Comments by the author on Volume 12, article R67:

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As pointed out by John Talbot, the principal result stated in this note is known, and may be found in [3]. Lovász provides sharper lower and upper bounds on g(k), viz., $\binom{2k-3}{k-1} < g(k) \le (2r-1)\binom{2k-3}{k-1}$. He also makes reference to a prior (1964), weaker variant of this result due to Calczynska and Karlowicz. Lovász's result was subsequently sharpened by Tuza [5] as follows: $2\binom{2k-4}{k-2} < g(k) \le \binom{2k-1}{k-1} + \binom{2k-4}{k-1}$. Related papers include [1] and [4]; the latter indicates that g(3) = 7 and g(4) = 16 are the largest known values of g. Lovász's upper bound is also presented in [2].

So, the chief surviving novelties of R67 are the generalization to lattices, and the enumeration of pedestals for $k \leq 3$.

References

- [1] N. Alon and Z. Füredi, On the kernel of intersecting families, Graphs and Combinatorics 3(1987), 91-94.
- [2] Bela Bollobàs, Combinatorics: Set Systems, Hypergraphs, Families of Vectors, and Combinatorial Probability, Cambridge University Press 1986 pp.93-94.
- [3] L. Lovász, Combinatorial Problems and Exercises, Second Edition, North-Holland, Amsterdam, 1993, Exercise 13.27 (first published 1979).
- [4] J. Talbot, The number of k-intersections of an intersecting family of r-sets, J. Combin. Theory Ser. A, 106 2 (2004), 277-286.
- [5] Zs. Tuza, Critical hypergraphs and intersecting set-pair systems, J. Combin. Theory Ser. B 39 (1985) pp. 134-145.