

# Small Ramsey Numbers

Stanisław P. Radziszowski<sup>\*</sup>  
Department of Computer Science  
Rochester Institute of Technology  
Rochester, NY 14623, spr@cs.rit.edu

Submitted: June 11, 1994; Accepted: July 3, 1994  
Revision #4: July 16, 1997

**ABSTRACT:** We present data which, to the best of our knowledge, includes all known nontrivial values and bounds for specific graph, hypergraph and multicolor Ramsey numbers, where the avoided graphs are complete or complete without one edge. Many results pertaining to other more studied cases are also presented. We give references to all cited bounds and values, as well as to previous similar compilations. We do not attempt complete coverage of asymptotic behavior of Ramsey numbers, but concentrate on their specific values.

Mathematical Reviews Subject Number 05C55.

## 1. Scope and Notation

There is a vast literature on Ramsey type problems starting in 1930 with the original paper of Ramsey [Ram]. Graham, Rothschild and Spencer in their book [GRS] present an exciting development of Ramsey Theory. The subject has grown amazingly, in particular with regard to asymptotic bounds for various types of Ramsey numbers (see the survey paper [GrRb]), but the progress on evaluating the basic numbers themselves has been very unsatisfactory for a long time. In the last decade, however, considerable progress has been obtained in this area, mostly by employing computer algorithms. The few known exact values and several bounds for different numbers are scattered among many technical papers. This compilation is a fast source of references for the best results known for specific numbers. It is not supposed to serve as a source of definitions or theorems, but these can be easily accessed via the references gathered here.

---

\* - This paper updates and extends a technical report RIT-TR-93-009 by the author [Ra4].

Ramsey Theory studies conditions when a combinatorial object contains necessarily some smaller given objects. The role of Ramsey numbers is to quantify some of the general existential theorems in Ramsey Theory.

Let  $G_1, G_2, \dots, G_m$  be graphs or  $s$ -uniform hypergraphs ( $s$  is the number of vertices in each edge).  $R(G_1, G_2, \dots, G_m; s)$  denotes the  $m$ -color Ramsey number for  $s$ -uniform graphs/hypergraphs, avoiding  $G_i$  in color  $i$  for  $1 \leq i \leq m$ . It is defined as the least integer  $n$  such that, in any coloring with  $m$  colors of the  $s$ -subsets of a set of  $n$  elements, for some  $i$  the  $s$ -subsets of color  $i$  contain a sub-(hyper)graph isomorphic to  $G_i$  (not necessarily induced). The value of  $R(G_1, G_2, \dots, G_m; s)$  is fixed under permutations of the first  $m$  arguments.

If  $s = 2$  (standard graphs) then  $s$  can be omitted. If  $G_i$  is a complete graph  $K_k$ , then we can write  $k$  instead of  $G_i$ , and if  $G_i = G$  for all  $i$  we can use the abbreviation  $R_m(G)$  (or  $R_m(G; s)$ ). For  $s = 2$ ,  $K_k - e$  denotes a  $K_k$  without one edge, and for  $s = 3$ ,  $K_k - t$  denotes a  $K_k$  without one triangle (hyperedge).  $P_i$  is a path on  $i$  vertices,  $C_i$  is a cycle of length  $i$ , and  $W_i$  is a wheel with  $i-1$  spokes, i.e. a graph formed by some vertex  $x$ , connected to all vertices of some cycle  $C_{i-1}$ .  $K_{n,m}$  is a complete  $n$  by  $m$  bipartite graph, in particular  $K_{1,n}$  is a star graph. The book graph  $B_i = K_2 + \bar{K}_i = K_1 + K_{1,i}$  has  $i+2$  vertices, and can be seen as  $i$  triangular pages attached to a single edge. For a graph  $G$ ,  $n(G)$  and  $e(G)$  denote the number of vertices and edges, respectively. Finally let  $\chi(G)$  be the chromatic number of  $G$ , and let  $nG$  denote  $n$  disjoint copies of  $G$ .

Section 2 contains the data for the classical two color Ramsey numbers  $R(k, l)$  for complete graphs, and section 3 for the two color case when the avoided graphs are complete or have the form  $K_k - e$ , but not both are complete. Section 4 lists the most studied two color cases for other graphs. The multicolor and hypergraph cases are gathered in sections 5 and 6, respectively. If some new bound has been not yet published, we also give a reference to the best published previous result, if any. Finally, section 7 gives pointers to cumulative data and to some previous surveys, especially those containing data not subsumed by this compilation.

## 2. Classical Two Color Ramsey Numbers

We split the data into the table of values and a table with corresponding references. Known exact values appear as centered entries, lower bounds as top entries, and upper bounds as bottom entries.

All the critical graphs for the numbers  $R(k, l)$  (graphs on  $R(k, l) - 1$  vertices without  $K_k$  and without  $K_l$  in the complement) are known for  $k = 3$  and  $l = 3, 4, 5, 6$  [Ka2], 7 [RK3, MZ], and there are 1, 3, 1, 7 and 191 of them, respectively. There exists a unique critical graph for  $R(4, 4)$  [Ka2]. There are 4 such graphs known for  $R(3, 8)$  [RK2], 1 for  $R(3, 9)$  [Ka2] and 350904 for  $R(4, 5)$  [MR5], but there might be more of them. In [MR4] evidence is given for the conjecture that  $R(5, 5) = 43$  and that there exist precisely 656 critical graphs on 42 vertices.

$l$	3	4	5	6	7	8	9	10	11	12	13	14	15
$k$													
3	6	9	14	18	23	28	36	40 43	46 51	51 60	59 69	66 78	73 89
4		18	25	35 41	49 61	53 84	69 115	80 149	96 191	128 238	131 291	136 349	145 417
5			43 49	58 87	80 143	95 216	114 316	118 442	140	150	158	182	198
6				102 165	298	495	780	1171					
7					205 540	1031	1713	2826					
8						282 1870	3583	6090					
9							565 6625	12715					
10								798 23854					

Table I. Known nontrivial values and bounds for two color Ramsey numbers  $R(k, l) = R(k, l; 2)$ .

$l$	3	4	5	6	7	8	9	10	11	12	13	14	15
$k$													
3	GG	GG	GG	Ka2	Ka2 GY	GR MZ	Ka2 GR	Ex2 RK2	Ka2 RK2	Ex8 RK2	Piw3 RK2	Ex7 RK2	WW Ra1
4		GG	Ka1 MR5	Ex8 MR4	Ex3 Mac	Ex3 Mac	RK1 Mac	Piw3 Mac	Piw3 Spe	SLL Spe	Ea3 Spe	Ea3 Spe	Ea3 Spe
5			Ex4 MR4	Ex8 Spe/HZ	CET Spe	Piw3 Spe	CET Mac	Ea3 Mac	SLL	SLW	SLW	SLW	SLW
6				Ka1 Mac	Mac	Mac	Mac	Mac					
7					Ma/S1 Mac	Mac	HZ	Mac					
8						BR Mac	Ea1	HZ					
9							Ma/S1 Mac	Ea1					
10								Ma/S1 Mac					

References for Table I.

Chokbownpaishand and Longani [ChLo] established  $R(3,20) \geq 98$ . Wang and Wang in [WW] showed that  $R(3,16) \geq 79$ , and [WWY] gives  $R(3,17) \geq 92$ ,  $R(3,18) \geq 98$ ,  $R(3,19) \geq 106$ ,

$R(3,20) \geq 109$ ,  $R(3,21) \geq 122$ ,  $R(3,22) \geq 125$  and  $R(3,23) \geq 136$ . Most of the lower bounds for  $R(4,n)$  presented by Bannani in [Ba], and two other results  $R(3,13) \geq 58$  [Ka2] and  $R(5,8) \geq 94$  [RK1], were improved in [Piw1, Piw3] by Piwakowski. The bound  $R(3,13) \geq 60$  [XZ] cited in the 1995 version of this survey was shown to be incorrect in [Piw3]. The previously best published upper bound for  $(k,l) = (5,6)$  of 94 can be found in [Wa2].

By taking a disjoint union of two critical graphs one can easily see that  $R(k,p) \geq s$  and  $R(k,q) \geq t$  imply  $R(k,p+q-1) \geq s+t-1$ . For example, this gives trivially a lower bound  $R(4,15) \geq 145$  with  $p=4$ ,  $q=12$ . Higher lower bounds implied this way are not shown. Some upper bounds implied by  $R(k,l) \leq R(k-1,l) + R(k,l-1)$ , or by its slight improvement with strict inequality when both terms on the right hand side are even, are marked [Ea1]. There are obvious generalizations of these inequalities for graphs other than complete.

The bound  $R(6,6) \leq 166$  is an immediate consequence of theorem 1 in [Wa1] and  $R(4,6) \leq 41$ , in this case the best published bound of 169 is due to Giraud [Gi3]. T. Spencer [Spe], Mackey [Mac], and Huang and Zhang [HZ], using the bounds for minimum and maximum number of edges in  $(4,5)$  Ramsey graphs listed in [MR2, MR4], were able to establish new upper bounds for several higher Ramsey numbers, improving all the previous longstanding results of Giraud [Gi1, Gi3, Gi4]. Spencer and Mackey keep outdoing each other on these bounds, and when the situation stabilizes we will try to report the outcome of this competition. We have recomputed the bounds marked [HZ] using the method from the paper [HZ], because the bounds there relied on an overly optimistic personal communication from Spencer.

For a more in depth study of triangle-free graphs in relation to the case of  $R(3,k)$ , for which considerable progress has been obtained in recent years, see also [AKS, FL, Fra1, Fra2, Gri, Loc, KM, RK3, RK4, S2, Stat, Yu]. In 1995, Kim [Kim] obtained a breakthrough by proving that  $R(3,k)$  has order of magnitude exactly  $\Theta(k^2/\log k)$ . Good asymptotic bounds for  $R(k,k)$  can be found, for example, in [Chu3, McS] (lower bound) and [Tho] (upper bound), and for many other asymptotic bounds in the general case of  $R(k,l)$  consult [GRS, GrRö].

### 3. Two Colors - Dropping One Edge from Complete Graph

For the following numbers it was established that the critical graphs are unique:  $R(K_3, K_l - e)$  for  $l=3$  [Tr], 6 and 7 [Ra2],  $R(K_4 - e, K_4 - e)$  [FRS2],  $R(K_5 - e, K_5 - e)$  [Ra3] and  $R(K_4 - e, K_7 - e)$  [McR]. The marks " $\geq n$ " in the Table II indicate lower bounds.

$G$	$H$	$K_3-e$	$K_4-e$	$K_5-e$	$K_6-e$	$K_7-e$	$K_8-e$	$K_9-e$	$K_{10}-e$
$K_3-e$		3	5	7	9	11	13	15	17
$K_3$		5	7	11	17	21	25	31	36-39
$K_4-e$		5	10	13	17	28			
$K_4$		7	11	19	$\geq 27$	$\geq 35$			
$K_5-e$		7	13	22	$\geq 30$				
$K_5$		9	16	30-34	$\geq 43$				
$K_6-e$		9	17	$\geq 30$					
$K_6$		11	21	$\geq 35$					

Table II. Two types of Ramsey numbers  $R(G, H)$ , includes all known nontrivial values.

$G$	$H$	$K_3-e$	$K_4-e$	$K_5-e$	$K_6-e$	$K_7-e$	$K_8-e$	$K_9-e$	$K_{10}-e$
$K_3-e$		Tr	Tr	Tr	Tr	Tr	Tr	Tr	Tr
$K_3$		Tr	CH2	Clan	FRS1	GH	Ra2	Ra2	Ea3-Ra2
$K_4-e$		Tr	CH1	FRS2	McR	McR			
$K_4$		Tr	CH2	EHM1	Ex10	Ea1			
$K_5-e$		Tr	FRS2	CEHMS	Ea1				
$K_5$		Tr	BH	Ex7-Ea1	Ea1				
$K_6-e$		Tr	McR	Ea1					
$K_6$		Tr	McN	Ea1					

References for Table II.

