of rotations as in (1996a). (GG: Cov, T, Aut)

Portions concern double covering graphs of signed graphs. §7: “Antipodal and algebraically antipodal maps”. A map is “antipodal” if it is the orientable double covering of a nonorientable map; that is, as a graph it is the canonical double covering of an unbalanced signed graph. A partial algebraic criterion for a map to be antipodal. §9: “Regular embeddings of canonical double coverings of graphs”. See (1996a). (sg, P: Cov, T, Aut)


Cases in which the classification of (1996a) is necessarily incomplete are studied by taking larger voltage (i.e., gain) groups and twisting the rotations at covering vertices by taking a power that depends on the position of the vertex in its fiber. Main result: the (very special) conditions on twisting under which a regular map lifts to a regular map. (GG: Cov, T, Aut)

Toshio Nemoto
See K. Ando.

H. Nencka
See Ph. Combe.

Jaroslav Nešetřil
See J. Kratochvíl.

A. Neumaier

In the signed graph \((K_n, \sigma)\) of a two-graph (see D.E. Taylor 1977a), a “clique” is a vertex set that induces an antibalanced subgraph. A two-graph is “completely regular” if every clique of size \(i\) lies in the same number of cliques of size \(i + 1\), for all \(i\). Thm. 1.4 implies there is only a small finite number of completely regular two-graphs. (TG)

Sang Nguyen
See P.L. Hammer.

Juhani Nieminen

Peter Nijkamp
See F. Brouwer.

Robert Z. Norman
See also F. Harary.

Robert Z. Norman and Fred S. Roberts

Polygon (“cycle”) indices of imbalance: the proportion of polygons that are unbalanced, with polygons weighted nonincreasingly according to length.
Exposition and application of (1972a).

Beth Novick and András Sebő
The clutter of negative circuits of a signed binary matroid \((M, \sigma)\). Important are the lift and extended lift matroids, \(L(M, \sigma)\) and \(L_0(M, \sigma)\), defined as in signed graph theory. An elementary result: the clutter is signed-graphic iff \(L_0(M, \sigma)/e_0\) is graphic (which is obvious). There are also more substantial but complicated results. [See Cornuéjols (20xxa), §8.4.] \((S(M): M)\)


Cyriel van Nuffelen
The unoriented incidence matrix has rank = rank\((G(−Γ))\). [Because the matrix represents \(G(−Γ)\).] \((p: I, ec)\)

Summarizes (1973a). \((p: I, ec)\)

M. Ocio
See E. Vincent.

E. Olaru
See St. Antohe.

D.D. Olesky
See C.R. Johnson.

Kenji Onaga

Shmuel Onn
See also P. Kleinschmidt.

For “signability” see Kleinschmidt and Onn (1995a). A strong signing is an exact signing that satisfies a recursive condition on lower intervals. \((S, G)\)
Rikio Onodera  

The Open University  
Social sciences (pp. 21–23). Signed digraphs (pp. 50–52). [Published version: see Wilson and Watkins (1990a).] (SG, PsS, SD: Exp)

Peter Orlik and Louis Solomon  

James B. Orlin  
See also R.K. Ahuja, M. Kodialam, and R. Shull.

Problems on 1-dimensional periodic graphs (i.e., covering (di)graphs of Z-gain graphs Φ) that can be solved in Φ: connected components, strongly connected components, directed path from one vertex to another, Eulerian trail (directed or not), bicolorability, and spanning tree with minimum average cost. (GG, GD: Cov: Paths, Polygons, Col: Alg)


Charles E. Osgood and Percy H. Tannenbaum  

Eiji O'Shima  
See M. Iri.

James G. Oxley  
See also J.P.S. Kung and L.R. Matthews.

See Exercise 3.20. (Bic: Exp)  
§10.3, Exercise 3 concerns the Dowling lattices of GF(q)*. (gg: M: Exp)

Manfred W. Padberg  
See E.L. Johnson
Steven R. Pagano

Ch. 1: “Separability”. Graphical characterization of bias-matroid \( k \)-separations of a biased graph. Also, some results on the possibility of \( k \)- separations in which one or both sides are connected subgraphs. \( \text{(GG: M: Str)} \)

Ch. 2: “Representability”. The bias matroid of every signed graph is representable over all fields with characteristic \( \neq 2 \). For which signed graphs is it representable in characteristic 2 (and therefore representable over GF(4), by the theorem of Geoff Whittle, A characterization of the matroids representable over GF(3) and the rationals. J. Combin. Theory Ser. B 65 (1995), 222-261. MR 96m:05046. Zbl. 835.05015.)? Solved (for 3-connected signed graphs having vertex-disjoint negative polygons and hence nonregular matroid). There are two essentially different types: (i) two balanced graphs joined by three independent unbalanced digons; (ii) a cylindrical signed graph, possibly with balanced graphs adjoined by 3-sums. [See notes on Seymour (1995a) for definition of (ii) and for Lovász’s structure theorem in the case without vertex-disjoint negative polygons.] \( \text{(SG: M: I, Str, T)} \)

Ch. 3: “Miscellaneous results”.

20xxa Binary signed graphs. Submitted \( \text{(SG: M: I, Str)} \)

20xxb Signed graphic GF(4) forbidden minors. Submitted \( \text{(SG: M)} \)

20xxc GF(4)-representations of bias matroids of signed graphs: The 3-connected case. Submitted \( \text{(SG: M: I, Str, T)} \)

Edgar M. Palmer
See F. Harary and F. Kharari.

B.L. Palowitch, Jr.
See M.A. Kramer.

Christos H. Papadimitriou
See also E.M. Arkin and A.S. LaPaugh.

Christos H. Papadimitriou and Kenneth Steiglitz

See Ch. 10, Problems 6-7, p. 244, for bidirected graphs and flows in relation to the matching problem. \( \text{(sg: O: Flows)} \)


Giorgio Parisi
See M. Mézard.

Philippa Pattison

Ch. 8, pp. 258-9: “The balance model. The complete clustering model.” Embedded in a more general framework. \( \text{(SG, S: A, B, Cl: Exp)} \)

G.A. Patwardhan
See B.D. Acharya and M.K. Gill.
Charles Payan  
See F. Jaeger.

Edmund R. Peay  
(SD, WD: A: Gen)

Proposes an index of nonclusterability for signed graphs and generalizes to edges weighted by a linearly ordered set.  
(SG, Gen: Cl: Fr)

See mainly §3: “Structural consistency.”  
(sd: Gen: B, Cl)

Uri N. Peled  

Francisco Pereira  
See A.J. Hoffman.

M. Petersdorf  
Treats signed $K_n$’s. Satz 1: $\max(\Sigma) = [(n-1)^2/4]$ with equality iff $\Sigma$ is antibalanced. [From which follows easily the full Thm. 14 of Abelson and Rosenberg (1958a).] Also, some further discussion of antibalanced and unbalanced cases. [For extensions of this problem see notes on Erdős, Győri, and Simonovits (1992a).]  
(SG: Fr)

J.L. Phillips  
Proposes to measure imbalance of a signed (di)graph by largest eigenvalue of a matrix close to $I + A(\Sigma)$. (Cf. Abelson (1967a).) Possibly, means to treat only graphs that are complete aside from isolated vertices. [Somewhat imprecise.] Summary of Ph.D. thesis.  
(SG: B, Fr, A)

Nancy V. Phillips  
See F. Glover.

Jean-Claude Picard and H. Donald Ratliff  
A minor application of signed switching to a weighted graph arising from an integer linear program.  
(sg: sw)

P. Pincus  
See S. Alexander.

Tomaž Pisanski and Jože Vrabec  
Definition (see Pisanski, Shawe-Taylor, and Vrabec (1983a)), examples, superimposed structure, classification.  
(GG: Cov(Gen))
Tomaž Pisanski, John Shawe-Taylor, and Jože Vrabec
A graph bundle is, roughly, a covering graph with an arbitrary graph $F_v$ (the “fibre”) over each vertex $v$, so that the edges covering $e:vw$ induce an isomorphism $F_v \rightarrow F_w$.

**Michael Plantholt**
See F. Harary.

**M.D. Plummer**
See L. Lovász.

**Svatopluk Poljak**
See also Y. Crama.

**Svatopluk Poljak and Daniel Turzik**

Main Theorem: For a simple, connected signed graph of order $n$ and size $|E| = m$, the frustration index $l(\Sigma) \leq g(m,n) := \frac{1}{2}m - \frac{1}{2}\left\lfloor\frac{1}{2}(n - 1)\right\rfloor$. The proof is algorithmic, by constructing a (relatively) small deletion set. Dictionary: $\Sigma$ is an “edge-2-colored graph” $(G,c)$. $E_+$ and $E_-$ are called $E_1$ and $E_2$, a balanced subgraph is “generalized bipartite”, and $m - l(\Sigma)$ is what is calculated. [Thus for a connected, simple graph, $D(\Gamma) \leq g(m,n)$: see Akiyama, Avis, Chvátal, and Era (1981a).]


Generalizes (1982a), with application to signed graphs in Cor. 3.


The polytope $P_B(\Sigma)$ (the authors write $P_{BL}$) is the convex hull in $\mathbb{R}^E$ of incidence vectors of balanced edge sets. It generalizes the bipartite subgraph polytope $P_B(\Gamma) = P_B(-\Gamma)$ (see Barahona, Grötschel, and Mahjoub (1985a)), but is essentially equivalent to it according to Prop. 2: The negative-subdivision trick preserves facets of the polytope. Thm. 1 gives new facets, corresponding to certain circulant subgraphs. (They are certain unions of two Hamilton polygons, each having constant sign.)


Further development of (1987a) for all-negative $\Sigma$. The import for general signed graphs is not discussed.

**Svatopluk Poljak and Zsolt Tuza**

Surveys max-cut and weighted max-cut [that is, max. size balanced subgraph and max. weight balanced subgraph in all-negative signed graphs]. See
esp. §2.9: “Bipartite subgraph polytope and weakly bipartite graphs”. [The weakly bipartite classes announced by Gerards suggested that a signed-graph characterization of weakly bipartite graphs is called for. This is provided by Guenin (20xxa).]

§1.2, “Lower bounds, expected size, and heuristics”, surveys results for all-negative signed graphs that are analogous to results in Akiyama, Avis, Chvátal, and Era (1981a) (q.v.), etc. [Problem. Generalize any of these results, that are not already generalized, to signed simple graphs and to simply signed graphs.]

Y. Pomeau
See B. Derrida.

Dragos Popescu [Dragoș.Radu Popescu]
See Dragoș-Radu Popescu.

Dragoș-Radu Popescu [Dragos Popescu]

A signed $K_n$ is balanced or antibalanced or has a positive and a negative polygon of every length $k=3,\ldots,n$. For odd $n$, the signed $K_n$ if not balanced has at least $\frac{n-1}{2}$ Hamiltonian polygons. For even $n$, $-K_n$ does not maximize the number of negative polygons. A “polygon basis” is a set of the smallest number of polygons whose signs determine all polygon signs. This is proved to have $\binom{n-1}{2}$ members. Furthermore, there is a basis consisting of $k$-gons for each $k=3,\ldots,n$. [A polygon basis in this sense is the same as a basis of polygons for the binary cycle space. See Zaslavsky (1981b), Topp and Ulatowski (1987a).]


Ch. 1: “A-balance” (p. 91). Let $F$ be a spanning subgraph of $K_n$ and $A$ a signed $K_n$. The “product” of signed graphs is $\Sigma_1 \ast \Sigma_2$ whose underlying graph is $|\Sigma_1| \cup |\Sigma_2|$, signed as in $\Sigma_i$ for an edge in only one $\Sigma_i$ but with sign $\sigma_1(e)\sigma_2(e)$ if in both. Let $G_F$ denote the group of all signings of $F$; let $G_F(A)$ be the group generated by the set of restrictions to $F$ of isomorphs of $A$. A member of $G_F(A)$ is “A-balanced”; other members of $G_F$ are $A$-unbalanced.

