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Corollary 26 is known in another setting. If a graph X and its complement are vertex- and edge-transitive then X is a strongly regular graph (and its automorphism group is a so-called "rank-three" permutation group). Equation (26.2) is equivalent to a standard result in the theory of strongly regular graphs. It occurs, for example, in disguised form as the middle of the three identities in Theorem 1.3.1(iii) from Brouwer, Cohen and Neumaier *Distance-Regular Graphs* (Springer, New York) 1989. The 'disguise' is explained in the following.

A graph is strongly regular with parameters v, k, λ and μ if it is a k-regular graph on v vertices and the number of common neighbours of two distinct vertices x and y is λ or μ, according as x and y are adjacent or not. (Actually this is not quite right, we should also assume that the graph is neither complete nor empty.)

If u is a vertex in the strongly regular graph X then we can count the edges joining the neighbours to the non-neighbours of u in two ways, to obtain

$$k(k-1-\lambda) = (v-k-1)\mu. \qquad (*)$$

The eigenvalues of the adjacency matrix of a strongly regular graph are its valency k and the two zeros of the quadratic

$$x^2 - (\lambda-\mu)x - (k-\mu). \qquad (**)$$

(For this, see Theorem 1.3.1(i) of Brouwer et al.) If the eigenvalues of X are k, θ and τ then (**) implies that

$$\begin{aligned}(k-\theta)(k-\tau) &= k^2 - (\lambda-\mu)k + \mu - k\\ &= k(k-\lambda-1) - (k+1)\mu.\end{aligned}$$

Using (*) we see that the last line is equal to $v\mu$.

On the other hand, in our notation Equation (26.2) is

$$(k-\tau)(v-k-\theta) = -\tau(\theta+1)v,$$

which can be rewritten as

$$(k-\theta)(k-\tau) = v[k-\tau+\tau(\theta+1)].$$

From (**) we find that $\theta\tau = \mu - k$, consequently the right side here is also equal to $v\mu$.