#### 4. General Graph Numbers in Two Colors

This section includes data with respect to general graph results. We tried to include all nontrivial values and identities regarding exact results (or references to them), but only those out of general bounds and other results which, in our opinion, have a direct connection to the evaluation of specific numbers. If some small value cannot be found below, it may be covered by the cumulative data gathered in section 7, or be a special case of a general result listed in this section. Note that  $B_1 = C_3 = W_3 = K_3$ ,  $B_2 = K_4 - e$ ,  $P_3 = K_3 - e$ ,  $W_4 = K_4$  and  $C_4 = K_{2,2}$  imply other identities not mentioned explicitly.

Paths:

$$R(P_n, P_m) = n + \lfloor m/2 \rfloor - 1 \quad \text{for all } n \geq m \geq 2 \quad [\text{GeGy}]$$

Cycles:

$$R(C_3, C_3) = 6 \quad [\text{GG}]$$

$$R(C_4, C_4) = 6 \quad [\text{CH1}]$$

Result obtained independently in [Ros] and [FS1]

$$R(C_n, C_m) = \begin{cases} 2n - 1 & \text{for } 3 \leq m \leq n, m \text{ odd}, (n, m) \neq (3, 3) \\ n - 1 + m/2 & \text{for } 4 \leq m \leq n, m \text{ and } n \text{ even}, (n, m) \neq (4, 4) \\ \max\{n - 1 + m/2, 2m - 1\} & \text{for } 4 \leq m < n, m \text{ even and } n \text{ odd} \end{cases}$$

$$R(nC_3, mC_3) = 3n + 2m \quad \text{for } n \geq m \geq 1, n \geq 2 \quad [\text{BES}]$$

Unions of cycles [MS, Den]

Wheels:

$$R(W_3, W_5) = 11 \quad [\text{Clan}]$$

$$R(W_3, W_n) = 2n - 1 \quad \text{for all } n \geq 6 \quad [\text{BE2}]$$

All critical colorings for  $R(W_3, W_n)$  for all  $n \geq 3$  [RaJi]

$$R(W_4, W_5) = 17 \quad [\text{He3}]$$

$$R(W_5, W_5) = 15 \quad [\text{HM2, He2}]$$

$$R(W_4, W_6) = 19, R(W_5, W_6) = 17 \quad \text{and } R(W_6, W_6) = 17,$$

and all critical colorings (2, 1 and 2) for these numbers [FM]

Books:

$$R(B_1, B_n) = 2n + 3 \quad \text{for all } n > 1 \quad [\text{RS1}]$$

$$R(B_3, B_3) = 14 \quad [\text{RS1, HM2}]$$

$$R(B_2, B_5) = 16, R(B_3, B_5) = 17, R(B_5, B_5) = 21,$$

$$R(B_4, B_4) = 18, R(B_4, B_6) = 22, R(B_6, B_6) = 26,$$

in general  $R(B_n, B_n) = 4n + 2$  for  $4n + 1$  a prime power,

and some other general equalities and bounds for  $R(B_n, B_m)$  [RS1].

Complete bipartite graphs:

$$R(K_{2,3}, K_{2,3}) = 10 \quad [\text{Bu4}]$$

$$R(K_{2,3}, K_{2,4}) = 12 \quad [\text{ER}]$$

$$R(K_{2,3}, K_{1,7}) = 13 \quad [\text{Par4}]$$

$$R(K_{2,3}, K_{3,3}) = 13 \quad \text{and } R(K_{3,3}, K_{3,3}) = 18 \quad [\text{HM3}]$$

$$R(K_{2,2}, K_{2,8}) = 15 \quad \text{and } R(K_{2,2}, K_{2,11}) = 18 \quad [\text{HM}]$$

$$R(K_{2,2}, K_{1,15}) = 20 \text{ [La2]}$$

$$R(nK_{1,3}, mK_{1,3}) = 4n + m - 1 \text{ for } n \geq m \geq 1, n \geq 2 \text{ [BES]}$$

$R(K_{1,n}, K_{1,m}) = n + m - \varepsilon$ , where  $\varepsilon = 1$  if both  $n$  and  $m$  are even and  $\varepsilon = 0$  otherwise [Ha1]. It is also a special case of multicolor numbers for stars obtained in [BuRo1].

$R(K_{2,n}, K_{2,n}) \leq 4n - 2$  for all  $n \geq 2$ , exact values 6, 10, 14, 18, 21, 26, 30, 33, 38, 42, 46, 50, 54, 57 and 62 of  $R(K_{2,n}, K_{2,n})$  for  $2 \leq n \leq 16$ , respectively. The first open case is  $65 \leq R(K_{2,17}, K_{2,17}) \leq 66$  [EHM2].

Triangle versus other graphs:

$$R(3, k) = \Theta(k^2 / \log k) \text{ [Kim]}$$

Explicit construction for  $R(3, 4k + 1) \geq 6R(3, k + 1) - 5$ , for all  $k \geq 1$  [CCD]

Explicit triangle-free graphs with independence  $k$  on  $\Omega(k^{3/2})$  vertices [Alon2]

$R(K_3, G) = 2n(G) - 1$  for any connected  $G$  on at least 4 vertices and with at most  $(17n(G) + 1)/15$  edges, in particular for  $G = P_i$  and  $G = C_i$ , for all  $i \geq 4$  [BEFRS1]

$R(K_3, G) \leq 2e(G) + 1$  for any graph  $G$  without isolated vertices [Sid3]

$R(K_3, G) \leq n(G) + e(G)$  for all  $G$ , a conjecture [Sid2]

$R(K_3, K_n)$ , see section 2

$R(K_3, K_n - e)$ , see section 3

$R(K_3, G)$  for all connected  $G$  up to 9 vertices, see section 7

See also [AKS, FL, Fra1, Fra2, Gri, Loc, KM, RK3, RK4, S2, Stat, Yu]

Cycles versus complete graphs:

$$R(C_4, K_3) = R(C_4, C_3) = 7$$

$$R(C_4, K_4) = 10 \text{ [CH2]}$$

$$R(C_4, K_5) = 14 \text{ [GG][He2, LRZ]}$$

$$R(C_4, K_6) = 18 \text{ [Ex9][RoJa]}$$

$$21 \leq R(C_4, K_7) \leq 22 \text{ [JR1]}$$

$$R(C_5, K_3) = R(C_5, C_3) = 9$$

$$R(C_5, K_4) = 13 \text{ and } R(C_5, K_5) = 17 \text{ [He2, JR2]}$$

$C_4$  versus  $K_n$  [LRZ]

Cycles versus  $K_n$  [BoEr, EFRS2]

Cycles versus other graphs:

$C_4$  versus stars [Par3, Gua]

$C_4$  versus trees [EFRS4, Bu6, Gua]

$C_4$  versus  $K_{m,n}$  [HM]

$R(C_4, B_n) = 7, 9, 11, 12, 13, 16$  and  $16$  for  $2 \leq n \leq 8$ , respectively [FRS6]

$R(C_5, W_6) = 13$  [CS]

Cycles versus paths [FLPS, BEFRS2]

Cycles versus stars [La1, Clark, see Par5]

Cycles versus trees [FSS]  
 Cycles versus books [FRS5, FSR6]  
 Cycles versus wheels [Zhou]  
 See bipartite graphs for  $K_{2,2}$

Mixed special cases:

$R(B_3, K_4) = 14$  [He3]  
 $R(C_5 + e, K_5) = 17$  [He5]  
 $R(W_5, K_5 - e) = 17$  [He2][YH]  
 $20 \leq R(B_3, K_5) \leq 22$  [He2]  
 $27 \leq R(W_5, K_5) \leq 29$  [He2]  
 $25 \leq R(K_5 - P_3, K_5) \leq 28$  [He2]  
 $26 \leq R(K_{2,2,2}, K_{2,2,2})$  (octahedron) [Ex7]

General cases (exact results and bounds):

Paths versus stars [Par2, BEFRS2]  
 Paths versus books [RS2]  
 Paths versus cycles [FLPS, BEFRS2]  
 Paths versus  $K_n$  [Par1]  
 Paths versus  $K_{n,m}$  [Häg]  
 Paths and cycles versus trees [FSS]  
 Books versus trees [EFRS7]  
 Books and  $(K_1 + tree)$  versus  $K_n$  [LR1]  
 Unicyclic graphs [Gro1, Kbh]  
 $K_{2,n}$  versus some stars [Par4]  
 $nK_3$  versus  $mK_3$  [BES]  
 $nK_3$  versus  $mK_4$  [LorMu]  
 $R(nK_4, nK_4) = 7n + 4$  for large  $n$  [Bu7]  
 Stars versus trees [Bu1, GV]  
 Stars versus stripes, stripes [CL, Lor]  
 Stars versus books [CRSPS, RS2]  
 Stars versus  $K_{n,m}$  [Stev, Par3]  
 Stars versus  $K_n - tK_2$  [Hua1, Hua2]  
 Union of two\* stars [Gro2]  
 Double stars [GHK]  
 Fans  $K_1 + nK_2$  versus  $K_n$  [LR3]  
 Trees [EG, GRS, FSS, GV]  
 Trees versus  $K_n$  [Chv]

---

\* - A double star is a union of two stars with their centers joined by an edge.



Trees versus  $K_n + \bar{K}_m$  [RS2, FSR]  
 Trees versus bipartite graphs [EFRS6]  
 Trees versus almost complete graphs [GJ2]  
 Trees versus small ( $n(G) \leq 5$ ) connected  $G$  [FRS4]  
 Linear forests [BuRo2, FS3]  
 Forests versus  $K_n$  [Stahl]  
 Forests versus almost complete graphs [CGP]  
 Sparse graphs versus paths and cycles [BEFRS2]  
 Multipartite complete graphs [BEFRS3, EFRS4, FRS3, Stev]  
 Multipartite complete graphs versus trees [EFRS8, BEFRSGJ]  
 Disconnected graphs versus any graph [GJ1]  
 Graphs with long tails [Bu2, BG]  
 Brooms<sup>+</sup> [EFRS3]

#### General results:

- [Wa1]  $R(k, k) \leq 4R(k, k-2) + 2$ .
- [Chv]  $R(K_n, T_m) = (n-1)(m-1) + 1$  for any tree  $T$  on  $m$  vertices.
- [CH2]  $R(G, H) \geq (\chi(G) - 1)(c(H) - 1) + 1$ , where  $\chi(G)$  is the chromatic number of  $G$ , and  $c(H)$  is the size of the largest connected component of  $H$ .
- [BE1]  $R(G, G) \geq \lfloor (4n(G) - 1)/3 \rfloor$  for any connected  $G$ .
- [BE2] Graphs yielding  $R(K_n, G) = (n-1)(n(G) - 1) + 1$  and related results (see also [EFRS5]).
- [BES] Study of Ramsey numbers for multiple copies of graphs (see also [Bu1, LorMu]).
- [Zeng]  $R(nK_3, nG)$  for all isolate-free graphs  $G$  on 4 vertices.
- [Bu7, Bu8] Study of Ramsey numbers for large disjoint unions of graphs, in particular  $R(nK_k, nK_l) = n(k+l-1) + R(K_{k-1}, K_{l-1}) - 2$ , for  $n$  large enough.
- [BEFRS4] Graphs  $H$  yielding  $R(G, H) = (\chi(G) - 1)(n(H) - 1) + s(G)$ , where  $s(G)$  is a chromatic surplus of  $G$ , defined as the minimum number of vertices in some color class under all vertex colorings in  $\chi(G)$  colors (such  $H$ 's are called  $G$ -good). This idea, initiated in [Bu2], is a basis of a number of exact results for  $R(G, H)$  for large and sparse graphs  $H$  [BG, BEFRS2, Bu5, FS, EFRS4, FRS3, BEFRSGJ]. A survey of this area appeared in [FRS7].
- [BEFS] Bounds for the difference between consecutive Ramsey numbers.
- [Par3, Par4] Relations between some Ramsey graphs and block designs.