We let $\hat{\Sigma}$ denote the coset of $\Sigma$ and $\approx$ the “isomorphism” of cosets induced by graph isomorphism, i.e., cosets are isomorphic if they have isomorphic members. Let $\hat{\Sigma}$ be the isomorphism class of $\Sigma$, $\hat{\Sigma}$ the isomorphism class of $\hat{\Sigma}$, and $\hat{\Sigma} := \bigcup \hat{\Sigma}$. Now choose a system of representatives of the coset isomorphism classes, $R = \{\Sigma_1,\ldots,\Sigma_l\}$. Prop. 1.4.1. Each $\hat{\Sigma}$ intersects exactly one $\Sigma_i$. Let $R_i = \{\Sigma_{i_1},\ldots,\Sigma_{i_{a_i}}\}$ be a system of representatives of $\hat{\Sigma}_i/\cong$, arranged so that $|E_-(\Sigma_{ij})|$ is a minimum when $j = 1$. This minimum value is the “[line] index of $A$-imbalance” of each $\Sigma \in \hat{\Sigma}_i$ and is denoted by $\delta_A(\Sigma)$. (§2.1: Taking $A$ to be $K_n$ with one vertex star all negative makes this equal the frustration index $I(\Sigma)$.)

Prop. 1.5.1. $\delta_A(\Sigma)$ is the least number of edges whose sign needs to be changed to make $\Sigma A$-balanced. Prop. 1.5.2. $\delta_A(\Sigma) = |E_-(\Sigma)|$ iff $|E_-(\Sigma) \cap E_-(F,\beta)| \leq \frac{1}{2}|E_-(F,\beta)|$ for every signing $\beta$.
of $F$. Finally, for each $\Sigma \in \mathcal{G}_F$ define the “$\Sigma$-relation” on coset isomorphism classes $\hat{\Sigma}$, to be the relation generated by negating in $\Sigma_1$ all the edges of $E_-(\Sigma)$, extended by isomorphism and transitivity. This is well defined (Prop. 1.6.1) and symmetric (Prop. 1.6.2) and is preserved under negation of coset isomorphism classes (Prop. 1.6.4, 1.6.5). Self-negative classes, such that $\hat{\Sigma} \approx -\hat{\Sigma}$, are the subject of Prop. 1.6.3.

Ch. 2: “Signed complete graphs” (p. 106). §2.5: “$H$-graphs”. If $H$ is a signed $K_h$, a “standard $H$-graph” $\Sigma$ is a signed $K_n$ such that $\Sigma_+ \cong H \cup K_{n-h}^c$. Prop. 2.5.3. Assume certain hypotheses on $n$, $|X_0|$ for $X_0 \subseteq V(\Sigma)$, and a quantity $D^-(H)$ derived from negative degrees. Then $|E_-(\Sigma)| = l(\Sigma) \Rightarrow$ the induced subgraph $G:X_0$ is a standard $H$-graph with $|E_-(\Sigma:X_0)| = l(\Sigma:X_0)$. The cases $H_+ = K_1$, $K_2$, and a 2-edge path are worked out. For the former, Prop. 2.5.3 reduces to Sozański’s (1976a) Thm. 3.

Ch. 3: “Frustration index” (p. 158). Some upper bounds.

Ch. 4: “Evaluations, divisibility properties” (p. 174). Similar to parts of (1996a) and Popescu and Tomescu (1996b).

Ch. 5: “Maximal properties” (p. 198). §5.1: “Minimum number and maximum number of negative stars, resp. 2-stars”. §5.2 is a special case of Popescu and Tomescu (1996a), Thm. 2. §5.3: “On the maximum number of negative cycles in some signed complete graphs”. Shows that Conjecture 1 is false for even $n \geq 6$. Some results on the odd case.

Conjecture 1 (Tomescu). A signed complete graph of odd order has the most negative polygons iff it is antibalanced. (Partial results are in §5.3.) [This example maximizes $l(\Sigma)$. A somewhat related conjecture is in Zaslavsky (1997b).] Conjecture 2. See (1993a). Conjecture 3. Given $k$ and $m$, there is $n(k,m)$ so that for any $n \geq n(k,m)$, a signed $K_n$ with $m$ negative edges has (a) the most negative $k$-gons iff the negative edges are pairwise nonadjacent; (b) the fewest iff the negative edges form a star.

(SG: B(Gen), K, Fr, E: Polygons, Paths)


The numbers of negative subgraphs, especially polygons and paths of length $k$, in an arbitrarily signed $K_n$. Formulas and divisibility and congruence properties. Extends part of Popescu and Tomescu (1996a).

(SG: K, E: Polygons, Paths)

Dragoș-Radu Popescu and Ioan Tomescu


The number $c_p$ of negative polygons of length $p$ in a signed $K_n$ with $s$ negative edges. Thm. 1. For $n$ sufficiently large compared to $p$ and $s$, $c_p$ is minimized if $E_-$ is a star (iff, when $s > 3$) and is maximized iff $E_-$ is a matching. Thm. 2. $c_p$ is divisible by $2^{p-2-\lfloor \log_2(p-1) \rfloor}$. Thm. 3. If $s \sim \lambda n$ and $p \sim \mu n$ and the negative-subgraph degrees are bounded
(this is essential), then asymptotically the fraction of negative $p$-gons is
\[ \frac{1}{2}(1 - e^{-4\lambda^2}). \]

(SG: K: Fr, E: Polygons)


A much earlier version of (1996a) with delayed publication. Contains part of (1996a): a version of Thm. 1 and a restricted form of Thm. 3.

(SG: K: Fr, E: Polygons)

Alexander Postnikov


§4.2 mentions the lift matroid of the integral poise gains of a transitively oriented complete graph. [See also Stanley (1996a).] (GG: M, G)

Alexander Postnikov and Richard P. Stanley

20xxa Deformations of Coxeter hyperplane arrangements. Submitted.

Geert Prins

See F. Harary.

Sharon Pronchik

See L. Fern.

Andrzej Proskurowski

See A.M. Farley.

J. Scott Provan


§4: “Determinacy in a class of network models.” [Fig. 1 and Thm. 4.7 hint at possible digraph version of signed-graph or gain-graph bias matroid.]

(?sg, gg: m(?bases): gen)

Teresa M. Przytycka and Józef H. Przytycki


Generalizing concepts from Kauffman (1989a). [See also Traldi (1989a) and Zaslavsky (1992b).] (SGc: Gen: N, Knot)


A “chromatic graph” is a graph with edges weighted from the set $Z \times \{d, l\}$, $Z$ being [apparently] an arbitrary set of “colors”. A “dichromatic graph” has $Z = \{+, -\}$. Such graphs have general dichromatic polynomials [see Przytycka and Przytycki (1988a), Traldi (1989a), and Zaslavsky (1992b)], as [partially] anticipated by Fortuin and Kasteleyn (1972a). I will not attempt to summarize this paper.

(SGc: N, Knot, Ref)

Jozef H. Przytycki

See K. Murasugi and T.M. Przytycka.
Vlastimil Ptak  
See M. Fiedler.

William R. Pulleyblank  
See J.-M. Bourjolly and M. Grötschel.

L. Pyber  
See L. Lovász.

Hongxun Qin  
See J.E. Bonin.

Louis V. Quintas  
See M. Gargano.

James P. Quirk  
See also L. Bassett and J.S. Maybee.


Comments by W.M. Gorman (pp. 175–189) and Eli Hellerman (pp. 191–192). Discussion: see pp. 193–196. (QM: Sta: sd, b: Exp)

James Quirk and Richard Ruppert  

W.M. Raike  
See A. Charnes.

R. Rammal  
See F. Barahona and I. Bieche.

M.R. Rao  
See Y.M.I. Dirickx.

S.B. Rao  
See also P. Das and [G.R.] Vijaya Kumar.


A complicated solution, with a polynomial-time algorithm, to the problem of characterizing consistency in vertex-signed graphs. Thm. 4.1 points out that graphs with signed vertices and edges can be easily converted to graphs with signed vertices only; thus harmony in graphs with signed vertices and edges is characterized as well. [See Hoede (1992a) for the last word.] (SG, VS: B, Alg)

S.B. Rao, N.M. Singhi, and K.S. Vijayan  
Thomas Raschle and Klaus Simon
Expounded by Mahadev and Peled (1995a), Sect. 8.5 (v.). (p: o, Alg)

H. Donald Ratliff
See J.-Cl. Picard.

Bertram H. Raven
See B.E. Collins.

D.K. Ray-Chaudhuri, N.M. Singhi, and G.R. Vijayakumar

Margaret A. Readdy
See R. Ehrenborg.

P. Reed
See A.J. Bray.

F. Regonati
See E. Damiani.

G. Reinelt
See M. Grötschel.

Victor Reiner
See also P. Edelman.
They are equivalent to acyclic bidirected graphs. (S, sg: O: Str, g)

Daniel J. Richman
See J.S. Maybee.

Robert G. Rieper
See J. Chen.

M.J. Rigby
See A.C. Day.

Chong S. Rim
See H. Choi.

R.D. Ringeisen
See also M.J. Lipman.

Gerhard Ringel
See also N. Hartsfield and M. Jungerman.
“Cascades”: see Youngs (1968b). (sg: O: Appl)


(Fred S. Roberts)


(Fred S. Roberts)


Ch. 9: “Balance theory and social inequalities.” Ch. 10: “Pulse processes and their applications.” Ch. 11: “Qualitative matrices.”

(Fred S. Roberts)


(Fred S. Roberts)


Russian edition of (1976b). (Fred S. Roberts)


§4: “Qualitative stability.” A fine, concise basic survey.

(QM: Fred S. Roberts)
§5: “Balanced signed graphs.” Another concise basic survey, and two open problems (p. 20).

[See Hoede (1992a).]

Fred S. Roberts and Thomas A. Brown


Neil Robertson, P.D. Seymour, and Robin Thomas
See also W. McCuaig.

‡20xxa Permanents, Pfaffian orientations, and even directed circuits. Submitted.

The main theorem is proved also in McCuaig (20xxa).

Robert W. Robinson
See also Harary, Palmer, Robinson, and Schwenk (1977a) and Harary and Robinson (1977a).


The “bilayered digraphs” of §7 are identical to simply signed, loop-free digraphs (where multiple arcs are allowed if they differ in sign or direction). Thm. 1: Their number $b_n =$ number of self-complementary digraphs of order $2n$. Cor. 1: Equality holds if the vertices are signed and $k$-colored. In §8, Cor. 2 concerns vertex-signed and 2-colored digraphs; Cor. 3 concerns vertex-signed tournaments. Assorted remarks on previous signed enumerations, mainly from Harary, Palmer, Robinson, and Schwenk (1977a), are scattered about the article.

Y. Roditty
See I. Krasikov.

Vojtěch Rödl

Milton J. Rosenberg
See also R.P. Abelson.

Milton J. Rosenberg and Robert P. Abelson

An attempt to test structural balance theory experimentally. The test involves, in effect, a signed $K_4$ [an unusually large graph for such an experiment]. Conclusion: there is a tendency to balance but it competes with other forces.

Seymour Rosenberg

M. Rosenfeld
See W.D. McCuaig.

Gian-Carlo Rota
See P. Doubilet.

Uriel G. Rothblum and Hans Schneider


Bernard Roy


Gordon F. Royle
See M.N. Ellingham.

G. Rozenberg
See A. Ehrenfeucht.

Arthur L. Rubin
See P. Erdős.

Richard Ruppert
See J. Quirk.

Herbert J. Ryser
See Richard A. Brualdi.

Rachid Saad

Thm.: In a bidirected all-negative complete graph with a suitable extra hypothesis, the maximum length of a coherent polygon equals the maximum order of a coherent degree-2 subgraph. More or less generalizes part of Bánkfalvi and Bánkfalvi (1968a) (q.v.). [Generalized in Bang-Jensen and Gutin (1998a).] [Problem. Generalize to signed complete graphs or further.]

(p: o: Paths, Alg)

Horst Sachs
See D.M. Cvetković.

Bruce Sagan
See also C. Bennett, A. Björner, A. Blass, F. Harary, and T. Józefiak.


