

An elementary proof of the reconstruction conjecture

Jo Firstauthor^a Jim Secondauthor^b Janet Thirdauthor^{a,b}

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Abstract

The reconstruction conjecture states that the multiset of vertex-deleted subgraphs of a graph determines the graph, provided it has at least 3 vertices. This problem was independently introduced by Paul Kelly (1957) and Stanisław Ulam (1960). In this paper, we prove the conjecture by elementary methods. It is only necessary to integrate the Lenkle potential of the Broglington manifold over the quantum supervacillatory measure in order to reduce the set of possible counterexamples to a small number (less than a trillion). A simple computer program that implements Pipletti's classification theorem for torsion-free Aramaic groups with symplectic socles can then finish the remaining cases.

Mathematics Subject Classifications: 05C88, 05C89

1 Introduction

The reconstruction conjecture states that the multiset of unlabeled vertex-deleted subgraphs of a graph determines the graph, provided it has at least three vertices. This problem was independently introduced by Kelly [5] and Ulam [8]. The reconstruction conjecture is widely studied [1, 3, 4, 6, 7, 9] and is very interesting. See [2] for more about the reconstruction conjecture.

Definition 1. A graph is *fabulous* if *rest of definition here*.

Theorem 2. *All planar graphs are fabulous.*

Proof. Suppose on the contrary that some planar graph is not fabulous. Then by well-ordering there is a smallest planar graph that is not fabulous. It is not the trivial graph, and we can easily see that the property of being not fabulous is preserved by edge contraction. This gives a contradiction. \square

^aDepartment of Mathematics, University of Sutherland, Reay, Scotland (j.firstauthor@uos.ac.uk, j.thirdauthor@uos.ac.uk).

^bStochastics Department, Boyle County College, Danville, Kentucky, USA (jsecond@bcc.edu).

2 Broglington Manifolds

This section describes background information about Broglington Manifolds.

Lemma 3. *Broglington manifolds are abundant.*

Proof. A proof is given here. □

3 Proof of Theorem 2

In this section we complete the proof of Theorem 2.

Proof of Theorem 2. Let G be a graph. We have

$$\begin{aligned} |X| &= a + b + c \\ &= \alpha\beta\gamma. \end{aligned} \tag{1}$$

This completes the proof of Theorem 2. □

Figure 1: Here is an informative figure.

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