---

+ - A broom is a star with a path attached to its center.

- [Bra3]  $R(G, H) > h(G, d)n(H)$  for all nonbipartite  $G$  and almost every  $d$ -regular  $H$ , for some  $h$  unbounded in  $d$ .
- [CSRT]  $R(G, G) \leq c_d n(G)$  for all  $G$ , where constant  $c_d$  depends only on the maximum degree  $d$  in  $G$ .
- [GS]  $R(G, G) \leq c_d n$  for all  $d$ -arrangeable graphs  $G$  on  $n$  vertices.
- [EFRS9] Study of graphs  $G$  for which there exists a constant  $C$  such that for all  $H$  with no isolates  $R(G, H) \leq Ce(H)$ .
- [Alon1]  $R(G, G) \leq 12n$  for all  $n$ -vertex graphs  $G$ , in which no two vertices of degree at least 3 are adjacent.
- [FSS] Discussion of the conjecture that  $R(T_1, T_2) \leq n(T_1) + n(T_2) - 2$  holds for all trees  $T_1, T_2$ .
- [FM]  $R(W_6, W_6) = 17$  and  $\chi(W_6) = 4$ . This gives a counterexample  $G = W_6$  to the Erdős conjecture (see [GRS])  $R(G, G) \geq R(K_{\chi(G)}, K_{\chi(G)})$ .
- [LR2] Bounds on  $R(H + K_n, K_n)$  for general  $H$ .
- [-] Special cases of multicolor results listed in section 5.
- [-] See also surveys listed in section 7.

## 5. Multicolor Graph Numbers

The only known value of a multicolor classical Ramsey number:

$$R(3,3,3) = R(3,3,3; 2) = 17 \quad \text{[GG]}$$

2 critical colorings [KS, LayMa]

Bounds for multicolor classical numbers:

$$51 \leq R(3,3,3,3) \leq 64 \quad \text{[Chu1] [San]}$$

$$162 \leq R(3,3,3,3,3) \leq 317 \quad \text{[Ex11] [Wh, Ea1]}$$

$$500 \leq R(3,3,3,3,3,3) \quad \text{[Ex11]}$$

$$128 \leq R(4,4,4) \leq 236 \quad \text{[HI] [Ea2]}$$

$$458 \leq R(4,4,4,4) \quad \text{[Su]}$$

$$30 \leq R(3,3,4) \leq 31 \quad \text{[Ka2] [PR]}$$

$$45 \leq R(3,3,5) \leq 57 \quad \text{[Ex9, KLR] [Ea2]}$$

$$55 \leq R(3,4,4) \leq 79 \quad \text{[KLR] [Ea2]}$$

$$84 \leq R(3,3,3,4) \leq 155 \quad \text{[Ex9] [Ea2]}$$

$$R(3,4,5) \leq 161 \quad \text{[Ea2]}$$

The result by Sanchez-Flores [San], 1995, improved a very old bound  $R(3,3,3,3) \leq 65$  obtained by Folkman [Fo] in 1974. The result in [PR] improved the bound from [Piw2] by 1.

The upper bounds marked [Ea1] and [Ea2] are easy implications of basic inequalities. Out of several lower bounds on  $R_k(4)$  in [Su], the one above for  $k=4$  is the only bound better than those implied by general results in [Song] (see below).

Multicolor general graphs:

$R(C_4, C_4, C_4) = 11$	[BS, see also Clap]
$R(C_5, C_5, C_3) = 17$	[YR1]
$R(C_6, C_6, C_6) = 12$	[YR3]
$R_4(C_4) \geq 18$ and $R_5(C_4) \geq 25$	[Ex9]
$R(C_4, C_4, K_3) = 12$	[Schu]
$R(C_4, K_3, K_3) = 17$	[ER]
$R(K_4 - e, K_4 - e, P_3) = 11$	[Ex6]
$28 \leq R(K_4 - e, K_4 - e, K_4 - e) \leq 30$	[Ex6] [Piw4]
$R(C_4, C_4, C_4, T) = 16$ for $T = P_4$ and $T = K_{1,3}$	[ER]

All colorings on at least 14 vertices for  $(K_3, K_3, K_3)$ , and all colorings for  $(K_4 - e, K_4 - e, P_3)$  were found in [Piw4].

General multicolor results:

- General bounds for  $R_k(G)$  [CH3].
- Bounds for  $R_k(3)$  [Fre, Chu1, Chu2, ChGri, GrRö].
- Formulas for  $R_k(G)$  for  $G$  being  $P_3$ ,  $2K_2$  and  $K_{1,3}$  for all  $k$ , and for  $P_4$  if  $k$  is not divisible by 3 [Ir]. Wallis [Wal] showed  $R_6(P_4) = 13$ , which already implied  $R_{3t}(P_4) = 6t + 1$ , for all  $t \geq 2$ . Independently, the case  $R_k(P_4)$  for  $k \neq 3^m$  was completed by Lindström in [Lin], and later Bierbrauer proved  $R_{3^m}(P_4) = 2 \cdot 3^m + 1$  for all  $m \geq 1$ .
- $R_k(4) \geq 3 \cdot 5^{k-1} + 1$  and  $R_k(5) \geq 4 \cdot 6.48^{k-1} + 1$  for all  $k$ , and other general lower bounds on  $R_k(n)$  [Song].
- $R_k(C_4) \leq k^2 + k + 1$  for all  $k \geq 1$ , and  $R_k(C_4) \geq k^2 - k + 2$  for all  $k-1$  a prime power [Ir, Chu2, ChGra]. For small  $k$  some improvements on the latter are known:  $R_3(C_4) = 11$  [BS],  $R_4(C_4) \geq 18$  and  $R_5(C_4) \geq 25$  [Ex9].
- Bounds for the bipartite graphs  $R_k(K_{s,t})$ , in particular for  $K_{2,2} = C_4$  [ChGra].
- Formulas for  $R(C_n, C_m, C_k)$  and  $R(C_n, C_m, C_k, C_l)$  for  $n$  sufficiently large [EFRS1].
- Formulas for  $R(P_{n_1}, \dots, P_{n_k})$ , except few cases [FS2].
- Monotone paths and cycles [Lef].

- Formulas for  $R(S_1, \dots, S_k)$ , where  $S_i$ 's are arbitrary stars [BuRo1].
- Formulas for  $R(S_1, \dots, S_k, K_n)$ , where  $S_i$ 's are arbitrary stars [Jac].
- Formulas for  $R(S_1, \dots, S_k, nP_2)$ , where  $S_i$ 's are arbitrary stars [CL2].
- Formulas for  $R(pP_3, qP_3, rP_3)$  and  $R(pP_4, qP_4, rP_4)$  [Scob].
- Cockayne and Lorimer [CL1] found the exact formula for  $R(n_1P_2, \dots, n_kP_2)$ , and later Lorimer [Lor] extended it to a more general case of  $R(K_m, n_1P_2, \dots, n_kP_2)$ . Still more general cases of the latter, with multiple copies of the complete graph and forests, were studied in [Stahl, LorSe, LorSo].
- If  $G$  is connected and  $R(K_k, G) = (k-1)(n(G)-1) + 1$ , in particular if  $G$  is any tree, then  $R(K_{n_1}, \dots, K_{n_k}, G) = (R(K_{n_1}, \dots, K_{n_k}) - 1)(n(G)-1) + 1$  [BE2]. A generalization for connected  $G_1, \dots, G_n$  in place of  $G$  appeared in [Jac].
- Study of  $R(S, G_1, \dots, G_k)$  for large sparse  $S$  [EFRS1, Bu3].
- Bounds for trees  $R_k(T)$  and forests  $R_k(F)$  [EG, GRS, BB, GT, Bra2].
- See also surveys listed in section 7.

## 6. Hypergraph Numbers

The only known value of a classical Ramsey number for hypergraphs:

$$R(4,4;3) = 13 \quad \text{[MR1]}$$

more than 200000 critical colorings

Other hypergraph cases:

$$R(K_4 - t, K_4 - t; 3) = 7 \quad \text{[Ea4]}$$

$$R(K_4 - t, K_4; 3) = 8 \quad \text{[Sob, Ex1, MR1]}$$

$$14 \leq R(K_4 - t, K_5; 3) \quad \text{[Ex1]}$$

$$63 \leq R(5, 5; 3) \quad \text{[Ea1]}$$

$$32 \leq R(4, 5; 3) \quad \text{[Ex7]}$$

$$34 \leq R(5, 5; 4) \quad \text{[Ex10]}$$

$$56 \leq R(4, 4, 4; 3) \quad \text{[Ex7]}$$

$$13 \leq R(K_4 - t, K_4 - t, K_4 - t; 3) \leq 17 \quad \text{[Ex1] [Ea1]}$$

The computer evaluation of  $R(4,4;3)$  in [MR1] consisted of an improvement of the upper bound from 15 to 13, which followed an extensive theoretical study of this number in [Gi2, Is1, Sid1]. Exoo in [Ex1] announced the bounds  $R(4,5;3) \geq 30$  and  $R(5,5;4) \geq 27$  without presenting the constructions. The best published bound of  $R(4,5;3) \geq 24$  was

obtained by Isbell [Is2]. Shastri in [Sha] shows a weak bound  $R(5, 5; 4) \geq 19$ , nevertheless his lemmas and those in [Ka3, Abb, GRS] can be used to derive other lower bounds for higher numbers. Study of lower bounds on  $R_m(k; s)$  can be found in [DLR], and on  $R(p, q; 4)$  in [SWY]. In [AS] it is shown that for some  $a, b$  the numbers  $R(m, a, b; 3)$  are at least exponential in  $m$ .

Theoretical results on hypergraph numbers are gathered in [GrRö, GRS].

## 7. Cumulative Data and Surveys

Cumulative data for two colors:

- [CH1]  $R(G, G)$  for all graphs  $G$  without isolates on at most 4 vertices.
- [CH2]  $R(G, H)$  for all graphs  $G$  and  $H$  without isolates on at most 4 vertices.
- [Clan]  $R(G, H)$  for all graphs  $G$  on at most 4 vertices and  $H$  on 5 vertices, except five entries.
- [He4] All critical colorings for  $R(G, H)$ , for isolate-free graphs  $G$  and  $H$  as in [Clan] above.
- [Bu4]  $R(G, G)$  for all graphs  $G$  without isolates and with at most 6 edges.
- [He1]  $R(G, G)$  for all graphs  $G$  without isolates and with at most 7 edges.
- [HM2]  $R(G, G)$  for all graphs  $G$  on 5 vertices and with 7 or 8 edges.
- [He2]  $R(G, H)$  for all graphs  $G$  and  $H$  on 5 vertices without isolates, except 7 entries.
- [FRS4]  $R(G, T)$  for all connected graphs  $G$  on at most 5 vertices and all (except some cases) trees  $T$ .
- [FRS1]  $R(K_3, G)$  for all connected graphs  $G$  on 6 vertices.
- [Jin]  $R(K_3, G)$  for all connected graphs  $G$  on 7 vertices. Some errors in this paper were corrected by [SchSch].
- [Brin]  $R(K_3, G)$  for all connected graphs  $G$  on at most 8 vertices. The numbers for  $K_3$  versus sets of graphs with fixed number of edges, on at most 8 vertices, were presented in [KM].
- [BBH]  $R(K_3, G)$  for all connected graphs  $G$  on 9 vertices.