In Section 4, coloring of a signed graph $\Sigma$, especially of $\pm K_n^*$ and $\pm K_n$, is used to calculate and factor the characteristic polynomial of $G(\Sigma)$. Presents the geometrical reinterpretation and generalization by Blass and Sagan (1998a). In Sections 5 and 6, other methods of calculation and factorization are applied to some signed graphs (in their geometrical representation).

**Michael Saks**  
See P.H. Edelman.

**Nicolau C. Saldanha**  
A generalized Kasteleyn matrix is the left-right adjacency matrix of a bipartite gain graph with the complex units as gain group. (A Kasteleyn matrix has for gain group the sign group.) The object is to interpret combinatorially the singular values. The approach is cohomological (cf. Cameron 1977b).

**E. Sampathkumar**  
See *Graph Theory Newsletter* 2, No. 2 (Nov., 1972), Abstract No. 7.

**E. Sampathkumar** and **V.N. Bhave**  
Group-weighted graphs, both in general and where the group has exponent 2 (so all $x^{-1} = x$). Analogs of elementary theorems of Harary and Flamant. Here balance of a polygon means that the weight product around the polygon, taking for each edge either $w(e)$ or $w(e)^{-1}$ arbitrarily, equals 1 for some choice of where to invert.  

**E. Sampathkumar and L. Nanjundaswamy**  
Given a permutation of $\{1, 2, \ldots, n\}$, sign $K_n$ so edge $ij$ is negative if the permutation reverses the order of $i$ and $j$ and is positive otherwise. Kendall’s
measure $\tau$ of correlation of rankings (i.e., permutations) $A$ and $B$ equals 
$$(\left| E_+ \right| - \left| E_- \right|)/|E|$$
in the signature due to $AB^{-1}$. (SG: K)

B. David Saunders
See also A. Berman.

B. David Saunders and Hans Schneider


R.H. Schelp
See P. Erdős.

Baruch Schieber
See L. Cai.

Rüdiger Schmidt

The “count” matroids of graphs (see Whiteley (1996a)) and an extensive further generalization of bicircular matroids that includes bias matroids of biased graphs. His “partly closed set” is a linear class of circuits in an arbitrary “count” matroid. (GG: M, Bic, EC: Gen)

Hans Schneider

Alexander Schrijver
See also A.M.H. Gerards.


Remark 21.2 (p. 308) cites Truemper’s (1982a) definition of balance of a $0, \pm 1$-matrix. (sg: p: I: Exp)


Assume $\Sigma$ embedded in the Klein bottle. If $\Sigma$ is bipartite, negative girth = max. number of disjoint balancing edge sets. If $\Sigma$ is Eulerian, frustration index = max. number of edge-disjoint negative polygons. Proved via polyhedral combinatorics. (SG: T, G, Fr)


§3: “Edge-disjoint paths and multicommodity flows,” pp. 334 ff. [This work suggests there may be a signed-graph generalization with the theorems discussed corresponding to all-negative signatures.] (p: Paths: Exp)


Michelle Schultz
See G. Chartrand.

W. Schwärzler and D.J.A. Welsh

Tutte and dichromatic polynomials of signed matroids, generalized from Kauffman (1989a); this is the 2-colored case of Zaslavsky’s (1992b) strong Tutte functions of colored matroids. [For terminology see Zaslavsky 1992b.] Applications to knot theory.

§2, “A matroid polynomial”, is foundational. Prop. 2.1 characterizes strong Tutte functions of signed matroids by two equations connecting their parameters and their values on signed coloops and loops. [If the function is 0 on positive coloops, the proof is incomplete and the functions $= 0$ except on $M = \emptyset$ are missed.] Prop. 2.2: The Tutte (basis-expansion) polynomial of a function $W$ of signed matroids is well defined iff $W$ is a strong Tutte function. Eq. (2.8) says $W = \text{the rank generating polynomial } Q_{\Sigma}$ (here also called $W$) if certain variables are nonzero; (2.9) shows there are only 3 essential variables since, generically, only the ratio of parameters is essential [an observation that applies to general strong Tutte functions]. Prop. 2.5 computes $Q_{\Sigma}$ of a 2-sum.

§3 adapts $Q_{\Sigma}$ to Kauffman’s and Murasugi’s (1989a) signed-graph polynomials and simplifies some of the latter’s results (esp. his chromatic degree). §4, “The anisotropic Ising model”, concerns the Hamiltonian of a state of a signed graph. The partition function is essentially an evaluation of $Q_{\Sigma}$. §5, “The bracket polynomial”, and §6, “The span of the bracket polynomial”: Certain substitutions reduce $Q_{\Sigma}$ to 1 variable; its properties are examined, esp. in light of knot-theoretic questions. Thm. 6.4 characterizes signed matroids with “full span” (a degree property). §7, “Adequate and semi-adequate link diagrams”, generalizes those notions to signed matroids. §8, “Zero span matroids”: when does $\text{span}($bracket$) = 0$? Yes if $M = M(\Sigma)$ where $\Sigma$ reduces by Reidemeister moves to $K_1$, but the converse is open (and significant if true). (Sc(M), SGc: N, Knot, Phys)

Allen J. Schwenk

András Sebő
See also B. Novick.

1990a Undirected distances and the postman-structure of graphs. *J. Combin. Theory*
See A. Frank (1996a).

J.J. Seidel

See also F.C. Bussemaker, P.J. Cameron, P.W.H. Lemmens, and J.H. van Lint.


Same as (1979a), with photograph. (TG: A)


Reprints many articles on two-graphs and related systems. (TG: Sw, G)


§3.2: “Equidistant sets in elliptic \((d-1)\)-space.” §3.3: “Regular two-graphs.” (TG: A, G: Exp)
J.J. Seidel and D.E. Taylor

J.J. Seidel and S.V. Tsaranov

A group $Ts(\Sigma)$ is defined from a signed complete graph $\Sigma$: its generators are the vertices and its relations are $(uv - \sigma(uv))^2 = 1$ for each edge $uv$. It is invariant under switching, hence determined by the two-graph of $\Sigma$. A certain subgraph of a Coxeter group of a tree $T$ is isomorphic to $Ts(\Sigma)$ for suitable $\Sigma_T$ constructed from $T$. [Generalized in Cameron, Seidel, and Tsaranov (1994a). More on $\Sigma_T$ under Tsaranov (1992a). The construction of $\Sigma_T$ is simplified in Cameron (1994a).] (TG: A, G)

Charles Semple and Geoff Whittle

§7: “Dowling group geometries”. A Dowling geometry of a group $G$ has a partial-field representation iff $G$ is abelian and has at most one involution. (gg: M: I)

E.C. Sewell

See Johnson and Padberg (1982a) for definitions. §2, “Equivalence to stable set problem”: Optimization on the bidirected stable set polytope is reduced to optimization on a stable set polytope with no more variables. Results of Bourjolly (1988a) and Hochbaum, Megiddo, Naor, and Tamir (1993a) can thereby be explained. §3, “Perfect bigraphs”, proves the conjectures of Johnson and Padberg (1982a): a transitively closed bidirection of a simple graph is perfect iff its underlying graph is perfect. [Also proved by Ikebe and Tamura (20xxa).] Dictionary: “Bigraph” = bidirected graph $B$. “Stable” set in $B$ = vertex set inducing no introverted edge. (SG: O: I, G, sw)

P.D. Seymour

See also Gerards, Lovász, et al. (1990a), W. McCuaig, and N. Robertson.


The central example is $Q_6 = C_\cdot(-K_4)$, the clutter of (edge sets of) negative polygons in $-K_4$. P. 199: the extended lift matroid $L_0(-K_4) = F_7^*$, the dual Fano matroid. Result (3.4) readily generalizes (by the negative-subdivision trick) to: every $C_\cdot(\Sigma)$ is a binary clutter, that is, a port of a binary matroid. [This is also immediate from the construction of $L_0(\Sigma)$.] P. 200, (i)–(iii): amongst minor-minimal binary clutters without the “weak MFMC property” are the circuit clutter of $F_7^*$ and $C(-K_5)$ and its blocker.
Main Thm. (§5): A binary clutter is “Mengerian” (I omit the definition) iff it does not have \(C_\infty(-K_4)\) as a minor. (See p. 200 for the antecedent theorem of Gallai.)

[See Cormuéjols (20xxa), Guenin (1998b) for more.]


Conjecture (based on (1977a)). A binary clutter has the weak MFMC property iff no minor is either the circuit clutter of \(F_7\) or \(C_\infty(-K_5)\) or its blocker.


In Thm. 6.6, p. 546, interpreting \(G\) as a signed graph and an “odd-\(K_4\)” as a subdivision of \(-K_4\) gives the signed graph generalization, due to Gerards and Schrijver (1986a) [also Gerards (1990a), Thm. 3.2.3]. Let \(\Sigma\) be a signed simple, 3-connected graph in which no 3-separation has \(>4\) edges on both sides. Then \(\Sigma\) has no \(-K_4\) minor iff either (i) deleting some vertex makes it balanced (the complete lift matroid of this type is graphic); or (ii) it is cylindrical: it can be drawn on a cylindrical surface that has a lengthwise red line so that an edge is negative iff it crosses the red line an odd number of times [Note: the extended lift matroid of this type is cographic, as observed by, I think, Gerards and Schrijver or by Lovász]. [See Pagano (1998a) for another use of cylindrical signed graphs.]

[Problem. Find the forbidden topological subgraphs, link minors, and \(Y\Delta\) graphs for cylindrical signed graphs.] [Question. Embed a signed graph in the plane with \(k\) distinguished faces so that a polygon’s sign is the parity of the number of distinguished faces it surrounds. Cylindrical embedding is \(k=1\). For each \(k\), which signed graphs are so embeddable?]

Thm. 6.7, pp. 546–547, generalizes to signed graphs, interpreting \(G\) as a signed graph and an “odd cycle” as a negative polygon. Take a signed simple, 3-connected, internally 4-connected graph. It has no two vertex-disjoint negative polygons iff it is one of four types: (i) deleting some vertex makes it balanced; (ii) deleting the edges of an unbalanced triangle makes it balanced; (iii) it has order \(\leq 5\); (iv) it can be orientation-embedded in the projective plane. This is due to Lovász; see, if you can, Gerards et al. (1990a). [A 2-connected \(\Sigma\) has no vertex-disjoint negative polygons iff \(G(\Sigma)\) is binary iff \(G(\Sigma)\) is regular iff the lift matroid \(L(\Sigma)\) is regular. See Pagano (1998a) for classification of \(\Sigma\) with vertex-disjoint negative polygons according to representability of the bias matroid.]

Paul Seymour and Carsten Thomassen


“Even” means every signing contains a positive cycle. A digraph is even iff it contains a subdigraph that is obtained from a symmetric odd-polygon digraph by subdivision and a vertex-splitting operation. [Cf. Thomassen (1985a).]

L. de Sèze

See J. Vannimenus.
Bryan L. Shader  
See Richard A. Brualdi.

Jia-Yu Shao  
See R. Manber.

John Shawe-Taylor  
See T. Pisanski.

F.B. Shepherd  
See A.M.H. Gerards.

Ronald G. Sherwin  

A very simple [but not efficient] matrix algorithm for counting different types of polygons in a signed (di)graph. [“Valence” means sign, unfortunately.]

Jeng-Horng Sheu  
See I. Gutman.

Elizabeth G. Shrader and David W. Lewit  

For $\Gamma \subset K_n$ and signing $\sigma$ of $\Gamma$, “plausibility” = mean and “differentiability” = standard deviation of $f(K_n; \sigma')$ over all extensions of $\sigma$ to $K_n$, where $f$ is any function that measures degree of balance. Proposed: tendency toward balance is high when plausibility and differentiability are high. A specific $f$, based on triangles and quite complicated, is studied for $n = 4$, with experiments.

Alan Shuchat  
See R. Shull.

Randy Shull, James B. Orlin, Alan Shuchat, and Marianne L. Gardner  

[See Coullard, del Greco, and Wagner (1991a).]  
(Bic(Bases))

Randy Shull, Alan Shuchat, James B. Orlin, and Marianne Lepp  


E.E. Shult  
See P.J. Cameron.

B. Simeone  

Slobodan K. Simić  
See also D.M. Cvetković.


