# Comment on "On the chromatic number of simple triangle-free triple systems" by Jeff Cooper, Alan Frieze, Dhruv Mubayi

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We have found several errors in the paper [1] and the goal here is to present corrections to all of them. Equational references with square brackets [..] are with respect to the published version. Those with round brackets (..) are with respect to this comment. The notation is from [1].

There was a substantial error in the proof of [15] (in Section 11.4.1 of [1]) and a trivial error in the calculation to the proof of [8] (in Section 9 of [1]). These are corrected in Sections 1 and 2, respectively of the current note.

## 1 Correction to the proof of [15]

Observe that

$$\begin{aligned}
f'_u &= \sum_c \sum_{v \in N(u)} p'_u(c)p'_v(c)1_{\kappa'(uv)=c} + \sum_c \sum_{\substack{\kappa(uv)=0 \\ \kappa'(uv)=c}} p'_u(c)p'_v(c) \\
&\leq \sum_c \sum_{\kappa(uv)=c} p'_u(c)p'_v(c)1_{v \in U'} \\
&+ \sum_c \sum_{uvw \in H} (p'_u(c)p'_v(c)1_{\gamma_w(c)=1} + p'_u(c)p'_w(c)1_{\gamma_v(c)=1}) \\
&:= S_1 + S_2.
\end{aligned}$$

We will bound each term separately.

### 1.1 $S_1$

Recall that

$$S_1 = \sum_c \sum_{\kappa(uv)=c} p'_u(c)p'_v(c)1_{v\in U'}.$$

For each color $c$, let $D_c$ be the event that $\gamma_v(c) = 1$ for at most $\Delta\hat{p}$ vertices $v \in N(u)$. Since $\mathsf{P}[\gamma_v(c) = 1] \leq \hat{p}\theta$,

$$\mathsf{P}[\bar{D}_c] \leq \binom{\Delta}{\Delta\hat{p}}(\hat{p}\theta)^{\Delta\hat{p}} \leq (\frac{e}{\hat{p}})^{\Delta\hat{p}}(\hat{p}\theta)^{\Delta\hat{p}} = (e\theta)^{\Delta\hat{p}} < e^{-\Delta^{1/2}}.$$

Let $D$ denote the event that $D_c$ holds for all $c$. By the union bound,

$$\mathsf{P}[\bar{D}] \leq qe^{-\Delta^{1/2}}.$$

By (1) below,

$$\begin{aligned}\mathbf{E}[S_1] &= \sum_c \sum_{\kappa(uv)=c} \mathbf{E}[p'_u(c)p'_v(c)1_{v\in U'}] \\ &\leq \sum_c \sum_{\kappa(uv)=c} p_u(c)p_v(c)(1-\theta(1-6\epsilon)) \\ &= f_u(1-\theta(1-6\epsilon)).\end{aligned}$$

Therefore,

$$\begin{aligned}\mathbf{E}[S_1|D] &= (\mathbf{E}[S_1] - \mathbf{E}[S_1|\bar{D}]\mathsf{P}[\bar{D}])/\mathsf{P}[D] \\ &\leq \mathbf{E}[S_1]/\mathsf{P}[D] \\ &= f_u(1-\theta(1-6\epsilon))/(1-qe^{-\Delta^{1/2}}) \\ &\leq f_u(1-\theta(1-7\epsilon)).\end{aligned}$$

For a vertex subset $X$, let $N(X) = \{v : \exists x \in X \text{ and } w \text{ with } xvw \in H\}$. Let $T_c$ denote the set of color trials for color $c$ at all vertices in $\{u\} \cup N(u) \cup N(N(u))$. Then the trials $T_1, \dots, T_q$ determine the variable $S_1$. Observe that $T_c$ affects every term of the form $p'_u(c)p'_v(c)1_{v\in U'}$. For $d \neq c$, $T_c$ affects $p'_u(d)p'_v(d)1_{v\in U'}$ only if $\gamma_v(c) = 1$; this is because if $\gamma_v(c) = 0$, the trials for color $c$ have no impact on whether or not $v \in U'$. Thus, given that $\gamma_v(c) = 1$ for at most $\Delta\hat{p}$ of the variables in $T_c$, changing the values in $T_c$ can change $S_1$ by at most $d(u,c)\hat{p}^2 + 2(\Delta\hat{p})\hat{p}^2$.