Chvátal and Harary [CH1, CH2] formulated several simple but very useful observations how to discover values of some numbers. All five missing entries in the tables of Clancy [Clan] have been solved. Out of 7 open cases in [He2] 2 have been solved, the bounds for 2 were improved, and the status of the other 3 did not change. Section 4 of this survey under "Mixed special cases" lists 4 of them (labeled [He2], 1 solved, 3 open).  $R(4, 5) = R(G_{19}, G_{23}) = 25$  is the second solved case. The other 2 open entries are  $K_5$  versus

$K_5$  (see section 2) and  $K_5$  versus  $K_5 - e$  (see section 3).

Cumulative data for three colors:

- [YR2]  $R_3(G)$  for all graphs  $G$  with at most 4 edges and no isolates.
- [YR1]  $R_3(G)$  for all graphs  $G$  with 5 edges and no isolates, except  $K_4 - e$ . The case of  $R_3(K_4 - e)$  remains open (see section 5).
- [YY]  $R_3(G)$  for all graphs  $G$  with 6 edges and no isolates, except 10 cases.
- [AKM]  $R(F, G, H)$  for most triples of isolate-free graphs with at most 4 vertices.

Surveys:

- [Bu1] A general survey of results in Ramsey graph theory by S.A. Burr (1974)
- [Par5] A general survey of results in Ramsey graph theory by T.D. Parsons (1978)
- [Ha2] Summary of progress by Frank Harary (1981)
- [ChGri] A general survey of bounds and values by F.R.K. Chung and C.M. Grinstead (1983)
- [JGT] Special volume of *the Journal of Graph Theory* (1983)
- [Rob] Nice textbook-type review of Ramsey graph theory for newcomers (1984)
- [Bu6] What can we hope to accomplish in generalized Ramsey Theory ? (1987)
- [GrRö] Survey of asymptotic problems by R.L. Graham and V. Rödl (1987)
- [GRS] An excellent book by R.L. Graham, B.L. Rothschild and J.H. Spencer, second edition (1990)
- [FRS7] Survey of graph goodness results, i.e. conditions for the formula  $R(G, H) = (\chi(G) - 1)(n(H) - 1) + s(G)$  (1991)
- [Neš] A chapter in *Handbook of Combinatorics* (1996)

The surveys by S.A. Burr [Bu1] and T.D. Parsons [Par5] contain extensive chapters on general exact results in graph Ramsey theory. F. Harary presented the state of the theory in 1981 in [Ha2], where he also gathered many references including seven to other survey papers. A decade ago, Chung and Grinstead in their survey paper [ChGri] gave less data than in this note, but included a broad discussion of different methods used in Ramsey computations in the classical case. S.A. Burr, one of the most experienced researchers in Ramsey graph theory, formulated in [Bu6] seven conjectures on Ramsey numbers for sufficiently large and sparse graphs, and reviewed the evidence for them found in the literature.

For newer extensive presentations see [GRS, GrRö, FRS7, Neš], though these focus on asymptotic theory not on the numbers themselves. Finally, this compilation could not pretend to be complete without mentioning a special volume of the *Journal of Graph Theory* [JGT] dedicated entirely to Ramsey theory. Besides a number of research papers, it includes historical notes and presents to us Frank P. Ramsey (1903-1930) as a person.

## 8. Concluding Remarks

This compilation does not include information on numerous variations of Ramsey numbers, nor related topics, like size Ramsey numbers, zero-sum Ramsey numbers, irredundant Ramsey numbers, local Ramsey numbers, connected Ramsey numbers, chromatic Ramsey numbers, avoiding sets of graphs in some colors, coloring graphs other than complete, or the so called Ramsey multiplicities. Interested reader can find such information in the surveys listed in section 7 here.

The author apologizes for any omissions or other errors in reporting results belonging to the scope of this work. Suggestions for any kind of corrections and/or additions will be greatly appreciated.

## Acknowledgement

I would like to thank Brendan McKay, Geoffrey Exoo and Heiko Harborth for their help in gathering data for earlier versions of this survey.

## References

We mark the papers containing results obtained with the help of computer algorithms with stars. We identify two categories of such papers: marked with \* involving some use of computers, where the results are easily verifiable with some computations, and those marked with \*\*, where cpu intensive algorithms have to be implemented to replicate or verify the results. The first category contains mostly constructions done by algorithms, while the second mostly nonexistence results or claims of complete enumerations of special kinds of graphs.

- [Abb] H.L. Abbot, A Theorem Concerning Higher Ramsey Numbers, in *Infinite and Finite Sets*, (A. Hajnal, R. Rado and V.T. Sós eds.) North Holland, (1975) 25-28.
- [AS] H.L. Abbot and M.J. Smuga-Otto, Lower Bounds for Hypergraph Ramsey Numbers, *Discrete Applied Mathematics*, 61 (1995) 177-180.
- [AKS] M. Ajtai, J. Komlós and E. Szemerédi, A Note on Ramsey Numbers, *Journal of Combinatorial Theory, Series A*, 29 (1980) 354-360.
- [Alon1] N. Alon, Subdivided Graphs Have Linear Ramsey Numbers, *Journal of Graph Theory*, 18 (1994) 343-347.
- [Alon2] N. Alon, Explicit Ramsey Graphs and Orthonormal Labelings, *The Electronic Journal of Combinatorics*, <http://ejc.math.gatech.edu:8080/Journal/ejc-wce.html>, #R12, 1 (1994) 8 pages.
- [AKM] J. Arste, K. Klamroth and I. Mengersen, Three Color Ramsey Numbers for Small Graphs, *Utilitas Mathematica*, 49 (1996) 85-96.
- [Ba]\* F. Bannani, Bounds on Classical Ramsey Numbers, *Ph.D. thesis*, Carleton University, Ottawa, November 1988.

- [BS] A. Bialostocki and J. Schönheim, On Some Turán and Ramsey Numbers for  $C_4$ , in *Graph Theory and Combinatorics* (ed. B. Bollobás), Academic Press, London, (1984) 29-33.
- [Bier] J. Bierbrauer, Ramsey Numbers for the Path with Three Edges, *European Journal of Combinatorics*, 7 (1986) 205-206.
- [BB] J. Bierbrauer and A. Brandis, On Generalized Ramsey Numbers for Trees, *Combinatorica*, 5 (1985) 95-107.
- [BH] R. Bolze and H. Harborth, The Ramsey Number  $r(K_{4-x}, K_5)$ , in *The Theory and Applications of Graphs*, (Kalamazoo, MI, 1980), John Wiley & Sons, New York, (1981) 109-116.
- [BoEr] J.A. Bondy and P. Erdős, Ramsey Numbers for Cycles in Graphs, *Journal of Combinatorial Theory, Series B*, 14 (1973) 46-54.
- [-] A. Brandis, see [BB].
- [Bra1] S. Brandt, Subtrees and Subforests in Graphs, *Journal of Combinatorial Theory, Series B*, 61 (1994) 63-70.
- [Bra2] S. Brandt, Sufficient Conditions for Graphs to Contain All Subgraphs of a Given Type, *Ph.D. thesis*, Freie Universität Berlin, 1994.
- [Bra3] S. Brandt, Expanding Graphs and Ramsey Numbers, *to appear*.
- [BBH]\*\* S. Brandt, G. Brinkmann and T. Harmuth, All Ramsey Numbers  $r(K_3, G)$  for Connected Graphs of Order 9, *in preparation*.
- [Brin]\*\* G. Brinkmann, All Ramsey Numbers  $r(K_3, G)$  for Connected Graphs of Order up to 8, *to appear*.
- [-] G. Brinkmann, see also [BBH].
- [BR]\* J.P. Burling and S.W. Reyner, Some Lower Bounds of the Ramsey Numbers  $n(k, k)$ , *Journal of Combinatorial Theory, Series B*, 13 (1972) 168-169.
- [Bu1] S.A. Burr, Generalized Ramsey Theory for Graphs - a Survey, in *Graphs and Combinatorics* (R. Bari and F. Harary eds.), Springer LNM 406, Berlin, (1974) 52-75.
- [Bu2] S.A. Burr, Ramsey Numbers Involving Graphs with Long Suspended Paths, *Journal of the London Mathematical Society* (2), 24 (1981) 405-413.
- [Bu3] S.A. Burr, Multicolor Ramsey Numbers Involving Graphs with Long Suspended Path, *Discrete Mathematics*, 40 (1982) 11-20.
- [Bu4] S.A. Burr, Diagonal Ramsey Numbers for Small Graphs, *Journal of Graph Theory*, 7 (1983) 57-69.
- [Bu5] S.A. Burr, Ramsey Numbers Involving Powers, *Ars Combinatoria*, 15 (1983) 163-168.
- [Bu6] S.A. Burr, What Can We Hope to Accomplish in Generalized Ramsey Theory?, *Discrete Mathematics*, 67 (1987) 215-225.
- [Bu7] S.A. Burr, On the Ramsey Numbers  $r(G, nH)$  and  $r(nG, nH)$  When  $n$  Is Large, *Discrete Mathematics*, 65 (1987) 215-229.
- [Bu8] S.A. Burr, On Ramsey Numbers for Large Disjoint Unions of Graphs, *Discrete Mathematics*, 70 (1988) 277-293.
- [BE1] S.A. Burr and P. Erdős, Extremal Ramsey Theory for Graphs, *Utilitas Mathematica*, 9 (1976) 247-258.
- [BE2] S.A. Burr and P. Erdős, Generalizations of a Ramsey-Theoretic Result of Chvátal, *Journal of Graph Theory*, 7 (1983) 39-51.
- [BEFRS1] S.A. Burr, P. Erdős, R.J. Faudree, C.C. Rousseau and R.H. Schelp, An Extremal Problem in Generalized Ramsey Theory, *Ars Combinatoria*, 10 (1980) 193-203.
- [BEFRS2] S.A. Burr, P. Erdős, R.J. Faudree, C.C. Rousseau and R.H. Schelp, Ramsey Numbers for the Pair Sparse Graph-Path or Cycle, *Transactions of the American Mathematical Society*, 269 (1982) 501-512.
- [BEFRS3] S.A. Burr, P. Erdős, R.J. Faudree, C.C. Rousseau and R.H. Schelp, On Ramsey Numbers Involving Starlike Multipartite Graphs, *Journal of Graph Theory*, 7 (1983) 395-409.