R. Simion and D.-S. Cao


J.M.S. Simões-Pereira


J.G. Schmidt

R. Stei
dom
1996a A remarkable generalization of Schmidt’s (1979a) remarkable generalization in §4.4. (GG: M, Bic, EC: Gen: Exp, Exr, Ref)
Klaus Simon  
See T. Raschle.

M. Simonovits  
See B. Bollobás, J.A. Bondy, and P. Erdős.

N.M. Singhi  

N.M. Singhi and G.R. Vijayakumar  

Jozef Širáň  
See also D. Archdeacon.

A signed graph orientation-embeds in only one surface iff any two polygons are vertex disjoint. (SG: T)

Richard A. Duke (The genus, regional number, and Betti number of a graph. *Canad. J. Math.* 18 (1966), 817–822. MR 33 #4917.) proved that the (orientable) genus range of a graph forms a contiguous set of integers. Stahl (1978a) proved the analog for nonorientable embeddings. Širáň shows this need not be the case for the demigenus range of an unbalanced signed graph. However, any gaps consist of a single integer each. The main examples with gaps are vertex amalgamations of balanced and uniquely embeddable unbalanced signed graphs, but a 3-connected example is $+W_6$ together with the negative diameters of the rim. *Question 1* (Širáň). Do all gaps occur at the bottom of the demigenus range? [*Question 2.* Can one in some way derive almost all signed graphs with gaps from balanced ones?] (SG: T)

Jozef Širáň and Martin Škoviera  
The maximum demigenus $d_M(\Sigma) =$ the largest demigenus of a closed surface in which $\Sigma$ orientation-embeds. Two formulas are proved for $d_M(\Sigma)$: one a minimum and the other a maximum of readily computable numbers. Thus $d_M(\Sigma)$ has a “good” (polynomial) characterization. Along the way, several results are proved about single-face embeddings. *Problem* (§11). Characterize those edge-2-connected $\Sigma$ such that $\Sigma$ and all $\Sigma \setminus e$ have single-face embeddings. [A complex and lovely paper.] (SG: T)

A. Skhreiver [A. Schrijver]  
See A. Schrijver.

Martin Škoviera  
See also R. Nedela and J. Širáň.


Automorphisms of covering projections of canonical covering graphs of gain graphs. (GG: T, Cov, Aut, Sw)


The model: each edge is selected with probability $p$, positive with probability $s$. Under mild hypotheses on $p$ and $s$, $\Sigma$ is almost surely unbalanced and almost surely has a 1-face orientation embedding. (SG: Rand, E, T)

N.J.A. Sloane
See P.C. Fishburn, R.L. Graham, and C.L. Mallows.

J. Laurie Snell
See J. Berger and J.G. Kemeny.

Patrick Solé and Thomas Zaslavsky

Among other things, improves some results in Akiyama, Avis, Chvátal, and Era (1981a). Thm. 1: For a loopless graph with $c$ components, $D(\Gamma) \geq \frac{1}{2}m - \sqrt{\frac{1}{2}\ln 2}m(n-c)$. Thm. 2: For a simple, bipartite graph, $D(\Gamma) \leq \frac{1}{2}(m-\sqrt{m})$. Conjecture. The best general asymptotic lower bound is $D(\Gamma) \geq \frac{1}{2}m - c_1\sqrt{mn} + o(\sqrt{mn})$ where $c_1$ is some constant between $\sqrt{\frac{1}{2}\ln 2}$ and $\frac{1}{2}\pi$. Question. What is $c_1$ for, e.g., $k$-connected graphs? Thm. 4 gives girth-based upper bounds on $D(\Gamma)$. §5, “Embedded graphs”, has bounds for several examples obtained by surface duality. All proofs are via covering radius of the cutset code of $\Gamma$. (SG: Fr, T)

Louis Solomon
See P. Orlik.

Tadeusz Sozański

$\Sigma$ denotes a signed $K_n$. The “level of balance” (“indice du niveau d’équilibre”) $\rho(\Sigma) :=$ maximum order of a balanced subgraph. [Complement of the vertex deletion number.] Define distance $d(\Sigma_1, \Sigma_2) := |E_1 \triangle E_2|$. Say $\Sigma$ is $p$-clusterable if $\Sigma_+ \subset$ consists of $p$ disjoint cliques [its “clusters”]. Thm. 1 evaluates the frustration index of a $p$-clusterable $\Sigma$. Thm. 2 bounds $l(\Sigma)$ in terms of $n$ and $\rho(\Sigma)$. A negation set $U$ for $\Sigma$ “conserves” a balanced induced subgraph if they are edge-disjoint; it is “(strongly) conservative” if it conserves some (resp., every) maximum-order balanced induced subgraph. Thm. 3: Every minimum negation set conserves every balanced induced subgraph of order $> \frac{2}{3}n$. Thm. 4: A minimum negation set can be ordered so that, successively negating its edges one by one, $\rho$ never decreases. (SG: K: Fr, Cl)

“Weak isomorphism” = switching isomorphism. Principal results: The number of switching nonisomorphic signed $K_n$’s. (Cf. Mallows and Sloane (1975a).) The number that are switching isomorphic to their negations. The number of nonisomorphic (not switching nonisomorphic!) balanced signings of a given graph.


Joel Spencer
See T.A. Brown.

Murali K. Srinivasan
Decomposes the Dowling lattice $Q_n(\mathcal{G})$ into Boolean algebras, indexed in part by integer compositions, that are cover-preserving and centered above the middle rank.

Saul Stahl
A generalized embedding scheme for a graph is identical to a rotation system for a signing of the graph. Thm. 2: Signed rotation systems describe all cellular embeddings of a graph. Thm. 4: Embeddings are homeomorphic iff their signed rotation systems are switching equivalent. Thm. 5: An embedding is orientable iff its signature is balanced. Compare Ringel (1977a). Dictionary: $\lambda$ is the signature. “$\lambda$-trivial” means balanced.

Richard P. Stanley
Ch. 3, “Partially ordered sets”: Exercise 51, pp. 165 and 191, concerns the Dowling lattices of a group and mentions Zaslavsky’s generalizations [signed and biased graphs].

Russian translation of Stanley (1986).


All-negative complete graphs (implicit in §3) and signed colorings (§4) are used to find the number of ordered degree sequences of $n$-vertex graphs and to study their convex hull.


Deformed braid hyperplane arrangements, i.e., hyperplane representations of $\text{Lat}^b(K_n, \varphi)$ with gains $\varphi(ij) = l_i \in \mathbb{Z}$ where $i < j$. ($\text{Lat}^b$ denotes the geometric semilattice of balanced flats of the bias or lift matroid. Write $\varphi_l$ if all $l_i = l$.) In particular (§4), $\varphi = \varphi_1$. Also (§5), the “Shi” arrangement, which represents $\text{Lat}^b \Phi$ where $\Phi = (K_n, \varphi_0) \cup (K_n, \varphi_1)$.


Additional exercises and some updating.


Kenneth Steiglitz

See C.H. Papadimitriou.

R. Stenli [Richard P. Stanley]

See R.P. Stanley.

Daniel L. Stern


Informally describes frustration in spin glasses in terms of randomly ferromagnetic and antiferromagnetic interactions (see Toulouse (1977a)) and gives some history and applications.

B.M. Stewart


Allen H. Stix


J. Randolph Stonesifer


The second kind of Whitney numbers of a Dowling lattice are binomially concave, hence strongly logarithmically concave, hence unimodal. [Famous Problem (Rota). Generalize this.]
Bernd Sturmfels  
See A. Björner.

J. Stutz  
See F. Glover.

Benjamin Sudakov  
See G. Gutin.

Janusz Szczypula  
See P. Doreian.

E. Szemerédi  
See B. Bollobás.

Z. Szigeti  
See A.A. Ageev.

Irving Tallman  

Arie Tamir  
See also D. Hochbaum.


Akihisa Tamura  
See also Y.T. Ikebe and D. Nakamura.


20xxa Perfect (0, ±1)-matrices and perfect bidirected graphs. Submitted. (sg: O: G, Alg)

Percy H. Tannenbaum  
See C.E. Osgood.

Éva Tardos and Kevin D. Wayne  

Michael Tarsi  
See F. Jaeger.

D.E. Taylor  
See also J.J. Seidel.
Introducing two-graphs and regular two-graphs (defined by G. Higman, unpublished). [See Seidel (1976a) etc. for more.] A “two-graph” is the class \( C_{3-} \) of negative triangles of a signed complete graph \((K_n, \sigma)\). (See §2. p. 258, where the group is \( \mathbb{Z}_2 \cong \{+,-\} \) and the definition is in terms of the 2-coboundary operator.) Two-graphs and switching classes of signed complete graphs are equivalent concepts. (Stated in terms of Seidel switching in §2, p. 260.) A two-graph is “regular” if every edge lies in the same number of negative triangles. Thm.: \( C_{3-} \) is regular iff \( A(K_n, \sigma) \) has at most two eigenvalues. Various parameters of regular two-graphs are calculated.

(TG: A. G)

Herbert Taylor
See P. Erdős.

Howard F. Taylor

A thorough and pleasantly written survey of psychological theories of balance, including formalizations by signed graphs (Chs. 3 and 6), experimental tests and critical evaluation of the formalisms, and so forth. Ch. 2: “Substantive models of balance”, takes the perspective of social psychology. §2.2: “Varieties of balance theory”, reviews the theories of Heider (1946a) (the source of Harary’s (1953a) invention of signed graphs), Osgood and Tannenbaum (1955a), and others. §2.2e: “The Rosenberg-Abelson modifications”, discusses their introduction of the “cost” of change of relations, which led them (Abelson and Rosenberg 1958a) to propose the frustration index as a measure of imbalance.

(PsS, SG, WG: Exp, Ref)

Ch. 3: “Formal models of balance”, reviews various graph-theoretic models: signed and weighted signed, different ways to weigh imbalance, etc., the relationship to theories in social psychology being constantly kept in mind.

§3.1: “Graph theory and balance theory”, presents the basics of balance, measures of degree of balance by polygons (Cartwright and Harary (1956a)), polygons with strengths of edges (Morrisette (1958a)), local balance and \( N \)-balance (Harary (1955a)), edge deletion and negation (Abelson and Rosenberg (1958a), Harary (1959b)), vertex elimination number (Harary (1959b)).

§3.2: “Evaluation of formalizations: strong points”, and §3.3: “Evaluation of formalizations: weak points”, judged from the applied standpoint. §3.3a: “Discrepancies between cycles or subsets of cycles”, suggests that differing degrees of imbalance among certain different subsets of the vertices may be significant [Is this reasonable?] and proposes measures, e.g., a variance measure (p. 71), of this “discrepancy”.

(SG, WG: B, Fr: Exp)

Ch. 6: “Issues involving formalization”, goes into more detail. §6.1: “Indices of balance”, compares five indices, in particular Phillips’ (1967a) eigenvalue index (also in Abelson (1967a)) with examples to show that the index differentiates among different balanced signings of the same graph. §6.2: “Extra-balance properties”, discusses Davis’s (1967a) clustering (§6.2b) and indices of clustering (§6.2c). §6.3: “The problem of cycle length and non-local cycles”. Are long polygons less important? Do polygons at a distance from an actor (that is, a vertex) have less effect on the actor in balancing processes?

(SG: Fr, A: Exp)
Hidetaka Terasaka
See S. Kinoshita.

Morwen B. Thistlethwaite

A 1-variable Tutte-style polynomial $\Gamma_\Sigma$ of a sign-colored graph. Fix an edge ordering. For each spanning tree $T$ and edge $e$, let $\mu_T(e) = -A^{3\tau_T(e)\sigma(e)}$ if $e$ is active with respect to $T$, $A^{\tau_T(e)\sigma(e)}$ if it is inactive, where $\tau_T(e) = +1$ if $e \in T$, $-1$ if $e \notin T$. Then $\Gamma_\Sigma(A) = \sum_T \prod_{e \in T} \mu_T(e)$. [In the notation of Zaslavsky (1992a), $\Gamma_\Sigma(A) = Q_\Sigma$ with $a_\epsilon = A^{-\epsilon}, b_\epsilon = A^{-\epsilon}$ for $\epsilon = \pm 1$ and $u = v = -(A^2 + A^{-2})$.] §§3 and 4 show $\Gamma_\Sigma$ is independent of the ordering. Other sections derive consequences for knot theory. [This marks the invention of a Tutte-style polynomial of a colored, or parametrized or weighted, graph or matroid, developed in Kauffman (1989a) and successors.]

