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1) In the proposition as stated at the top of page 7 the use of "uniform" is incorrect. The correct proposition is

Proposition. *The Gaussian polynomial* $\binom{n}{t}_q$ *is the product of exactly those cyclotomic polynomials* $F_j(q)$ *for which* $\bar{t} > \bar{n}$, *modulo* j.

This suggests splitting the Fourier transform sum into a sum of sums over the roots of the cyclotomic polynomials. Let us define

$$\bar{f}(j,n,t,m) = \sum_{F_d(\omega)=0} \omega^{t(t+1)/2-j} \binom{n}{t}_\omega$$

so

$$f(j,n,t,m) = \frac{1}{m} \sum_{d|m} \bar{f}(j,n,t,d)$$

Now the proposition implies that if $t \bmod d > n \bmod d$ then $\binom{n}{t}_\omega$ equals zero for all ω that are roots of the d'th cyclotomic polynomial. So $\bar{f}(i,n,t,d) = 0$ for all i.

This allows us to answer question 4b by saying that if (n,t,m) has only one bad divisor d then

$$\begin{aligned} f(i,n,t,m) &= \frac{1}{m}(\bar{f}(i,n,t,1) + \bar{f}(i,n,t,d)) \\ &= \frac{d}{m}(1/d)(\bar{f}(i,n,t,1) + \bar{f}(i,n,t,d)) \\ &= \frac{d}{m} f(i,n,t,d) \end{aligned}$$

2) The general conjecture as stated is false, as can be seen by considering $f(16,6,5)$. There the bad divisors are 3 and 5. The general conjecture would then imply that f(16,6,15) is a linear combination of $f(15,6,3)$ and $f(15,6,5)$ (extended appropriately). Computation reveals that the combination must be $(1/3)f(15,6,5)+(1/5)f(15,6,3)$, further computation shows that this combination does not work.

The General Conjecture might be true if we allow 1 to be included in the set of bad divisors.