- [BEFRS4] S.A. Burr, P. Erdős, R.J. Faudree, C.C. Rousseau and R.H. Schelp, The Ramsey Number for the Pair Complete Bipartite Graph-Graph of Limited Degree, in *Graph Theory with Applications to Algorithms and Computer Science*, (Y. Alavi et al. eds.), John Wiley & Sons, New York, (1985) 163-174.
- [BEFRSGJ] S.A. Burr, P. Erdős, R.J. Faudree, C.C. Rousseau, R.H. Schelp, R.J. Gould and M.S. Jacobson, Goodness of Trees for Generalized Books, *Graphs and Combinatorics*, 3 (1987) 1-6.
- [BEFS] S.A. Burr, P. Erdős, R.J. Faudree and R.H. Schelp, On the Difference between Consecutive Ramsey Numbers, *Utilitas Mathematica*, 35 (1989) 115-118.
- [BES] S.A. Burr, P. Erdős and J.H. Spencer, Ramsey Theorems for Multiple Copies of Graphs, *Transactions of the American Mathematical Society*, 209 (1975) 87-99.
- [BG] S.A. Burr and J.W. Grossman, Ramsey Numbers of Graphs with Long Tails, *Discrete Mathematics*, 41 (1982) 223-227.
- [BuRo1] S.A. Burr and J.A. Roberts, On Ramsey Numbers for Stars, *Utilitas Mathematica*, 4 (1973) 217-220.
- [BuRo2] S.A. Burr and J.A. Roberts, On Ramsey Numbers for Linear Forests, *Discrete Mathematics*, 8 (1974) 245-250.
- [CET]\* N.J. Calkin, P. Erdős and C.A. Tovey, New Ramsey Bounds from Cyclic Graphs of Prime Order, *to appear*.
- [CGP] G. Chartrand, R.J. Gould and A.D. Polimeni, On Ramsey Numbers of Forests versus Nearly Complete Graphs, *Journal of Graph Theory*, 4 (1980) 233-239.
- [CRSPS] G. Chartrand, C.C. Rousseau, M.J. Stewart, A.D. Polimeni and J. Sheehan, On Star-Book Ramsey Numbers, in *Proceedings of the Fourth International Conference on the Theory and Applications of Graphs*, (Kalamazoo, MI 1980), John Wiley & Sons, (1981) 203-214.
- [ChLo]\* V. Chokbowornpaishand and V. Longani, Lower Bounds for Some Ramsey Numbers, *Southeast Asian Bulletin of Mathematics*, 15 (1991) 105-107.
- [Chu1] F.R.K. Chung, On the Ramsey Numbers  $N(3,3,\dots,3; 2)$ , *Discrete Mathematics*, 5 (1973) 317-321.
- [Chu2] F.R.K. Chung, On Triangular and Cyclic Ramsey Numbers with  $k$  Colors, in *Graphs and Combinatorics* (R. Bari and F. Harary eds.), Springer LNM 406, Berlin, (1974) 236-241.
- [Chu3] F.R.K. Chung, A Note on Constructive Methods for Ramsey Numbers, *Journal of Graph Theory*, 5 (1981) 109-113.
- [CCD] F.R.K. Chung, R. Cleve and P. Dagum, A Note on Constructive Lower Bounds for the Ramsey Numbers  $R(3,t)$ , *Journal of Combinatorial Theory, Series B*, 57 (1993) 150-155.
- [ChGra] F.R.K. Chung and R.L. Graham, On Multicolor Ramsey Numbers for Complete Bipartite Graphs, *Journal of Combinatorial Theory, Series B*, 18 (1975) 164-169.
- [ChGri] F.R.K. Chung and C.M. Grinstead, A Survey of Bounds for Classical Ramsey Numbers, *Journal of Graph Theory*, 7 (1983) 25-37.
- [Chv] V. Chvátal, Tree-Complete Graph Ramsey Numbers, *Journal of Graph Theory*, 1 (1977) 93.
- [CH1] V. Chvátal and F. Harary, Generalized Ramsey Theory for Graphs, II. Small Diagonal Numbers, *Proceedings of American Mathematical Society*, 32 (1972) 389-394.
- [CH2] V. Chvátal and F. Harary, Generalized Ramsey Theory for Graphs, III. Small Off-Diagonal Numbers, *Pacific Journal of Mathematics*, 41 (1972) 335-345.
- [CH3] V. Chvátal and F. Harary, Generalized Ramsey Theory for Graphs, I. Diagonal Numbers, *Periodica Mathematica Hungarica*, 3 (1973) 115-124.
- [CRST] V. Chvátal, V. Rödl, E. Szemerédi and W.T. Trotter Jr., The Ramsey Number of a Graph with Bounded Maximum Degree, *Journal of Combinatorial Theory, Series B*, 34 (1983) 239-243.
- [CS] V. Chvátal and A. Schwenk, On the Ramsey Number of the Five-Spoked Wheel, in *Graphs and Combinatorics* (R. Bari and F. Harary eds.), Springer LNM 406, Berlin, (1974) 247-261.
- [Clan] M. Clancy, Some Small Ramsey Numbers, *Journal of Graph Theory*, 1 (1977) 89-91.

- [Clap] C. Clapham, The Ramsey Number  $r(C_4, C_4, C_4)$ , *Periodica Mathematica Hungarica*, 18 (1987) 317-318.
- [CEHMS] C. Clapham, G. Exoo, H. Harborth, I. Mengersen and J. Sheehan, The Ramsey Number of  $K_5 - e$ , *Journal of Graph Theory*, 13 (1989) 7-15.
- [Clark] L. Clark, On Cycle-Star Graph Ramsey Numbers, *Congressus Numerantium*, 50 (1985) 187-192.
- [-] R. Cleve, see [CCD].
- [CL1] E.J. Cockayne and P.J. Lorimer, The Ramsey Number for Stripes, *Journal of the Australian Mathematical Society, Series A*, 19 (1975) 252-256.
- [CL2] E.J. Cockayne and P.J. Lorimer, On Ramsey Graph Numbers for Stars and Stripes, *Canadian Mathematical Bulletin*, 18 (1975) 31-34.
- [-] P. Dagum, see [CCD].
- [Den] T. Denley, The Ramsey Numbers for Disjoint Unions of Cycles, *Discrete Mathematics*, 149 (1996) 31-44.
- [DLR] D. Duffus, H. Lefmann and V. Rödl, Shift Graphs and Lower Bounds on Ramsey Numbers  $r_k(l; r)$ , *Discrete Mathematics*, 137 (1995) 177-187.
- [Ea1] Easy to obtain by simple combinatorics from other results.
- [Ea2] Easy to obtain using  $R(3,3,4) \leq 31$ .
- [Ea3] Easy to obtain using graphs (or their disjoint unions) establishing lower bounds with smaller parameters.
- [Ea4] Unique 2-(6,3,2) design gives lower bound 7, upper bound is easy.
- [EFRS1] P. Erdős, R.J. Faudree, C.C. Rousseau and R.H. Schelp, Generalized Ramsey Theory for Multiple Colors, *Journal of Combinatorial Theory, Series B*, 20 (1976) 250-264.
- [EFRS2] P. Erdős, R.J. Faudree, C.C. Rousseau and R.H. Schelp, On Cycle-Complete Graph Ramsey Numbers, *Journal of Graph Theory*, 2 (1978) 53-64.
- [EFRS3] P. Erdős, R.J. Faudree, C.C. Rousseau and R.H. Schelp, Ramsey Numbers for Brooms, *Congressus Numerantium*, 35 (1982) 283-293.
- [EFRS4] P. Erdős, R.J. Faudree, C.C. Rousseau and R.H. Schelp, Multipartite Graph-Sparse Graph Ramsey Numbers, *Combinatorica*, 5 (1985) 311-318.
- [EFRS5] P. Erdős, R.J. Faudree, C.C. Rousseau and R.H. Schelp, A Ramsey Problem of Harary on Graphs with Prescribed Size, *Discrete Mathematics*, 67 (1987) 227-233.
- [EFRS6] P. Erdős, R.J. Faudree, C.C. Rousseau and R.H. Schelp, Extremal Theory and Bipartite Graph-Tree Ramsey Numbers, *Discrete Mathematics*, 72 (1988) 103-112.
- [EFRS7] P. Erdős, R.J. Faudree, C.C. Rousseau and R.H. Schelp, Book-Tree Ramsey Numbers, *Scientia A: Math*, 1 (1988) 111-117.
- [EFRS8] P. Erdős, R.J. Faudree, C.C. Rousseau and R.H. Schelp, Multipartite Graph-Tree Graph Ramsey Numbers, in *Graph Theory and Its Applications: East and West, Proceedings of the First China-USA International Graph Theory Conference*, Annals of the New York Academy of Sciences, 576 (1989) 146-154.
- [EFRS9] P. Erdős, R.J. Faudree, C.C. Rousseau and R.H. Schelp, Ramsey Size Linear Graphs, *Combinatorics, Probability and Computing*, 2 (1993) 389-399.
- [EG] P. Erdős and R.L. Graham, On Partition Theorems for Finite Sets, in *Infinite and Finite Sets*, (A. Hajnal, R. Rado and V.T. Sós eds.) North Holland, (1975) 515-527.
- [-] P. Erdős, see also [BoEr, BE1, BE2, BEFRS1, BEFRS2, BEFRS3, BEFRS4, BEFRSGJ, BEFS, BES, CET].
- [Ex1]\* G. Exoo, Ramsey Numbers of Hypergraphs, *Journal of Combinatorial Mathematics and Combinatorial Computing*, 2 (1987) 5-11.

- [Ex2]\* G. Exoo, On Two Classical Ramsey Numbers of the Form  $R(3, n)$ , *SIAM Journal of Discrete Mathematics*, 2 (1989) 488-490.
- [Ex3]\* G. Exoo, Applying Optimization Algorithm to Ramsey Problems, in *Graph Theory, Combinatorics, Algorithms, and Applications* (Y. Alavi ed.), SIAM Philadelphia, (1989) 175-179.
- [Ex4]\* G. Exoo, A Lower Bound for  $R(5, 5)$ , *Journal of Graph Theory*, 13 (1989) 97-98.
- [Ex5]\* G. Exoo, A Lower Bound for  $r(K_5 - e, K_5)$ , *Utilitas Mathematica*, 38 (1990) 187-188.
- [Ex6]\* G. Exoo, On the Three Color Ramsey Number of  $K_4 - e$ , *Discrete Mathematics*, 89 (1991) 301-305.
- [Ex7]\* G. Exoo, Indiana State University, *personal communication* (1992).
- [Ex8]\* G. Exoo, Announcement: On the Ramsey Numbers  $R(4, 6)$ ,  $R(5, 6)$  and  $R(3, 12)$ , *Ars Combinatoria*, 35 (1993) 85.
- [Ex9]\* G. Exoo, Constructing Ramsey Graphs with a Computer, *Congressus Numerantium*, 59 (1987) 31-36.
- [Ex10]\* G. Exoo, Indiana State University, *personal communication* (1997).
- [Ex11]\* G. Exoo, A Lower Bound for Schur Numbers and Multicolor Ramsey Numbers of  $K_3$ , *The Electronic Journal of Combinatorics*, <http://ejc.math.gatech.edu:8080/Journal/ejc-wce.html>, #R8, 1 (1994) 3 pages.
- [EHM1] G. Exoo, H. Harborth and I. Mengersen, The Ramsey Number of  $K_4$  versus  $K_5 - e$ , *Ars Combinatoria*, 25A (1988) 277-286.
- [EHM2] G. Exoo, H. Harborth and I. Mengersen, On Ramsey Number of  $K_{2,m}$ , in *Graph Theory, Combinatorics, Algorithms, and Applications* (Y. Alavi, F.R.K. Chung, R.L. Graham and D.F. Hsu eds.), SIAM Philadelphia, (1989) 207-211.
- [ER]\* G. Exoo and D.F. Reynolds, Ramsey Numbers Based on  $C_5$ -Decompositions, *Discrete Mathematics*, 71 (1988) 119-127.
- [-] G. Exoo, see also [CEHMS].
- [FLPS] R.J. Faudree, S.L. Lawrence, T.D. Parsons and R.H. Schelp, Path-Cycle Ramsey Numbers, *Discrete Mathematics*, 10 (1974) 269-277.
- [FM]\*\* R.J. Faudree and B.D. McKay, A Conjecture of Erdős and the Ramsey Number  $r(W_6)$ , *Journal of Combinatorial Mathematics and Combinatorial Computing*, 13 (1993) 23-31.
- [FRS1] R.J. Faudree, C.C. Rousseau and R.H. Schelp, All Triangle-Graph Ramsey Numbers for Connected Graphs of Order Six, *Journal of Graph Theory*, 4 (1980) 293-300.
- [FRS2] R.J. Faudree, C.C. Rousseau and R.H. Schelp, Studies Related to the Ramsey Number  $r(K_5 - e)$ , in *Graph Theory and Its Applications to Algorithms and Computer Science*, (Y. Alavi et al. eds.), John Wiley and Sons, New York, (1985) 251-271.
- [FRS3] R.J. Faudree, C.C. Rousseau and R.H. Schelp, Generalizations of the Tree-Complete Graph Ramsey Number, in *Graphs and Applications*, (F. Harary and J.S. Maybee eds.), John Wiley and Sons, New York, (1985) 117-126.
- [FRS4] R.J. Faudree, C.C. Rousseau and R.H. Schelp, Small Order Graph-Tree Ramsey Numbers, *Discrete Mathematics*, 72 (1988) 119-127.
- [FRS5] R.J. Faudree, C.C. Rousseau and J. Sheehan, Cycle-Book Ramsey Numbers, *Ars Combinatoria*, 31 (1991) 239-248.
- [FRS6] R.J. Faudree, C.C. Rousseau and J. Sheehan, More from the Good Book, in *Proceedings of the Ninth Southeastern Conference on Combinatorics, Graph Theory, and Computing*, Utilitas Mathematica Publ., (1978) 289-299.
- [FRS7] R.J. Faudree, C.C. Rousseau and R.H. Schelp, A Good Idea in Ramsey Theory, in *Graph Theory, Combinatorics, Algorithms, and Applications* (San Francisco, CA 1989), SIAM Philadelphia, PA (1991) 180-189.
- [FS1] R.J. Faudree and R.H. Schelp, All Ramsey Numbers for Cycles in Graphs, *Discrete Mathematics*, 8 (1974) 313-329.