A.D. Thomas
See F.W. Clarke.

Robin Thomas
See W. McCuaig and N. Robertson.

Carsten Thomassen
See also Paul Seymour.


It is an NP-complete problem to decide whether a given signed digraph has a positive but not all-positive cycle, even if there are only 2 negative arcs. This follows from Lemma 3 of Steven Fortune, John Hopcroft, and James Wyllie, The directed subgraph homeomorphism problem (*Theoret. Computer Sci.* 10 (1980), 111–121. MR 81e:68079. Zbl. 419.05028.) by the simple argument in the proof of Prop. 2.1 here.

To decide whether a specified arc of a digraph lies in an even cycle, or in an odd cycle, are NP-complete problems (Prop. 2.1). To decide existence of an even cycle [hence, by the negative subdivision trick, of a positive cycle in a signed digraph] is difficult [but is solvable in polynomial time; see Robertson, Seymour, and Thomas (20xxa)], although existence of an odd cycle [resp., of a negative cycle] is easy, by a trick here attributed to Edmonds (unpublished). Prop. 2.2: Deciding existence of a positive cycle in a signed digraph is polynomial-time solvable if $|E_-|$ is bounded. Thm. 3.2: If the outdegrees of a digraph are all $> \log_2 n$, then every signing has a positive cycle, and this bound is best possible; restricting to the all-negative signature, the lower bound might (it’s not known) go down by a factor of up to 2, but certainly (Thm. 3.1) a constant minimum on outdegree does not imply existence of an even cycle. [See (1992a) for the effect of connectivity.]


(SD, P: B, Alg)

§8: “Even directed circuits and sign-nonsingular matrices.”

(SD, QM: B, Sol: Exp)

§§8–10 treat even cycles in digraphs.

(SD: B: Exp)

[General Problem. Generalize even-cycle and odd-cycle results to positive and negative cycles in signed digraphs, the unsigned results corresponding to all-negative signatures.]


(QM, SD: Sol, A)


§5 describes the “fundamental cycle method”, a simple algorithm for a shortest unbalanced polygon in a biased graph (Thm. 5.1). Thus the method finds a shortest noncontractible polygon (Thm. 5.2). A noteworthy linear class: the surface-separating (“II-separating”) polygons (p. 166). Dictionary: “3-path-condition” on a class $F$ of polygons = property that $F^c$ is a linear class. “Möbius cycle” = negative polygon in the signature induced by a nonorientable embedding.

(gg, sg: Alg, T)


A digraph that is strongly connected and has all in- and out-degrees $\geq 3$ contains an even cycle.

(sd: p: b)


A polynomial-time algorithm for deciding the existence of an even cycle in a planar digraph.

(sd: p: b: Alg)

G.L. Thompson
See V. Balachandran.

R.L. Tobin


Bjarne Toft
See T.R. Jensen.

Ioan Tomescu
See also D.R. Popescu.


Independent proof of Petersdorf’s (1966a) Satz 1. Also, treats similarly a variation on the frustration index.

(SG: Fr)


(SG: B, Cl)


The parity of the number of negative triangles = that of $n|E_-|$. The number of negative $t$-gons is even when $n, t \geq 4$ [strengthened in Popescu (1991a), (1996a)].

(SG: B)

C.B. Tompkins
See I. Heller.

J. Topp and W. Ulatowski
An additive real gain graph is balanced iff every polygon in a polygon basis is balanced, iff the gains are induced by a vertex labelling [in effect, switch to 0], iff every two paths with the same endpoints have the same gains. A digraph is gradable (Harary, Norman, and Cartwright (1965a); also see Marcu (1980a)) iff \( \varphi_1 \) is balanced, where for each arc \( e, \varphi_1(e) = 1 \in \mathbb{Z} \) (Thm. 3). The Windy Postman Problem (Thms. 4, 5). (GG, GD: B)

Aleksandar Torgašev
See also D.M. Cvetković.
An infinite analog of Doob’s (1973a) characterization via the even-cycle matroid of when a line graph has \(-2\) as an eigenvalue. [Problem. Generalize to line graphs of infinite signed graphs.] (p: A(LG))

An infinite graph is a generalized line graph iff its least “limit” eigenvalue \( \geq -2 \). [Problem. Generalize to line graphs of infinite signed graphs.] (p: A(LG))

Gérard Toulouse
See also B. Derrida and J. Vannimenus.
Introduces the notion of imbalance (“frustration”) of a signed graph to account for inherent disorder in an Ising model (here synonymous with a signed graph, usually a lattice graph). (Positive and negative edges are called “ferromagnetic and antiferromagnetic bonds”.) Observes that switching the edge signs from all positive (the model of D.D. Mattis, Phys. Lett. 56A (1976), 421–?) makes no essential difference. In a planar lattice [or any plane graph] frustration of face boundaries (“plaquettes”) can be thought of as curvature, i.e., failure of flatness. Proposes two kinds of asymptotic behavior of frustration as a polygon encloses more plaquettes. The planar-duality approach for finding the states with minimum frustration (i.e., switchings with fewest negative edges); the number of such states is the “ground-state degeneracy” and is important. Ideas are sketched; no proofs. (SG: Phys, Sw, B)

Gérard Toulouse and Jean Vannimenus

Popular exposition of the elements of frustration in relation to the Ising model [evidently based on Toulouse (1977a)]. Briefly mentions the social psychology application. [See also Stern (1989a).]

Lorenzo Traldi

Generalizing Kauffman’s (1989a) Tutte polynomial of a sign-colored graph, Traldi’s “weighted dichromatic polynomial” \( Q(\Gamma; t, z) \) is the \( Q(1, w; t, z) \) of Zaslavsky (1992b), in which the deletion-contraction parameters \( a_e = 1 \) and \( b_e = w(e) \), the weight of \( e \). Thm. 2 gives the Tutte-style spanning-tree expansion. Thm. 4: Kauffman’s Tutte polynomial \( Q(\Sigma)(A, B, d) = d^{-1}A^{E=1}B^{E=1}Q(1, w; d, d) \) for connected \( \Sigma \), with \( w(e) = (AB^{-1})^{\sigma(e)} \).

[SSee Kauffman (1989a) for other generalizations. Traldi gives perhaps too much credit to Fortuin and Kasteleyn (1972a).]

Marian Trenkler
See S. Jezný.

Nenad Trinajstić
See also A. Graovac.


[SG: Chem, A: Exp]


Ch. 3, §V.B: “Möbius graphs.” Ch. 4, §I: “The adjacency matrix”: see pp. 42–43. Ch. 5: “The characteristic polynomial of a graph”, §II.B: “The extension of the Sachs formula to Möbius systems”; §III.D: “Möbius cycles”. Ch. 6, §VIII: “Eigenvalues of Möbius annulenes” (i.e., unbalanced polygons); §IX: “A classification scheme for monocyclic systems” (i.e., characteristic polynomials of polygons).

[SG: A, Chem]

Ch. 7: “Topological resonance energy,” §V.C: “Möbius annulenes”; §V.G: “Aromaticity in the lowest excited state of annulenes”.

K. Truemper
See also Gerards et al. (1990a).


[gg: GN, sg: B, Sw]


A 0,±1-matrix is called “balanced” if it contains no submatrix that is the incidence matrix of a negative polygon. More generally, $\alpha$-balance of a 0,±1-matrix corresponds to prescribing the signs of holes in a signed graph. Main theorem characterizes the sets of holes (chordless polygons) in a graph that can be the balanced holes in some signing. [A major result. See Conforti and Kapoor (1998a) for a new proof and discussion of applications.] (sg: B, I)


**Marcello Truzzi**

See F. Harary.

**S.V. Tsaranov**

See also F.C. Bussemaker, P.J. Cameron, and J.J. Seidel.


A two-graph whose points are the edges of a tree $T$ and whose triples are the nonseparating triples of edges of $T$ (from Seidel and Tsaranov (1990a) via Cameron (1994a)). An associated signed complete graph $\Sigma_T$ on vertex set $E(T)$ is obtained by orienting $T$ arbitrarily, then taking $\sigma_T(ef) = +$ or $-$ depending on whether $e$ and $f$ are similarly or oppositely oriented in the path of $T$ that contains both. Reorienting edges corresponds to switching $\Sigma_T$. Thm.: Letting $n = |V(T)|$, the matrices $3I_n + A(\Sigma_T)$ and $2I_{n+1} - A(T)$ have the same numbers of zero and negative eigenvalues. (TG: A, G)


New proof of theorem on the group (Seidel and Tsaranov 1990a) of the two-graph (Tsaranov 1992a) of a tree. (TG: A, G)

**Michael Tsatsomeros**

See C.R. Johnson.

**Thomas W. Tucker**

See J.L. Gross.

**Vanda Tulli**

See A. Bellacicco.
Edward C. Turner  
See R.Z. Goldstein.

Daniel Turzik  
See S. Poljak.

W.T. Tutte  

Zsolt Tuza  
See S. Poljak.

J.P. Uhry  
See F. Barahona and I. Bieche.

Włodzimierz Ulatowski  
See also J. Topp.

Examines injective, nowhere zero, balanced gains (called “graceful labelings”) from $Z_{m+1}$, $m = |E|$, on arbitrarily oriented polygons and variously oriented paths.  

*Question.* Does this work generalize to bidirected polygons and paths?  

(sg: EC, D)

N.B. Ul’janov [N.B. Ul’yanov]  
See N.B. Ul’yanov.

N.B. Ul’yanov  
See D.O. Logofet.

M.E. Van Valkenburg  
See W. Mayeda.

Pauline van den Driessche  
See J. Bélair, C. Jeffries, C.R. Johnson, and V. Klee.

Jean Vannimenus  
See also B. Derrida and G. Toulouse.

J. Vannimenus, J.M. Maillard, and L. de Sèze  

(Phys: SG: Fr)

J. Vannimenus and G. Toulouse  

(SG: Phys: Fr)

Vijay V. Vazirani and Mihalis Yannakakis  
Slightly abridged version of (1989a).  

(SD: A, B: Alg)
Vijay V. Vazirani and Milhalis [Mihalis] Yannakakis

“Evenness” of a digraph (i.e., every signing contains a positive cycle) is polynomial-time equivalent to evaluableity of a certain 0–1 permanent by a determinant and to parts of the existence and recognition problems for Pfaffian orientations of a graph. Briefly expounded in Brundage (1996a).

(SD: A, B: Alg)

G.K. Vijayakumar
See G.R. Vijayakumar.

G.R. Vijayakumar
See also P.D. Chawathe, D.K. Ray-Chaudhuri, and N.M. Singhi.


G.R. Vijayakumar and N.M. Singh

G.R. Vijayakumar (as “Vijaya Kumar”), S.B. Rao, and N.M. Singh

K.S. Vijayan
See S.B. Rao.

Jacques Villain


Andrew Vince
See Theorem 6.1.

E. Vincent, J. Hammann, and M. Ocío
Recent Progress in Random Magnets, pp. 207–236. World Scientific, Singapore, 

Surveys experiments with spin glass materials, especially their aging behavior. Interprets results as tending to support the Parisi-type model (see notes 
on Mézard, Parisi, and Virasoro (1987a)).

Miguel Angel Virasoro
See M. Mézard.

Jože Vrabek
See T. Pisanski.

Kristina Vušković
See M. Conforti.

Donald K. Wagner
See also V. Chandru and C.R. Coullard.

MR 87c:05041. Zbl. 584.05019.

Prop. 1 and Thm. 2 show that \( n \)-connectivity of the bicircular matroid \( B(\Gamma) \) 
is equivalent to \( "n\)-biconnectivity” of \( \Gamma \).