Let $\pi(t_i) = \mathsf{P}(T_i = t_i \mid D)$ for $i = 1, 2, \dots, q$ and let

$$\rho(t_i, t_{i+1}, \dots, t_q) = \pi(t_i)\pi(t_{i+1})\cdots\pi(t_q) = \mathsf{P}(T_j = t_j, j = i, i+1, \dots, q \mid D).$$

Here we use the fact that conditioning on $D$ still leaves the choices $t_1, t_2, \ldots, t_q$ for the distinct sets of colors $T_1, T_2, \ldots, T_c$ independent of each other. Thus,

$$\begin{aligned}
&|\,\mathbf{E}[S_1|D, T_1 = t_1, \ldots, T_c = t_c] - \mathbf{E}[S_1|D, T_1 = t_1, \ldots, T_{c-1} = t_{c-1}, T_c = t'_c]| \\
&= \left|\sum_{t_{c+1},\ldots,t_q} [S_1(t_1, \ldots, t_{c-1}, t_c, t_{c+1}, \ldots, t_q) - S_1(t_1, \ldots, t_{c-1}, t'_c, t_{c+1}, \ldots, t_q)]\rho(t_{c+1}, \ldots, t_q)\right| \\
&\le d(u, c)\hat{p}^2 + 2\Delta\hat{p}^3 \\
&\le 2t_0\theta\Delta\hat{p}^3 + 2\Delta\hat{p}^3 \\
&\le 3t_0\theta\Delta\hat{p}^3.
\end{aligned}$$

Since

$$\sum_c (3t_0\theta\Delta\hat{p}^3)^2 = 9qt_0^2\theta^2\Delta^2\hat{p}^6 \le 9t_0^2\theta^2\Delta^{2+1/2-66/24} \le \Delta^{-5/24},$$

the Azuma-Hoeffding inequality implies

$$\begin{aligned}
\mathsf{P}[S_1 > f_u(1 - \theta(1 - 7\epsilon)) + \Delta^{-1/12}|D] &\le \mathsf{P}[S_1 > \mathbf{E}[S_1|D] + \Delta^{-1/12}|D] \\
&\le e^{-\Delta^{5/24-2/12}} \\
&= e^{-\Delta^{1/24}}.
\end{aligned}$$

Thus

$$\begin{aligned}
\mathsf{P}[S_1 > f_u(1 - \theta(1 - 7\epsilon)) + \Delta^{-1/12}] &\le \mathsf{P}[S_1 > f_u(1 - \theta(1 - 7\epsilon)) + \Delta^{-1/12}|D]\mathsf{P}[D] + \mathsf{P}[\bar{D}] \\
&\le e^{-\Delta^{1/24}}(1 - qe^{-\Delta^{1/2}}) + qe^{-\Delta^{1/2}} \\
&\le e^{-\Delta^{1/25}}.
\end{aligned}$$

### 1.1.1 Proof of (1)

We prove that if $\kappa(uv) = c$,

$$\mathbf{E}[p'_u(c)p'_v(c)1_{v \in U'}] \le p_u(c)p_v(c)(1 - \theta(1 - 6\epsilon)). \tag{1}$$

We first establish the following claim.

**Claim.** $\mathsf{P}[v \notin U'|c \notin L(v)] \ge \mathsf{P}[v \notin U'] \ge \theta(1 - 5\epsilon)$.

*Proof of claim.* The vertex $v$ is colored (i.e., not in $U'$) if and only if for some color $d \notin B(v)$, $\gamma_v(d) = 1$ and $d \notin L(v)$. Let $R_d$ denote the event that $\gamma_v(d) = 1$ and $d \notin L(v)$. If $c \in B(v)$, then

$$\mathsf{P}[v \notin U'|c \notin L(v)] = \mathsf{P}[v \notin U'].$$

Otherwise, since $\gamma_v(c) = 1$ is independent of $c \notin L(v)$ and $R_d$ is independent of $c \notin L(v)$ for $c \neq d$,

$$\begin{aligned}
\mathsf{P}[v \notin U' | c \notin L(v)] &= \mathsf{P}[(\cup_{d \notin B(v) \cup \{c\}} R_d) \cup (R_c) | c \notin L(v)] \\
&= \mathsf{P}[(\cup_{d \notin B(v) \cup \{c\}} R_d) \cup (\gamma_v(c) = 1 \cap c \notin L(v)) | c \notin L(v)] \\
&= \mathsf{P}[(\cup_{d \notin B(v) \cup \{c\}} R_d) \cup (\gamma_v(c) = 1)] \\
&\geq \mathsf{P}[(\cup_{d \notin B(v) \cup \{c\}} R_d) \cup R_c] \\
&= \mathsf{P}[v \notin U'].
\end{aligned}$$