- [FS2] R.J. Faudree and R.H. Schelp, Path Ramsey Numbers in Multicolorings, *Journal of Combinatorial Theory, Series B*, 19 (1975) 150-160.
- [FS3] R.J. Faudree and R.H. Schelp, Ramsey Numbers for All Linear Forests, *Discrete Mathematics*, 16 (1976) 149-155.
- [FSR] R.J. Faudree, R.H. Schelp and C.C. Rousseau, Generalizations of a Ramsey Result of Chvátal, in *Proceedings of the Fourth International Conference on the Theory and Applications of Graphs*, (Kalamazoo, MI 1980), John Wiley & Sons, (1981) 351-361.
- [FSS] R.J. Faudree, R.H. Schelp and M. Simonovits, On Some Ramsey Type Problems Connected with Paths, Cycles and Trees, *Ars Combinatoria*, 29A (1990) 97-106.
- [FS] R.J. Faudree and M. Simonovits, Ramsey Problems and Their Connection to Turán-Type Extremal Problems, *Journal of Graph Theory*, 16 (1992) 25-50.
- [-] R.J. Faudree, see also [BEFRS1, BEFRS2, BEFRS3, BEFRS4, BEFRSGJ, BEFS, EFRS1, EFRS2, EFRS3, EFRS4, EFRS5, EFRS6, EFRS7, EFRS8, EFRS9].
- [Fo] J. Folkman, Notes on the Ramsey Number  $N(3,3,3,3)$ , *Journal of Combinatorial Theory, Series A*, 16 (1974) 371-379.
- [Fra1] K. Fraughnaugh Jones, Independence in Graphs with Maximum Degree Four, *Journal of Combinatorial Theory, Series B*, 37 (1984) 254-269.
- [Fra2] K. Fraughnaugh Jones, Size and Independence in Triangle-Free Graphs with Maximum Degree Three, *Journal of Graph Theory*, 14 (1990) 525-535.
- [FL] K. Fraughnaugh and S.C. Locke, Finding Independent Sets in Triangle-free Graphs, *SIAM Journal of Discrete Mathematics*, 9 (1996) 674-681.
- [Fre] H. Fredrickson, Schur Numbers and the Ramsey Numbers  $N(3,3,\dots,3;2)$ , *Journal of Combinatorial Theory, Series A*, 27 (1979) 376-377.
- [GeGy] L. Gerencsér and A. Gyárfás, On Ramsey-Type Problems, *Ann. Univ. Sci. Budapest, Eötvös Sect. Math.*, 10 (1967) 167-170.
- [Gi1] G. Giraud, Nouvelles majorations des nombres de Ramsey binaires-bicolores, *C.R. Acad. Sc. Paris, A*, 268 (1969) 5-7.
- [Gi2] G. Giraud, Majoration du nombre de Ramsey ternaire-bicolore en (4,4), *C.R. Acad. Sc. Paris*, 269, 15 (1969) 620-622.
- [Gi3] G. Giraud, Une minoration du nombre de quadrangles unicolores et son application a la majoration des nombres de Ramsey binaires bicolores, *C.R. Acad. Sci., Paris, A*, 276 (1973) 1173-1175.
- [Gi4] G. Giraud, Sur le Probleme de Goodman pour les Quadrangles et la Majoration des Nombres de Ramsey, *Journal of Combinatorial Theory, Series B*, 27 (1979) 237-253.
- [-] A.M. Gleason, see [GG].
- [GJ1] R.J. Gould and M.S. Jacobson, Bounds for the Ramsey Number of a Disconnected Graph Versus Any Graph, *Journal of Graph Theory*, 6 (1982) 413-417.
- [GJ2] R.J. Gould and M.S. Jacobson, On the Ramsey Number of Trees Versus Graphs with Large Clique Number, *Journal of Graph Theory*, 7 (1983) 71-78.
- [-] R.J. Gould, see also [BEFRSGJ, CGP].
- [GrRö] R.L. Graham and V. Rödl, Numbers in Ramsey Theory, in *Surveys in Combinatorics*, (ed. C. Whitehead), Cambridge University Press, 1987.
- [GRS] R.L. Graham, B.L. Rothschild and J.H. Spencer, *Ramsey Theory*, John Wiley & Sons, 1990.
- [-] R.L. Graham, see also [ChGra, EG].
- [GY] J.E. Graver and J. Yackel, Some Graph Theoretic Results Associated with Ramsey's Theorem, *Journal of Combinatorial Theory*, 4 (1968) 125-175.
- [GG] R.E. Greenwood and A.M. Gleason, Combinatorial Relations and Chromatic Graphs, *Canadian Journal of Mathematics*, 7 (1955) 1-7.

- [GH] U. Grenda and H. Harborth, The Ramsey Number  $r(K_3, K_7 - e)$ , *Journal of Combinatorics, Information & System Sciences*, 7 (1982) 166-169.
- [Gri] J.R. Griggs, An Upper Bound on the Ramsey Numbers  $R(3, k)$ , *Journal of Combinatorial Theory, Series A*, 35 (1983) 145-153.
- [GR]\*\* C. Grinstead and S. Roberts, On the Ramsey Numbers  $R(3, 8)$  and  $R(3, 9)$ , *Journal of Combinatorial Theory, Series B*, 33 (1982) 27-51.
- [-] C. Grinstead, see also [ChGri].
- [Gro1] J.W. Grossman, Some Ramsey Numbers of Unicyclic Graphs, *Ars Combinatoria*, 8 (1979) 59-63.
- [Gro2] J.W. Grossman, The Ramsey Numbers of the Union of Two Stars, *Utilitas Mathematica*, 16 (1979) 271-279.
- [GHK] J.W. Grossman, F. Harary and M. Klawe, Generalized Ramsey Theory for Graphs, X: Double Stars, *Discrete Mathematics*, 28 (1979) 247-254.
- [-] J.W. Grossman, see also [BG].
- [Gua] Guantao Chen, A Result on  $C_4$ -Star Ramsey Numbers, *Discrete Mathematics*, 163 (1997) 243-246.
- [GS] Guantao Chen and R.H. Schelp, Graphs with Linearly Bounded Ramsey Numbers, *Journal of Combinatorial Theory, Series B*, 57 (1993) 138-149.
- [GV] Yubao Guo and L. Volkmann, Tree-Ramsey Numbers, *Australasian Journal of Combinatorics*, 11 (1995) 169-175.
- [GT] A. Gyárfás and Z. Tuza, An Upper Bound on the Ramsey Number of Trees, *Discrete Mathematics*, 66 (1987) 309-310.
- [-] A. Gyárfás, see also [GeGy].
- [Häg] R. Häggkvist, On the Path-Complete Bipartite Ramsey Number, *Discrete Mathematics*, 75 (1989) 243-245.
- [Ha1] F. Harary, Recent Results on Generalized Ramsey Theory for Graphs, in *Graph Theory and Applications*, (Y. Alavi et al. eds.) Springer, Berlin (1972) 125-138.
- [Ha2] F. Harary, Generalized Ramsey Theory I to XIII: Achievement and Avoidance Numbers, in *Proceedings of the Fourth International Conference on the Theory and Applications of Graphs*, (Kalamazoo, MI 1980), John Wiley & Sons, (1981) 373-390.
- [-] F. Harary, see also [CH1, CH2, CH3, GHK].
- [HM1] H. Harborth and I. Mengersen, An Upper Bound for the Ramsey Number  $r(K_5 - e)$ , *Journal of Graph Theory*, 9 (1985) 483-485.
- [HM2] H. Harborth and I. Mengersen, All Ramsey Numbers for Five Vertices and Seven or Eight Edges, *Discrete Mathematics*, 73 (1988/89) 91-98.
- [HM3] H. Harborth and I. Mengersen, The Ramsey Number of  $K_{3,3}$ , in *Combinatorics, Graph Theory, and Applications*, Vol. 2 (Y. Alavi, G. Chartrand, O.R. Oellermann and J. Schwenk eds.), John Wiley & Sons, (1991) 639-644.
- [-] H. Harborth, see also [BH, CEHMS, EHM1, EHM2, GH].
- [HM] M. Harborth and I. Mengersen, Some Ramsey Numbers for Complete Bipartite Graphs, *Australasian Journal of Combinatorics*, 13 (1996) 119-128.
- [-] T. Harmuth, see [BBH].
- [He1] G.R.T. Hendry, Diagonal Ramsey Numbers for Graphs with Seven Edges, *Utilitas Mathematica*, 32 (1987) 11-34.
- [He2] G.R.T. Hendry, Ramsey Numbers for Graphs with Five Vertices, *Journal of Graph Theory*, 13 (1989) 245-248.
- [He3] G.R.T. Hendry, The Ramsey Numbers  $r(K_2 + \bar{K}_3, K_4)$  and  $r(K_1 + C_4, K_4)$ , *Utilitas Mathematica*, 35 (1989) 40-54, addendum in 36 (1989) 25-32.