When do two 3-biconnected graphs have isomorphic bicircular matroids? \S 5 
proves that 3-biconnected graphs with > 4 vertices have isomorphic bicir-
cular matroids if one is obtained from the other by a sequence of operations 
called “edge rolling” and “3-star rotation”. This is the bicircular analog 
of Whitney’s polygon-matroid isomorphism theorem, but it is complicated. 
[An important theorem, generalized to all bicircular matroids in Coullard, 
bias matroids of biased graphs. Find the analog for lift matroids.]

(Bic: Str)


“Factor matroid” = even-cycle matroid \( G(-\Gamma) \). Decides when \( G(-\Gamma) \cong 
G(B) \) where \( B \) is a given bipartite, 4-connected graph.

(EC: Str)

Bronislaw Wajnryb
See R. Aharoni.

Derek A. Waller
See also F.W. Clarke.

MR 53 
#10662. Zbl. 318.05113.

(SG: Cov)

Egon Wanke
See also F. Höfting.

1993a Paths and cycles in finite periodic graphs. In: Andrzej M. Borzyszkowski and Ste-

Broadly resembles Höfting and Wanke (1994a) but omits those edges of \( \Phi \) 
that are affected by the modulus \( \alpha \).
20xxa Paths and cycles in finite periodic graphs. Submitted.

Full version of (1993a).

(GD(Cov): Alg)

G.H. Wannier

(Phys: P: Fr)

Stanley Wasserman and Katherine Faust

§1.2: “Historical and theoretical foundations.” A brief summary of various network methods in sociometry, signed graphs and digraphs among them.

§4.4: “Signed graphs and signed directed graphs.” Mathematical basics.

Ch. 6: “Structural balance and transitivity.” Application of balance of signed (di)graphs and of ensuing notions like clusterability, historically evolving into transitivity of unsigned digraphs. History and evaluation.

(PsS, SG, SD: B, Fr, Cl, Gen: Exp, Ref)

William C. Waterhouse


John J. Watkins
See R.J. Wilson.

Kevin D. Wayne
See Ŕ. Tardos.

Jeffrey R. Weeks and Kenneth P. Bogart

(GD(Cov): Alg)

Gerry M. Weiner
See J.S. Maybee.

Volkmar Welker

The arrangement is the affine part (that is, where \(x_0 = 1\)) of the projective representation of \(G(\Phi)\), where \(\Phi\) is the complex multiplicative gain graph \(\Phi = \{1\} K_{n+1} \cup \{re_{0i} : 1 \leq i \leq n \text{ and } 2 \leq r \leq s\}\. \) Here the vertex set is \(\{0, 1, \ldots, n\}\), \(s\) is any positive integer, and \(re_{0i}\) (in the paper, \(e_{0i}(r)\)) denotes an edge \(v_0v_i\) with gain \(r\). The topics of interest are those related to the complex complement. The study is based on the combinatorics of the intersection semilattice [that is, the geometric semilattice \(\text{Lat}^b \Phi\) of balanced flats, including the Poincaré polynomial of the arrangement [equivalent to the balanced chromatic polynomial of \(\Phi\)].

(gg: M, G, N)

Albert L. Wells, Jnr.
See also P.J. Cameron and Y. Cheng.


(SG: Sw, A, E, TG, G, Cov, Aut)


(SG: Sw, E, Aut)
D.J.A. Welsh [Dominic Welsh]
See also L. Lovász and W. Schwaärzler.


§11.4: “Partition matroids determined by finite groups”, sketches the most basic parts of Dowling (1973b).


The signed graph of a link diagram is employed to get an upper bound.


Includes very brief treatments of some appearances of signed graphs.

§2.2, “Tait colourings”, defines the signed graph of a link diagram, mentioned again in observation (2.3.1) on alternating links and Prop (5.2.16) on “states models” (from Schwärzler and Welsh (1993a)). §5.6, “Thistlethwaite’s non-triviality criterion”: the criterion depends on the signed graph.

§2.5, “The braid index and the Seifert index of a link”, defines the Seifert graph, a signed graph based on splitting the link diagram. (SGc, Knot)

§5.7, “Link invariants and statistical mechanics”, defines a relatively simple spin model for signed graphs, with an arbitrary finite number of possible spin values. The partition function is related to link diagrams.

§4.2, “The Ising model”, introduces the basic concepts in mathematical terms. §6.4, “The complexity of the Ising model”, “Computing ground states of spin systems”, pp. 105–107, discusses finding a ground state of the Ising model. This is described as the min-weight cut problem with weights the negatives [this is an error] of the Ising bond interaction values: that is, the weighted frustration index problem in the negative [erroneous] of the Ising graph. It is the max-cut problem when the Ising graph is balanced (ferromagnetic) [should be antibalanced (antiferromagnetic)]. For external magnetic field, follows Barahona (1982a).

§3.6, “Ice models”, counts “ice configurations” (certain graph orientations) via poise gains modulo 3, although the counting function is not gain-graphic.

§4.4: “The Ashkin–Teller–Potts model”. This treatment of the Potts model has a different Hamiltonian from that of Fischer and Hertz (1991a). [It does not seem that Welsh intends to admit edge signs but if they are allowed then the Hamiltonian (without edge weights) is $-\sum \sigma(e_{ij})(\delta(s_i, s_j) - 1)$. Up to halving and a constant term, this is Doreian and Mrvar’s (1996a) clusterability measure $P(\pi)$, with $\alpha = .5$, of the vertex partition induced by the state.] [Also cf. Fischer and Hertz (1991a).] (cl, Phys)


Link diagrams ↔ dual pairs of sign-colored plane graphs: based on Yajima and Kinoshita (1957a). Unsolved algorithmic problems about knots based on
link diagrams; in particular, triviality of diagrams is equivalent to Problem 4.2: A polynomial-time algorithm to decide whether the graphical Reide-
meister moves can convert a given signed plane graph to one with edges all
of one sign. 

(SGc: D, Knot: Alg, Exp)

1993c Knots and braids: some algorithmic questions. In: Neil Robertson and Paul Sey-
792.05058.

§1 presents the sign-colored graph of a link diagram and §5, “Reidemeister
graphs”, describes Schwärtzer and Welsh (1993a). §3 defines the sign-colored
Seifert graph. 

(SGc. Sc(M): N, Alg, Knot: Exp)

1997a Knots. In: Lowell W. Beineke and Robin J. Wilson, eds., Graph Connections:
Relationships between Graph Theory and other Areas of Mathematics, Ch. 12,
878.57001.

Mostly describes the signed graph of a link diagram and its relation to knot
theory, including knot properties deducible directly from the signed graph,
the Kauffman bracket and two-variable polynomials, etc. Similar to relevant
parts of (1993a). 

(D. de Werra
See C. Benzaken.

Arthur T. White

1984a Graphs, Groups and Surfaces. Completely revised and enlarged edn. North Hol-
551.05037.

Chapter 10: “Voltage graphs”.

(GG: T, Cov)

1994a An introduction to random topological graph theory. Combinatorics, Probability

Take a graph Γ with cyclomatic number k and randomly sign it so that each
edge is negative with probability p. The probability that (Γ, σ) is balanced
= 2−k if p = 1/2 [obvious] and ≤ [max(p, 1 − p)]k in general [not obvious]
(this has an interesting asymptotic consequence due to Gimbel, given in this
paper).

(SG: Rand, B)

Neil L. White

See also A. Björner.

1986a A pruning theorem for linear count matroids. Congressus Numerantium 54 (1986),

(Bic: Gen)

Neil White and Walter Whiteley

1983a A class of matroids defined on graphs and hypergraphs by counting properties.
Unpublished manuscript, 1983.

See Whiteley (1996a) for an exposition and extension. 

(Bic: Gen)

Walter Whiteley

See also N. White.

1996a Some matroids from discrete applied geometry. In: Joseph E. Bonin, James G.

Appendix: “Matroids from counts on graphs and hypergraphs”, which ex-
spounds and extends Loréa (1979a), Schmidt (1979a), and especially White
and Whiteley (1983a), describes matroids on the edge sets of graphs (and hypergraphs) that generalize the bicircular matroid. The definition: given $m \geq 0$ and $k \in \mathbb{Z}$, $S$ is independent iff $\emptyset \subset S' \subseteq S$ implies $|S'| \leq m|V(S')| + k$.

(Bic: Gen)(Ref)

Geoff Whittle
See also C. Semple.


A Dowling-lattice version of Crapo and Rota’s critical problem is developed. Some minimal matroids whose critical exponent is $k$ (i.e., tangential $k$-blocks) are given, one being $G(\pm K_n^\circ)$.

Robin J. Wilson and John J. Watkins

§3.2: “Social Sciences” (pp. 51–53) applies signed graphs. §5.1: “Signed digraphs” (pp. 96–98) discusses positive and negative feedback (i.e., positive and negative cycles) in applications. Based on Open University (1981a).

(SG, PsS, SD: Exp)

Shmuel Winograd
See R.M. Karp.

Wayland H. Winstead
See J.R. Burns.

H.S. Witsenhausen
See Y. Gordon.

C. Witzgall and C.T. Zahn, Jr.

A. Wongseelashote

Takeshi Yajima and Shin’ichi Kinoshita

Examines the relationship between the two dual sign-colored graphs, $\Sigma$ and $\Sigma'$, of a link diagram (Bankwitz 1930a), translating the Reidemeister moves into graph operations and showing that they will convert $\Sigma$ into $\Sigma'$.

(SGc: Knot)

Jing-Ho Yan, Ko-Wei Lih, David Kuo, and Gerard J. Chang

Net degree sequences of signed simple graphs. Theorem 2 improves the Havel–Hakimi-type theorem from Chartrand, Gavlas, Harary, and Schultz (1992a) by determining the length parameter. Theorem 7 characterizes the net degree sequences of signed trees. [There seems to be room to strengthen the characterization and generalize to weighted degree sequences: see notes on Chartrand et al.]

(SGw: N)
Mihalis Yannakakis
See Esther M. Arkin and V.V. Vazirani.

Mihalis Yannakakis [Mihalis Yannakakis]
See Mihalis Yannakakis.

Yeong-Nan Yeh

Anders Yeo
See G. Gutin.

J.W.T. Youngs

Introducing “cascades”: current graphs with bidirected edges. A “cascade” is a bidirected graph, not all positive, that is provided with both a rotation system (hence it is orientation embedded in a surface) and a current (which is a special kind of bidirected flow). Dictionary: “broken” means a negative edge. (sg: O: Appl, Flows)


“Cascades”: see Youngs (1968b). (sg: O: Appl)

Cheng-Ching Yu
See C.-C. Chang.

Raphael Yuster and Uri Zwick


For fixed even $k$, a very fast algorithm for finding a $k$-gon. Also, one for finding a shortest even polygon. [Question. Are these the all-negative cases of similarly fast algorithms to find positive $k$-gons, or shortest positive polygons, in signed graphs?] (p: Cycles: Alg)

C.T. Zahn, Jr.
See also C. Witzgall.


Robert B. Zajonc


Wenan Zang
An algorithm, based in part on Gerards (1994a), that, given an all-negative signed graph, finds a subdivided $-K_4$ subgraph or a 3-coloring of the underlying graph. Question. Is there a generalization to all signed graphs?

( sg: p: Col, Alg, Ref )

Thomas Zaslavsky

See also C. Greene, P. Hanlon, and P. Solé.


Being published, greatly expanded, in (1989a, 1991a, 1995b, 20xxg) and more; as well as (but restricted to signed graphs) in (1982a, 1982b).

(GG: M)


(GG: M, Bic)


Signed graphs correspond to arrangements of hyperplanes in $\mathbb{R}^n$ of the forms $x_i = x_j$, $x_i = -x_j$, and $x_i = 0$. Consequently, one can compute the number of regions of the arrangement from graph theory, esp. for arrangements corresponding to “sign-symmetric” graphs, i.e., having both or none of each pair $x_i = \pm x_j$. Simplified account of parts of (1982a, 1982b, 1982c), emphasizing geometry.

(SG: M, G, N)


Characterizes the sets of polygons that are the positive ones in some signing of a graph.

