Theorems 3.1, 3.7 of the paper assert that the energy of the unitary Cayley graph \( \text{Cay}(\mathbb{Z}_n, \mathbb{Z}_n^*) \) is \( 2^{\omega(n)} \phi(n) \). We give an essentially one-sentence proof of these theorems; we do not address other results in the paper.

The Cayley graph \( C_n = \text{Cay}(\mathbb{Z}_n, \mathbb{Z}_n^*) \) is a connected \( \phi(n) \)-regular graph and the eigenvalues of its adjacency matrix are the Ramanujan sums \( c(r, n) = \phi(n) \frac{\mu(n, (r, n))}{\varphi(n, (r, n))} \) for \( 1 \leq r \leq n \). The energy of the graph is defined to be the sum of the absolute values of the eigenvalues. Let us prove that the energy of \( C_n \) is \( 2^{\omega(n)} \phi(n) \) by proving the identity:

\[
\sum_{r=1}^{n} \left| \frac{\mu(n, (r, n))}{\varphi(n, (r, n))} \right| = 2^{\omega(n)}.
\]

For each divisor \( d \) of \( n \), call \( S_d := \{ r \leq n : (r, n) = d \} \). Note that \( |S_d| = \phi(n/d) \) as \( \{ r \leq n : (r, n) = d \} = \{ dR \leq n : (R, n/d) = 1 \} \). Now, writing \( n = p_1^{a_1} \cdots p_r^{a_r} \), we have \( |\mu(n/d)| = 1 \) if and only if \( n/d \) is square-free, which is so if and only if \( d = p_1^{b_1} \cdots p_r^{b_r} \) with each \( b_i = a_i \) or \( a_i - 1 \). Write \( T \) for such divisors; clearly \( |T| = 2^r \).

Therefore,

\[
\sum_{r=1}^{n} \left| \frac{\mu(n, (r, n))}{\varphi(n, (r, n))} \right| = \sum_{d|n} |S_d||\mu(n/d)| \sum_{d\in\mathbb{T}} 1 = |T| = 2^r.
\]

This completes the proof.