- [He4] G.R.T. Hendry, Critical Colorings for Clancy's Ramsey Numbers, *Utilitas Mathematica*, 41 (1992) 181-203.
- [He5] G.R.T. Hendry, Small Ramsey Numbers II. Critical Colorings for  $r(C_5 + e, K_5)$ , *Quaestiones Mathematica*, 17 (1994) 249-258.
- [-] G.R.T. Hendry, see also [YH].
- [HI]\* R. Hill and R.W. Irving, On Group Partitions Associated with Lower Bounds for Symmetric Ramsey Numbers, *European Journal of Combinatorics*, 3 (1982) 35-50.
- [Hua1] Huang Guotai, Some Generalized Ramsey Numbers (in Chinese), *Mathematica Applicata*, 1 (1988) 97-101.
- [Hua2] Huang Guotai, An Unsolved Problem of Gould and Jacobson (in Chinese), *Mathematica Applicata*, 9 (1996) 234-236.
- [HZ] Huang Yi Ru and Zhang Ke Min, An New Upper Bound Formula for Two Color Classical Ramsey Numbers, *to appear* (1993).
- [Ir] R.W. Irving, Generalised Ramsey Numbers for Small Graphs, *Discrete Mathematics*, 9 (1974) 251-264.
- [-] R.W. Irving, see also [HI].
- [Is1] J.R. Isbell,  $N(4,4;3) \geq 13$ , *Journal of Combinatorial Theory*, 6 (1969) 210.
- [Is2] J.R. Isbell,  $N(5,4;3) \geq 24$ , *Journal of Combinatorial Theory*, Series A, 34 (1983) 379-380.
- [Jac] M.S. Jacobson, On the Ramsey Number for Stars and a Complete Graph, *Ars Combinatoria*, 17 (1984) 167-172.
- [-] M.S. Jacobson, see also [BEFRSGJ, GJ1, GJ2].
- [JR1] C.J. Jayawardene and C.C. Rousseau, An Upper Bound on a Ramsey Number for Quadrilaterals versus a Complete Graph on Seven Vertices, *preprint*.
- [JR2] C.J. Jayawardene and C.C. Rousseau, Some Ramsey Numbers for a  $C_5$  vs. Small Complete Graphs, *preprint*.
- [-] C.J. Jayawardene, see also [RoJa].
- [Jin]\*\* Jin Xia, Ramsey Numbers Involving a Triangle: Theory & Applications, *Technical Report RIT-TR-93-019*, MS thesis, Department of Computer Science, Rochester Institute of Technology, 1993.
- [-] Jin Xia, see also [RaJi].
- [JGT] Special volume on Ramsey theory of *Journal of Graph Theory*, Volume 7, Number 1, (1983).
- [Ka1] J.G. Kalbfleisch, Construction of Special Edge-Chromatic Graphs, *Canadian Mathematical Bulletin*, 8 (1965) 575-584.
- [Ka2]\* J.G. Kalbfleisch, Chromatic Graphs and Ramsey's Theorem, *Ph.D. thesis*, University of Waterloo, January 1966.
- [Ka3] J.G. Kalbfleisch, On Robillard's Bounds for Ramsey Numbers, *Canadian Mathematical Bulletin*, 14 (1971) 437-440.
- [KS] J.G. Kalbfleisch and R.G. Stanton, On the Maximal Triangle-Free Edge-Chromatic Graphs in Three Colors, *Journal of Combinatorial Theory*, 5 (1968) 9-20.
- [Kim] J.H. Kim, The Ramsey Number  $R(3,t)$  has Order of Magnitude  $t^2/\log t$ , *Random Structures and Algorithms*, 7 (1995) 173-207.
- [KM] K. Klamroth and I. Mengersen, Ramsey Numbers of  $K_3$  versus  $(p,q)$ -Graphs, *Ars Combinatoria*, 43 (1996) 107-120.
- [-] K. Klamroth, see also [AKM].
- [-] M. Klawe, see [GHK].
- [Köb] W. Köhler, On a Conjecture by Grossman, *Ars Combinatoria*, 23 (1987) 103-106.

- [-] J. Komlós, see [AKS].
- [KLR]\* D.L. Kreher, Wei Li and S.P. Radziszowski, Lower Bounds for Multi-Colored Ramsey Numbers From Group Orbits, *Journal of Combinatorial Mathematics and Combinatorial Computing*, 4 (1988) 87-95.
- [-] D.L. Kreher, see also [RK1, RK2, RK3, RK4].
- [La1] S.L. Lawrence, Cycle-Star Ramsey Numbers, *Notices of the American Mathematical Society*, 20 (1973) Abstract A- 420.
- [La2] S.L. Lawrence, Bipartite Ramsey Theory, *Notices of the American Mathematical Society*, 20 (1973) Abstract A- 562.
- [-] S.L. Lawrence, see also [FLPS].
- [LayMa] C. Laywine and J.P. Mayberry, A Simple Construction Giving the Two Non-isomorphic Triangle-Free 3-Colored  $K_{16}$ 's, *Journal of Combinatorial Theory, Series B*, 45 (1988) 120-124.
- [Lef] H. Lefmann, Ramsey Numbers for Monotone Paths and Cycles, *Ars Combinatoria*, 35 (1993) 271-279.
- [-] H. Lefmann, see also [DLR].
- [-] Li Qiao, see [SLL].
- [LR1] Yusheng Li and C.C. Rousseau, On Book-Complete Graph Ramsey Numbers, *Journal of Combinatorial Theory, Series B*, 68 (1996) 36-44.
- [LR2] Yusheng Li and C.C. Rousseau, A Note on the Ramsey Number  $(H + K_n, K_n)$ , *Discrete Mathematics*, 171 (1997) 265-267.
- [LR3] Yusheng Li and C.C. Rousseau, Fan-Complete Graph Ramsey Numbers, *Journal of Graph Theory*, 23 (1996) 413-420.
- [LRZ] Yusheng Li, C.C. Rousseau and Y. Zhang, The Ramsey Function for a Quadrilateral vs. Complete Graphs, *to appear*.
- [Lin] B. Lindström, Undecided Ramsey-Numbers for Paths, *Discrete Mathematics*, 43 (1983) 111-112.
- [Loc] S.C. Locke, Bipartite Density and the Independence Ratio, *Journal of Graph Theory*, 10 (1986) 47-53.
- [-] S.C. Locke, see also [FL].
- [-] V. Longani, see [ChLo].
- [Lor] P.J. Lorimer, The Ramsey Numbers for Stripes and One Complete Graph, *Journal of Graph Theory*, 8 (1984) 177-184.
- [LorMu] P.J. Lorimer and P.R. Mullins, Ramsey Numbers for Quadrangles and Triangles, *Journal of Combinatorial Theory, Series B*, 23 (1977) 262-265.
- [LorSe] P.J. Lorimer and R.J. Segedin, Ramsey Numbers for Multiple Copies of Complete Graphs, *Journal of Graph Theory*, 2 (1978) 89-91.
- [LorSo] P.J. Lorimer and W. Solomon, The Ramsey Numbers for Stripes and Complete Graphs 1, *Discrete Mathematics*, 104 (1992) 91-97. Corrigendum in *Discrete Mathematics*, 131 (1994) 395.
- [-] P.J. Lorimer, see also [CL1, CL2].
- [-] Luo Haipeng, see [SLL, SLW].
- [Mac]\* J. Mackey, Combinatorial Remedies, *Ph.D. Thesis*, Department of Mathematics, University of Hawaii, 1994.
- [Ma]\* R. Mathon, Lower Bounds for Ramsey Numbers and Association Schemes, *Journal of Combinatorial Theory, Series B*, 42 (1987) 122-127.
- [-] J.P. Mayberry, see [LayMa].
- [McS] C. McDiarmid and A. Steger, Tidier Examples for Lower Bounds on Diagonal Ramsey Numbers, *Journal of Combinatorial Theory, Series A*, 74 (1996) 147-152.

- [MR1]\*\* B.D. McKay and S.P. Radziszowski, The First Classical Ramsey Number for Hypergraphs is Computed, *Proceedings of the Second Annual ACM-SIAM Symposium on Discrete Algorithms*, SODA'91, San Francisco, (1991) 304-308.
- [MR2]\*\* B.D. McKay and S.P. Radziszowski, Linear Programming in Some Ramsey Problems, *Journal of Combinatorial Theory*, Series B, 61 (1994) 125-132.
- [MR3]\* B.D. McKay and S.P. Radziszowski, New Upper Bound for the Ramsey Number  $R(5,5)$ , *Australian Journal of Combinatorics*, 5 (1992) 13-20.
- [MR4]\*\* B.D. McKay and S.P. Radziszowski, Subgraph Counting Identities and Ramsey Numbers, *Journal of Combinatorial Theory*, Series B, 69 (1997) 193-209.
- [MR5]\*\* B.D. McKay and S.P. Radziszowski,  $R(4,5) = 25$ , *Journal of Graph Theory*, 19 (1995) 309-322.
- [MZ]\*\* B.D. McKay and Zhang Ke Min, The Value of the Ramsey Number  $R(3,8)$ , *Journal of Graph Theory*, 16 (1992) 99-105.
- [-] B.D. McKay, see also [FM].
- [McN]\*\* J. McNamara, SUNY Brockport, *personal communication* (1995).
- [McR]\*\* J. McNamara and S.P. Radziszowski, The Ramsey Numbers  $R(K_4 - e, K_6 - e)$  and  $R(K_4 - e, K_7 - e)$ , *Congressus Numerantium*, 81 (1991) 89-96.
- [-] I. Mengersen, see [AKM, CEHMS, EHM1, EHM2, HM1, HM2, HM3, HM, KM].
- [MS] H. Mizuno and I. Sato, Ramsey Numbers for Unions of Some Cycles, *Discrete Mathematics*, 69 (1988) 283-294.
- [-] P.R. Mullins, see [LorMu].
- [Neš] J. Nešetřil, Ramsey Theory, chapter 25 in *Handbook of Combinatorics*, ed. R.L. Graham, M. Grötschel and L. Lovász, The MIT-Press, Vol. II, 1996, 1331-1403.
- [Par1] T.D. Parsons, The Ramsey Numbers  $r(P_m, K_n)$ , *Discrete Mathematics*, 6 (1973) 159-162.
- [Par2] T.D. Parsons, Path-Star Ramsey Numbers, *Journal of Combinatorial Theory*, Series B, 17 (1974) 51-58.
- [Par3] T.D. Parsons, Ramsey Graphs and Block Designs, I, *Transactions of the American Mathematical Society*, 209 (1975) 33-44.
- [Par4] T.D. Parsons, Ramsey Graphs and Block Designs, *Journal of Combinatorial Theory*, Series A, 20 (1976) 12-19.
- [Par5] T.D. Parsons, Ramsey Graph Theory, in *Selected Topics in Graph Theory*, (L.W. Beineke and R.J. Wilson eds.), Academic Press, (1978) 361-384.
- [-] T.D. Parsons, see also [FLPS].
- [Piw1]\* K. Piwakowski, Technical University of Gdańsk, paper presented at the conference "System-Modeling-Control", Zakopane, Poland (1993).
- [Piw2]\* K. Piwakowski, On Ramsey Number  $R(4,3,3)$  and Triangle-free Edge-chromatic Graphs in Three Colors, "The Second Kraków Conference on Graph Theory" (1994), *Discrete Mathematics*, 164 (1997) 243-249.
- [Piw3]\* K. Piwakowski, Applying Tabu Search to Determine New Ramsey Graphs, *The Electronic Journal of Combinatorics*, <http://ejc.math.gatech.edu:8080/Journal/ejc-wce.html>, #R6, 3 (1996) 4 pages.
- [Piw4]\*\* K. Piwakowski, Applying Algorithmic Techniques in Finding Lower and Upper Bounds for Ramsey Numbers (in Polish), *Ph.D. Thesis*, Technical University of Gdańsk, ETI 5/96, 1996.
- [PR]\*\* K. Piwakowski and S.P. Radziszowski, New Upper Bound for the Ramsey Number  $R(3,3,4)$ , *to appear*.
- [-] A.D. Polimeni, see [CGP, CRSPS].
- [Ra1]\* S.P. Radziszowski, used the same method as in [RK2], *unpublished* (1988).
- [Ra2]\*\* S.P. Radziszowski, The Ramsey Numbers  $R(K_3, K_8 - e)$  and  $R(K_3, K_9 - e)$ , *Journal of Combinatorial Mathematics and Combinatorial Computing*, 8 (1990) 137-145.