(SG: B)


(SG: T, M)


Basic results on the bias matroid $G(\Sigma)$, the signed covering graph $\tilde{\sigma}$, the matrix-tree theorem [different from that of Murasugi (1989a)], and vector representation [as multisubsets of root systems $B_n \cup C_n$]. Examples. Conjectures about the interrelation between representability in characteristic 2 and unique representability in characteristic 0 [since answered by Geoff Whittle (A characterisation of the matroids representable over GF(3) and the rationals. J. Combin. Theory Ser. B 65 (1995), 222–261. MR 96m:05046. Zbl. 835.05015) as developed by Pagano (1998a, 20xxc)].

(SG, GG: M, B, Sw, Cov, I, G; EC, K)


A “proper $k$-coloring” of $\Sigma$ partitions $V$ into a special “zero” part, possibly void, that induces a stable subgraph, and up to $k$ other parts (labelled from a set of $k$ colors), each of which induces an antibalanced subgraph. A “zero-free proper $k$-coloring” is similar but without the “zero” part. [The suggestion is that the signed analog of a stable vertex set is one that induces an antibalanced subgraph. Problem. Use this insight to develop generalizations
of stable-set notions, such as cliques and perfection. \textit{Example}. Let $\alpha(\Sigma)$, the “antibalanced vertex set number”, be the largest size of an antibalance-inducing vertex set. Then $\alpha(\Gamma) = \alpha([+\Gamma \cup -K_n])$ One gets two related chromatic polynomials. The chromatic polynomial, $\chi_{\Sigma}(2k+1)$, counts all proper $k$-colorings; it is essentially the characteristic polynomial of the bias matroid. It can often be most easily computed via the zero-free chromatic polynomial, $\chi_{\Sigma}^*(2k)$, which counts proper zero-free colorings: see (1982c).

(SG, GG: M, Col, N, Cov, O, G)


Continuation of (1982b). The fundamental balanced expansion formulas, that express the chromatic polynomial in terms of the zero-free chromatic polynomial. Many special cases, treated in great detail: antibalanced graphs, signed graphs that contain $+K_n$ or $-K_n$, signed $K_n$’s (a.k.a. two-graphs), etc.

(SG, GG: M, N, Col, Cov, O, G; EC, K)


The zero-free chromatic number, and in particular that of a complete signed graph (possibly with parallel edges).

(SG: Col)


The line graph of a switching class $[\Sigma]$ of signed graphs is a switching class of signed graphs; call it $[L'(\Sigma)]$. The reduced line graph $L$ is formed from $L'$ by deleting parallel pairs of oppositely signed edges. Then $A(L) = A(L') = 2I - MM^T$, where $M$ is the incidence matrix of $\Sigma$. Thm. 1: $A(L)$ has all eigenvalues $\leq 2$. Examples: For an ordinary graph $\Gamma$, $L(-\Gamma) = -L(\Gamma)$. Taking $-\Gamma$ and attaching any number of pendant negative digons to each vertex yields (the negative of) Hoffman’s generalized line graph. Additional results are claimed but there are no proofs. [See also 20xxb]. [This work is intimately related to that of Vijayakumar \textit{et al.}, which was then unknown to the author, and to Cameron (1980a) and Cameron, Goethals, Seidel, and Shult (1976a).]

(SG: LG: Sw, A, I)


Forbidden-minor and structural characterizations. The latter for signed graphs is superseded by a result of Pagano (1998a).

(GG: M)

Decompose $E(\Sigma)$ into the fewest balanced subsets (generalizing the biparticity of an unsigned graph), or balanced connected subsets. These minimum numbers are $\delta_0$ and $\delta_1$. Thm. 1: $\delta_0 = [\chi^*] + 1$, where $\chi^*$ is the zero-free chromatic number of $-\Sigma$. Thm. 2: $\delta_0 = \delta_1$ if $\Sigma$ is complete. Conjecture 1. $\Sigma$ partitions into $\delta_0$ balanced, connected, and spanning edge sets (whence $\delta_0 = \delta_1$) if it has $\delta_0$ edge-disjoint spanning trees. [Solved and generalized to basepointed matroids by D. Slilaty.] Conjecture 2 is a formula for $\delta_1$ in terms of $\delta_0$ of subgraphs. [It has been thoroughly disproved by Slilaty.]

SG: Fr


Such a vertex (also, a “balancing vertex”) is a vertex of an unbalanced graph whose removal leaves a balanced graph. Some elementary results. (GG: Fr)


An attempt to generalize two-graphs (here [alas?] called “unitogs”) in a way similar to that of Cameron and Wells (1986a) although largely independent. The notable new example is “Johnson togs”, based on the Johnson graph of $k$-subsets of a set. “Hamming togs” are based on a Hamming graph (that is, a Cartesian product of complete graphs) and generalize examples of Cameron and Wells. Other examples are as in (1984b). (SG: TG: Gen)


Fundamental concepts and lemmas of biased graphs. Bias from gains; switching of gains; characterization of balance [for which see also Harary, Lindstrom, and Zetterstrom (1982a)]. (GG: B, Sw)


Basic theory of the bias, lift, and complete lift matroids. Several questions and conjectures. (GG: M)


Oriented signed graph = bidirected graph. The oriented matroid of an oriented signed graph. A “cycle” in a bidirected graph is a bias circuit (a balanced polygon, or a handcuff with both circles negative) oriented to have no source or sink. Cycles in $\Sigma$ are compared with those in its derived covering graph $\Sigma^*$. The correspondences among acyclic orientations of $\Sigma$ and regions of the hyperplane arrangements of $\Sigma$ and $\Sigma^*$, and dually the faces of the acyclotope of $\Sigma$. Thm. 4.1: the net degree vector $d(\tau)$ of an
orientation \( \tau \) belongs to the face of the acyclotope that is determined by the union of all cycles. Cor. 5.3 (easy): a finite bidirected graph has a source or sink.

\[ \text{Orientation embedding of signed graphs. J. Graph Theory 16 (1992), 399–422. MR 93i:05056. Zbl. 778.05033.} \]

Positive polygons preserve orientation, negative ones reverse it. The minimal embedding surface of a one-point amalgamation of signed graphs. The formula is almost additive.


Suppose that a function of matroids with labelled points is defined that is multiplicative on direct sums and satisfies a Tutte-Grothendieck recurrence with coefficients (the “parameters”) that depend on the element being deleted and contracted, but not on the particular minor from which it is deleted and contracted: specifically, \( F(M) = a_e F(M \setminus e) + b_e F(M/e) \) if \( e \) is not a loop or coloop in \( M \). Thm. 2.1 completely characterizes such “strong Tutte functions” for each possible choice of parameters: there is one general type, defined by a rank generating polynomial \( R_M(a, b; u, v) \) (the “parametrized rank generating polynomial”) involving the parameters \( a = (a_e), \ b = (b_e) \) and the variables \( u, v \), and there are a few special types that exist only for degenerate parameters. All have a Tutte-style basis expansion; indeed, a function has such an expansion iff it is a strong Tutte function (Thms. 7.1, 7.2). The Tutte expansion is a polynomial within each type. If the points are colored and the parameters of a point depend only on the color, one has a multicolored matroid generalization of Kauffman’s (1989a) Tutte polynomial of a sign-colored graph. Kauffman’s particular choices of parameters are shown to be related to matroid and color duality.

For a graph the “parametrized dichromatic polynomial” \( Q_\Gamma = u^{\beta_0(\Gamma)} R_{G(\Gamma)}, \) where \( G = \text{graphic matroid} \) and \( \beta_0 = \text{number of connected components} \). A “portable strong Tutte function” of graphs is multiplicative on disjoint unions, satisfies the parametrized Tutte-Grothendieck recurrence, and has value independent of the vertex set. Thm. 10.1: Such a function either equals \( Q_\Gamma \) or is one of two degenerate exceptions. Prop. 11.1: Kauffman’s (1989a) polynomial of a sign-colored graph equals \( R_{G(\Sigma)}, \sigma(a, b, d; d) \) for connected \( \Sigma \), where \( a_+ = b_- = B \) and \( a_- = b_+ = A \). [Cf. Traldi 1989a.]

This paper differs from other generalizations of Kauffman’s polynomial, by Przytycka and Przytycki (1988a) and Traldi (1989a) (and partially anticipated by Fortuin and Kasteleyn (1972a)), who also develop the parametrized dichromatic polynomial of a graph, principally in that it characterizes all strong Tutte functions; also in generalizing to matroids and in having little to say about knots. Schwärzler and Welsh (1993a) generalize to signed matroids (and characterize their strong Tutte functions) but not to arbitrary colors.


Characterized by six forbidden minors or eight forbidden topological subgraphs, all small. A close analog of Kuratowski’s theorem; the proof even has much of the spirit of the Dirac-Schuster proof of the latter, and all but one of the forbidden graphs are simply derived from the Kuratowski graphs.
Paul Seymour showed me an alternative proof from Kuratowski’s theorem that explains this; but it uses sophisticated results, as yet unpublished, of Robertson, Seymour, and Shih.

(SG: T)

Related: “projective outer-planarity” (POP): embeddable in the projective plane with all vertices on a common face. I have found most of the 40 or so forbidden topological subgraphs for POP of signed graphs (finding the rest will be routine); the proof is long and tedious and will probably not be published. Problem. Find a reasonable proof.

(SG: T)


A simple matroidal characterization of the bias matroids of biased graphs.

(GG: M)


Introducing the signed Heawood problem: what is the largest signed, or zero-free signed, chromatic number of any signed graph that orientation embeds in the sphere with \( h \) crosscaps? Solved for \( h = 1, 2 \).

(SG: T, Col)


Polynomials of gain and biased graphs: the fundamental object is a four-variable polynomial, the “polychromatic” (“polychromatic polynomial”), that specializes to the chromatic, dichromatic, and Whitney-number polynomials. The polynomials come in two flavors: unrestricted and balanced, depending on the edge sets that appear in their defining sums. (They can be defined in the even greater abstraction of “two-ideal graphs”, which clarifies the most basic properties.)

\( \S 4: \) “Gain-graph coloring”. In \( \Phi = (\Gamma, \phi, \mathcal{G}) \), a “zero-free \( k \)-coloring” is a mapping \( f : V \rightarrow [k] \times \mathcal{G} \); it is “proper” if, when \( euv \) is a link or loop and \( f(v) = (i, g), f(w) = (i, h) \), then \( h \neq g \phi(e; v, w) \). A “\( k \)-coloring” is similar but the color set is enlarged by inclusion of a color 0; propriety requires the additional restriction that \( f(v) \) and \( f(w) \) are not both 0 (and \( f(v) \neq 0 \) if \( v \) supports a half edge). In particular, a “group-coloring” of \( \Phi \) is a zero-free 1-coloring (ignoring the irrelevant numerical part of the color). A “partial group-coloring” is a group-coloring of an induced subgraph [which can only be proper if the uncolored vertices form a stable set]. The unrestricted and balanced chromatic polynomials count, respectively, unrestricted and zero-free proper \( k \)-colorings; the two Whitney-number polynomials count all colorings, proper and improper, by their improper edge sets.

\( \S 5: \) “The matroid connection”. The various polynomials are, in essence, bias matroid invariants and closely related to corresponding lift matroid and extended lift matroid invariants.

Almost infinitely many identities, some of them (esp., the balanced expansion formulas in \( \S 6 \)) essential. Innumerable examples worked in detail. [The first half, to the middle of \( \S 6 \), is fundamental. The rest is more or less ornamental. Most of the results are, intentionally, generalizations of properties of ordinary graphs.]

(GG: N, M, Col)

The smallest surface that holds $K_n$ with loops, if odd polygons reverse orientation, even ones preserve it (this is parity embedding). That is, the demigenus $d(-K_n^o)$. 


Like (1996a), but without loops. *Conjecture* 1. The minimal surface for parity embedding $K_n$ is sufficient for orientation embedding of any signed $K_n$. *Conjectures* 3–4. The minimal surfaces of $\pm K_n^o$ and $\pm K_n$ are the smallest permitted by the lower bound obtained from Euler’s polyhedral formula. 


Basically, they are the antibalanced and bipartite signed graphs; but the exact description depends on the characterization one chooses for biparticity: whether it is evenness of polygons, closed walks, face boundaries in surface embeddings, etc. Characterization by chromatic number leads to a slightly more different list of analogs. 


Complete and annotated—or as nearly so as I can make it. In preparation in perpetuum. Hurry, hurry, write an article!


A complete (or so it is intended) terminological dictionary of signed, gain, and biased graphs and related topics; including necessary special terminology from ordinary graph theory and mathematical interpretations of the special terminology of applications.


[The solution implies that $(*)$ $f_0(m) \leq \lfloor 2^{m-1}(m-1)!/\sqrt{e} \rfloor$, where $f_0(m) = \text{the smallest } r \text{ such that every group of order } \geq r \text{ is a possible gain group for every contrabalanced gain graph of cyclomatic number equal to } m$. *Problem* 1. Find a good upper bound on $f_0$. ($(*)$ is probably weak.) *Problem* 2. Find a good lower bound. *Problem* 3. Estimate $f_0$ asymptotically.] (“Avoiding the identity” concerns not $f_0$ but a larger function $f$ corresponding to a simplified question.)

20xxa The largest demigenus of a bipartite signed graph. Submitted.

The smallest surface for orientation embedding of $\pm K_{r,s}$. 


Line graphs of signed graphs are, fundamentally, (bidirected) line graphs of bidirected graphs. Then the line graph of a signed graph is a polar graph, i.e., a switching class of bidirected graphs; the line graph of a polar graph is a signed graph; and the line graph of a sign-biased graph, i.e., of a switching class of signed graphs, is a sign-biased graph. In particular, the line graph of an antibalanced switching class is an antibalanced switching class. (Partly for this reason, ordinary graphs should usually be regarded as antibalanced, i.e., all negative, in line graph theory.) Since a digraph is an oriented all-positive signed graph, its line graph is a bidirected graph whose positive part is the Harary-Norman line digraph. Among the line graphs of signed graphs, some reduce by cancellation of parallel but oppositely signed edges to all-negative graphs; these are precisely Hoffman’s generalized line graph of ordinary graphs, a fact which explains their line-graph-like behavior. [Attempts at a completely descriptive line graph of a digraph were Muracchini and Ghirlanda (1965a) and Hemminger and Klerlein (1979a). The geometry of line graphs and signed graphs has been developed by Vijayakumar et al. See also (1984c).]

20xxc Perpendicular dissections of space. In preparation. (GG: M, G)


20xxe Geometric lattices of structured partitions: II. Lattices of group-valued partitions based on graphs and sets. Manuscript, 1985 et seq. (GG: M, N, col)


20xxg Universal and topological gains for biased graphs. In preparation. (GG: T)


Supersolvable biased-graph matroids, characterized by a form of simplicial vertex ordering (that is, reverse perfect vertex elimination scheme)—but with a few exceptions (it’s combinatorics!). Later sections treat examples. §4: “Near-Dowling and dowling lift lattices”. §5: “Group expansions and biased expansions”. §6: “An extension of Edelman and Reiner’s theorem” to general gain groups (see Edelman and Reiner (1994a)). §7: “Composed partitions and circular n-permutation polynomials”: the lattice of k-composed partial partitions and the meet subsemilattice of k-composed partitions. §8: “Bicircular matroids”.

20xxi Big flats in a box. In preparation. The naive approach to characteristic polynomials via lattice point counting (in characteristic 0) and Möbius inversion (as in Blass and Sagan (1998a)) can only work when one expects it to. [This is a theorem!]

Morris Zelditch, Jr.
See J. Berger.

Bohdan Zelinka
See also R.L. Hemminger.

See (1976a). [This appears to be a very brief abstract of a lecture.]

\textit{(sg: O, sw)}


Establishes correspondences between quasigroups, algebraic loops, and groups on one hand, and 1-factored complete digraphs on the other, and between automorphisms of the latter and autotopies of the former. \textit{(GG: Aut)}


See (1976a) for definitions. Railway tracks and switches modeled by edges and vertices of a polar graph. Forming its derived graph (see (1976d)), thence a digraph obtained therefrom by splitting vertices into two copies and adjusting arcs, the time for a train to go from one segment to another is found by a shortest path calculation in the digraph. A similar method is used to solve the problem for several trains. \textit{(sg: O, sw: LG: Appl)}


Basic definitions (Zitek (1972a)): “Polarized graph” $B = \text{bidirected graph}$ (with no negative loops and no parallel edges sharing the same bidirection).

“Polar graph” $P \cong \text{switching class of bidirected graphs}$ (that is, we forget which direction at a vertex is in and which is out—here called “north” and “south” poles—but we remember that they are different).

Thms. 1–6. Elementary results about automorphisms, including finding the automorphism groups of the “complete polarized” and polar graphs. (The “complete polarized graph” has every possible bidirected link and positive loop, without repetition.) Thm. 7: With small exceptions, any (ordinary) graph can be made polar as, say, $P$ so that Aut $P$ is trivial.

Thms. 8–10. Analogs of Whitney’s theorem that the line graph almost always determines the graph. The “pole graph” $B^*$ of $B$ or $[B]$: Split each vertex into an “in” copy and an “out” copy and connect the edges appropriately.

[Generalizes splitting a digraph into a bipartite graph. It appears to be a “twisted” signed double covering graph.] Thm. 8. The pole graph is determined, with two exceptions, by the edge relation $e \sim_1 f$ if both enter or both leave a common vertex. (A trivial consequence of Whitney’s theorem.)

Thm. 9. A polar graph $[B]$ with enough edges going in and out at each vertex is determined by the edge relation $e \sim_2 f$ if one enters and the other exits a common vertex (Examples show that too few edges going in and out leave $[B]$ undetermined.) Thm. 10. Knowing $\sim_1$, $\sim_2$, and which edges are parallel with the same sign, and if no component of the simplified underlying graph of $B$ is one of twelve forbidden graphs, then $[B]$ is determined. \textit{[Problem 1. Improve Thm. 10 to a complete characterization of the bidirected graphs that are reconstructible from their line graphs (which are to be taken as bidirected; see Zaslavsky (1984c, 20xxb)). In connection with this, see results on characterizing line graphs of bidirected (or signed) graphs by Vijayakumar (1987a). \textit{Problem 2. It would be interesting to improve Thm. 9.]} \textit{(sg: O, sw: Aut, lg)}

See (1976a) for basic definitions. Here is the framework of the 8 theorems. Given a bidirected or polar graph, $B$ or $P$, vertices $a$ and $b$, and a type $X$ of walk, let $s_X [s'_X] = \text{the fewest vertices [edges] whose deletion eliminates all } (a,b) \text{ walks of type } X$, and let $d_X [d'_X] = \text{maximum number of suitably pairwise internally vertex-disjoint [or, suitably pairwise edge-disjoint] walks of type } X \text{ from } a \text{ to } b$. [My notation.] By “suitably” I mean that a common internal vertex or edge is allowed in $P$ (but not in $B$) if it is used oppositely by the two walks using it. (See the paper for details.) Thms. 1–4 (there are two Theorems 4) concern all-positive and all-introverted walks in a bidirected (“polarized”) graph, and are simply the vertex and edge Menger theorems applied to the positive and introverted subgraphs. Thms. 4$_2$–7 concern polar graphs and have the form $s_X \leq d_X \leq 2s_X [s'_X \leq d'_X \leq 2s'_X]$, which is best possible. Thms. 4$_2$–5 concern type “heteropolar” (equivalently, directed walks in a bidirected graph). The proofs depend on Menger’s theorems in the double covering graph of the polar graph. [Since this has 2 vertices for each 1 in the polar graph, the range of $d_X [d'_X]$ is explained.] Thms. 6–7 concern type “homopolar” (i.e., antidirected walks). The proofs employ the pole graph (see (1976a)).


See (1976a) for basic definitions. An Eulerian trail in a bidirected graph is a directed trail containing every edge. [Equivalently, a heteropolar trail that contains all the edges in the corresponding polar graph.] It is closed if the endpoints coincide and the trail enters at one end and departs at the other. The fewest directed trails needed to cover a connected bidirected graph is the total of the absolute differences between in-degrees and out-degrees at all vertices, or 1 if in-degree = out-degree everywhere. (sg: O, sw: Paths)


See (1976a) for basic definitions. The “derived graph” of a bidirected graph [this is equivalent to the author’s terminology] is essentially the positive part of the bidirected line graph. The theorem can be restated, somewhat simplified: A finite connected bidirected graph $B$ is isomorphic to its derived graph iff $B$ is balanced and contains exactly one polygon. (sg: O, sw: Paths)


See (1976a) for basic definitions. A polar graph $PG(\mathcal{G}, A)$ of a group and a subset $A$ is defined. [It is the Cayley digraph.] In bidirected language: a (bi)directed graph is “homogeneous” if it has automorphisms that are transitive on vertices, both preserving and reversing the orientations of edges, and that induce an arbitrary permutation of the incoming edges at any given vertex, and similarly for outgoing edges. It is shown that the Cayley digraph $PG(\mathcal{G}, A)$, where $\mathcal{G}$ is a group and $A$ is a set of generators, is homogeneous if $A$ is both arbitrarily permutable and invertible by Aut $\mathcal{G}$. [Bidirection—i.e., the polarity—seems to play no part here.] (sg: O, sw: Aut)

Is a simple graph $\Gamma$ a double cover of some signing of a simple graph? An elementary answer in terms of involutions of $\Gamma$. Further: if there are two such involutions $\alpha_0, \alpha_1$ that commute, then $\Gamma/\langle \alpha_0, \alpha_1 \rangle$ has involution induced by $\alpha_{1-i}$, so is a double cover of $\Gamma/\langle \alpha_0, \alpha_1 \rangle$, which is not necessarily simple. [No properties of particular interest for signed covering are treated.] \( \text{(sg: Cov)} \)


The double covers here are those of all-negative simple graphs (hence are bipartite; denote them by $B(\Gamma)$). Some properties of these double covers are proved, then connections with a certain lattice (the “logic”) of a graph.

(p: Cov: Aut)


The second half of (1983a).

(p: Cov: Aut)


Harary and Sagan (1983a) asked: which signed graphs have the form $S(P)$ for some poset $P$? Zelinka gives a rather complicated answer for all-negative signed graphs, which has interesting corollaries. For instance, Cor. 3: If $S(P)$ is all negative, and $P$ has $0$ or $1$, then $S(P)$ is a tree. \( \text{(SG, S)} \)

Hans-Olov Zetterström

G.M. Ziegler
See A. Björner and L. Lovász.

Ping Zhang


Blass and Sagan’s (1998a) geometrical form of signed-graph coloring is used to calculate (I) characteristic polynomials of several versions of $k$-equal sub-space arrangements (these are the main results) and (II) [also in Zhang (20xxa)] the chromatic polynomials (in geometrical guise) of ordinary graphs extending $K_n$ by one vertex, signed graphs extending $\pm K_n^\circ$ by one vertex, and $\pm K_n$ with any number of negative loops adjoined. \( \text{(sg: N, G, col)} \)

20xxa The characteristic polynomials of interpolations between Coxeter arrangements. Submitted.

Uses signed-graph coloring (in geometrical guise) to evaluate the chromatic polynomials (in geometrical guise) of all signed graphs interpolating between (1) $+K_n$ and $+K_{n+1}$ [i.e., ordinary graphs extending a complete graph by one vertex]; (2) $\pm K_n^\circ$ and $\pm K_{n+1}^\circ$; (3) $\pm K_n$ and $\pm K_{n+1}$ [known already by several methods, including this one]; (4a) $\pm K_{n-1}$ and $\pm K_n \cup +K_n$; (4b) $\pm K_{n-1} \cup -K_n$ and $\pm K_n$; and certain signed graphs interpolating (by adding negative edges one vertex at a time, or working down and removing them one vertex at a time) between (5) $+K_n$ and $\pm K_n^\circ$; (6) $+K_n$ and $\pm K_n$. In cases (1)–(3) the chromatic polynomial depends only on how many edges are added [which is obvious from the coloring procedure]. \( \text{(sg: N, col, G)} \)
Xiankun Zhang  
See H.-J. Lai.

F. Zítek  
For definitions see Zelinka (1976a). For work on these objects see many papers of Zelinka.  

Uri Zwick  
See R. Yuster.

Ondřej Zýka  
See J. Kratochvíl.