- [Ra3]\*\* S.P. Radziszowski, On the Ramsey Number  $R(K_5 - e, K_5 - e)$ , *Ars Combinatoria*, 36 (1993) 225-232.
- [Ra4] S.P. Radziszowski, Small Ramsey Numbers, *Technical Report RIT-TR-93-009*, Department of Computer Science, Rochester Institute of Technology (1993).
- [RaJi] S.P. Radziszowski and Jin Xia, Paths, Cycles and Wheels in Graphs without Antitriangles, *Australian Journal of Combinatorics*, 9 (1994) 221-232.
- [RK1]\* S.P. Radziszowski and D.L. Kreher, Search Algorithm for Ramsey Graphs by Union of Group Orbits, *Journal of Graph Theory*, 12 (1988) 59-72.
- [RK2]\*\* S.P. Radziszowski and D.L. Kreher, Upper Bounds for Some Ramsey Numbers  $R(3, k)$ , *Journal of Combinatorial Mathematics and Combinatorial Computing*, 4 (1988) 207-212.
- [RK3]\*\* S.P. Radziszowski and D.L. Kreher, On  $R(3, k)$  Ramsey Graphs: Theoretical and Computational Results, *Journal of Combinatorial Mathematics and Combinatorial Computing*, 4 (1988) 37-52.
- [RK4] S.P. Radziszowski and D.L. Kreher, Minimum Triangle-Free Graphs, *Ars Combinatoria*, 31 (1991) 65-92.
- [-] S.P. Radziszowski, see also [KLR, MR1, MR2, MR3, MR4, MR5, McR, PR].
- [Ram] F.P. Ramsey, On a Problem of Formal Logic, *Proceedings of the London Mathematical Society*, 30 (1930) 264-286.
- [-] S.W. Reyner, see [BR].
- [-] D.F. Reynolds, see [ER].
- [Rob] F.S. Roberts, *Applied Combinatorics*, Prentice-Hall, Englewood Cliffs, 1984.
- [-] J.A. Roberts, see [BuRo1, BuRo2].
- [-] S. Roberts, see [GR].
- [-] V. Rödl, see [CRST, DLR, GrRö].
- [Ros] V. Rosta, On a Ramsey Type Problem of J.A. Bondy and P. Erdős, I & II, *Journal of Combinatorial Theory, Series B*, 15 (1973) 94-120.
- [-] B.L. Rothschild, see [GRS].
- [RoJa] C.C. Rousseau and C.J. Jayawardene, The Ramsey Number for a Quadrilateral vs. a Complete Graph on Six Vertices, *preprint*.
- [RS1] C.C. Rousseau and J. Sheehan, On Ramsey Numbers for Books, *Journal of Graph Theory*, 2 (1978) 77-87.
- [RS2] C.C. Rousseau and J. Sheehan, A Class of Ramsey Problems Involving Trees, *Journal of the London Mathematical Society (2)*, 18 (1978) 392-396.
- [-] C.C. Rousseau, see also [BEFRS1, BEFRS2, BEFRS3, BEFRS4, BEFRSGJ, CRSPS, EFRS1, EFRS2, EFRS3, EFRS4, EFRS5, EFRS6, EFRS7, EFRS8, EFRS9, FRS1, FRS2, FRS3, FRS4, FRS5, FRS6, FRS7, FRS, JR1, JR2, LR1, LR2, LR3, LRZ].
- [-] P. Rowlinson, see [YR1, YR2, YR3].
- [San] A. Sanchez-Flores, An Improved Bound for Ramsey Number  $N(3,3,3,3;2)$ , *Discrete Mathematics*, 140 (1995) 281-286.
- [-] I. Sato, see [MS].
- [-] R.H. Schelp, see [BEFRS1, BEFRS2, BEFRS3, BEFRS4, BEFRSGJ, BEFS, EFRS1, EFRS2, EFRS3, EFRS4, EFRS5, EFRS6, EFRS7, EFRS8, EFRS9, FLPS, FRS1, FRS2, FRS3, FRS4, FRS7, FS1, FS2, FS3, FSR, FSS, GS].
- [-] J. Schönheim, see [BS].
- [SchSch]\* A. Schelten and I. Schiermeyer, Ramsey Numbers  $r(K_3, G)$  for Connected Graphs  $G$  of Order Seven, *to appear*.
- [-] I. Schiermeyer, see [SchSch].

- [Schu] C. -U. Schulte, Ramsey-Zahlen für Bäume und Kreise, *Ph.D. thesis*, Heinrich-Heine-Universität Düsseldorf, (1992).
- [-] A. Schwenk, see [CS].
- [Scob] M.W. Scobee, On the Ramsey Number  $R(m_1P_3, m_2P_3, m_3P_3)$  and Related Results, ..., *MA thesis*, University of Louisville (1993).
- [-] R.J. Segegin, see [LorSe].
- [Sha] A. Shastri, Lower Bounds for Bi-Colored Quaternary Ramsey Numbers, *Discrete Mathematics*, 84 (1990) 213-216.
- [S1]\* J.B. Shearer, Lower Bounds for Small Diagonal Ramsey Numbers, *Journal of Combinatorial Theory*, Series A, 42 (1986) 302-304.
- [S2] J.B. Shearer, A Note on the Independence Number of Triangle-free Graphs II, *Journal of Combinatorial Theory*, Series B, 53 (1991) 300-307.
- [-] J. Sheehan, see [CRSPS, CEHMS, FRS5, FRS6, RS1, RS2].
- [Sid1] A.F. Sidorenko, On Turán Numbers  $T(n, 5, 4)$  and Number of Monochromatic 4-cliques in 2-colored 3-graphs (in Russian), *Voprosy Kibernetiki*, 64 (1980) 117-124.
- [Sid2] A.F. Sidorenko, An Upper Bound on the Ramsey Number  $R(K_3, G)$  Depending Only on the Size of the Graph  $G$ , *Journal of Graph Theory*, 15 (1991) 15-17.
- [Sid3] A.F. Sidorenko, The Ramsey Number of an  $N$ -Edge Graph Versus Triangle Is at Most  $2N + 1$ , *Journal of Combinatorial Theory*, Series B, 58 (1993) 185-196.
- [-] M. Simonovits, see [FSS, FS].
- [-] M.J. Smuga-Otto, see [AS].
- [Sob] A. Sobczyk, Euclidian Simplices and the Ramsey Number  $R(4, 4; 3)$ , *Technical Report #10*, Clemson University (1967).
- [-] W. Solomon, see [LorSo].
- [Song] Song Enmin, New Lower Bound Formulas for the Ramsey Numbers  $N(k, k, \dots, k; 2)$  (in Chinese), *Mathematica Applicata*, 6 (1993) suppl., 113-116.
- [SWY] Song Enmin, Weiguo Ye and Yanwu Liu, New Lower Bounds for Ramsey Number  $R(p, q; 4)$ , *Discrete Mathematics*, 145 (1995) 343-346.
- [-] J.H. Spencer, see [BES, GRS].
- [Spe]\* T. Spencer, University of Nebraska at Omaha, *personal communication* (1993), and, Upper Bounds for Ramsey Numbers via Linear Programming, *to appear*.
- [Stahl] S. Stahl, On the Ramsey Number  $R(F, K_m)$  where  $F$  is a Forest, *Canadian Journal of Mathematics*, 27 (1975) 585-589.
- [-] R.G. Stanton, see [KS].
- [Stat] W. Staton, Some Ramsey-type Numbers and the Independence Ratio, *Transactions of the American Mathematical Society*, 256 (1979) 353-370.
- [-] A. Steger, see [McS].
- [Stev] S. Stevens, Ramsey Numbers for Stars Versus Complete Multipartite Graphs, *Congressus Numerantium*, 73 (1990) 63-71.
- [-] M.J. Stewart, see [CRSPS].
- [Su] Su Wenlong, The Estimation of Lower Bounds about Some Ramsey Numbers  $R_3(n)$  and  $R_4(n)$  (in Chinese), *Guangxi Sciences*, 3, 3 (1996) 3-7.
- [SLL]\* Su Wenlong, Luo Haipeng and Li Qiao,  $R(5, 11) \geq 140$  and  $R(4, 12) \geq 128$ , *in preparation*, (1997).
- [SLW]\* Su Wenlong, Luo Haipeng and Wu Kang, New Lower Bounds for Some Ramsey Numbers, *in preparation*, (1997).

- [-] E. Szemerédi, see [AKS, CRST].
- [Tho] A. Thomason, An Upper Bound for Some Ramsey Numbers, *Journal of Graph Theory*, 12 (1988) 509-517.
- [-] C.A. Tovey, see [CET].
- [Tr] Trivial results.
- [-] W.T. Trotter Jr., see [CRST].
- [-] Z. Tuza, see [GT].
- [-] L. Volkmann, see [GV].
- [Wa1] K. Walker, Dichromatic Graphs and Ramsey Numbers, *Journal of Combinatorial Theory*, 5 (1968) 238-243.
- [Wa2] K. Walker, An Upper Bound for the Ramsey Number  $M(5,4)$ , *Journal of Combinatorial Theory*, 11 (1971) 1-10.
- [Wal] W.D. Wallis, On a Ramsey Number for Paths, *Journal of Combinatorics, Information & System Sciences*, 6 (1981) 295-296.
- [-] Wang Gongben, see [WW, WWY].
- [WW]\* Wang Qingxian and Wang Gongben, New Lower Bounds of Ramsey Numbers  $r(3,q)$  (in Chinese), *Acta Scientiarum Naturalium, Universitatis Pekinensis*, 25 (1989) 117-121.
- [WWY]\* Wang Qingxian, Wang Gongben and Yan Shuda, A Search Algorithm And New Lower Bounds for Ramsey Numbers  $r(3,q)$ , *to appear*.
- [-] Wei Li, see [KLR].
- [-] Weiguo Ye, see [SWY].
- [Wh] E.G. Whitehead, The Ramsey Number  $N(3,3,3,3;2)$ , *Discrete Mathematics*, 4 (1973) 389-396.
- [-] Wu Kang, see [SLW].
- [XZ]\* Xie Jiguo and Zhang Xiaoxian, A New Lower Bound for Ramsey Number  $r(3,13)$  (in Chinese), *Journal of Lanzhou Railway Institute*, 12 (1993) 87-89.
- [-] J. Yackel, see [GY].
- [-] Yan Shu-da, see [WWY].
- [YY]\*\* Yang Yuansheng, On the Third Ramsey Numbers of Graphs with Six Edges, *Journal of Combinatorial Mathematics and Combinatorial Computing*, 17 (1995) 199-208.
- [YH]\* Yang Yuansheng and G.R.T. Hendry, The Ramsey Number  $r(K_1 + C_4, K_5 - e)$ , *Journal of Graph Theory*, 19 (1995) 13-15.
- [YR1]\*\* Yang Yuansheng and P. Rowlinson, On the Third Ramsey Numbers of Graphs with Five Edges, *Journal of Combinatorial Mathematics and Combinatorial Computing*, 11 (1992) 213-222.
- [YR2]\* Yang Yuansheng and P. Rowlinson, The Third Ramsey Numbers for Graphs with at Most Four Edges, *Discrete Mathematics*, 125 (1994) 399-406.
- [YR3]\* Yang Yuansheng and P. Rowlinson, On Graphs without 6-Cycles and Related Ramsey Numbers, *Utilitas Mathematica*, 44 (1993) 192-196.
- [-] Yanwu Liu, see [SWY].
- [Yu]\* Yu Song-Nian, A Computer Assisted Number Theoretical Construction of  $(3,k)$ -Ramsey Graphs, *Annales Univ. Sci. Budapest., Sect. Comput.*, 10 (1989) 35-44.
- [Zeng] Zeng Wei-Bin, Ramsey Numbers for Triangles and Graphs of Order Four with No Isolated Vertex, *Journal of Mathematical Research & Exposition*, 6 (1986) 27-32.
- [-] Zhang Y., see [LRZ].
- [-] Zhang Ke Min, see [HZ, MZ].
- [-] Zhang Xiaoxian, see [XZ].

[Zhou] Zhou Huai Lu, The Ramsey Number of an Odd Cycle with Respect to a Wheel, *Journal of Mathematics - Wuhan*, 15 (1995) 119-120.

Out of 252 references gathered above 195 appeared in 37 different periodicals, among which most articles were published in: *Journal of Combinatorial Theory* (old, Series A and B) 36, *Discrete Mathematics* 33, and *Journal of Graph Theory* 31. The results of 59 references depend on computer algorithms.