In either case,

$$\begin{aligned}
\mathsf{P}[v \notin U' | c \notin L(v)] &\geq \mathsf{P}[v \notin U'] \\
&= \mathsf{P}[\cup_{d \notin B(v)} R_d] \\
&\geq \sum_{d \notin B(v)} \mathsf{P}[R_d] - \sum_{d,d' \notin B(v)} \mathsf{P}[R_d]\mathsf{P}[R_{d'}] \\
&= \sum_{d \notin B(v)} \theta p_v(d) q_v(d) - \sum_{d,d' \notin B(v)} \theta^2 p_v(d) p_v(d') q_v(d) q_v(d') \\
&\geq \theta \sum_{d \in C} p_v(d) q_v(d) - \theta \sum_{d \in B(v)} p_v(d) q_v(d) - \theta^2 \sum_{d,d' \notin B(v)} p_v(d) p_v(d') \\
&\geq \theta \sum_{d \in C} p_v(d) q_v(d) - \theta |B(v)| \hat{p} - \theta^2 \sum_{d,d' \notin B(v)} p_v(d) p_v(d').
\end{aligned}$$

Using the inequality $\prod_x (1-x) \geq 1 - \sum_x x$ (for $x \in [0,1]$), we obtain

$$\begin{aligned}
q_v(d) &= \prod_{uvw \in H} (1 - \theta^2 p_u(d) p_w(d)) \prod_{\substack{uv \in G \\ \kappa(uv) = d}} (1 - \theta p_u(d)) \\
&\geq 1 - \sum_{uvw \in H} \theta^2 p_u(d) p_w(d) - \sum_{\substack{uv \in G \\ \kappa(uv) = d}} \theta p_u(d) \\
&= 1 - \theta^2 \sum_{uvw \in H} p_u(d) p_w(d) - \theta \sum_{\substack{uv \in G \\ \kappa(uv) = d}} p_u(d) \\
&= 1 - \theta^2 e_v(d) - \theta f_v(d).
\end{aligned}$$

Since $\sum_{d \in C} p_v(c) = 1 + o(1)$,

$$\theta^2 \sum_{d,d' \notin B(v)} p_v(d) p_v(d') = \frac{1}{2} \theta^2 \sum_{d \in C} \sum_{d' \in C : d' \neq d} p_v(d) p_v(d') \leq \frac{1}{2} \theta^2 (\sum_{d \in C} p_v(d))^2 \leq \theta^2.$$

By [22], $|B(v)| < \epsilon / \hat{p}$. By [8], $f_v < 3\omega$, so $\theta f_v < 3\epsilon$. By [7] and [10] (see [19]), $e_v \leq \omega + \Delta^{-1/10}$, so $\theta^2 e_v < \epsilon/3$. Using these three inequalities and $\sum_{d \in C} p_v(c) \geq (1 - \epsilon/3)$, we finally obtain

$$\begin{aligned}
\mathsf{P}[v \notin U'] &\geq \theta \sum_d p_v(d)(1 - \theta^2 e_v(d) - \theta f_v(d)) - \theta|B(v)|\hat{p} - \theta^2 \\
&\geq \theta \sum_d p_v(d) - \theta^3 \sum_d p_v(d) e_v(d) - \theta^2 \sum_d p_v(d) f_v(d) - \theta\epsilon - \theta^2 \\
&= \theta \sum_d p_v(d) - \theta^3 e_v - \theta^2 f_v - \theta\epsilon - \theta^2 \\
&\geq \theta(1 - \epsilon/3) - \theta\epsilon/3 - 3\theta\epsilon - \theta\epsilon - \theta\epsilon/3 \\
&= \theta(1 - 5\epsilon).
\end{aligned}$$

$\square$

We now bound $\mathbf{E}[p'_u(c)p'_v(c)1_{v \in U'}]$. First assume that $p'_u(c)$ and $p'_v(c)$ are determined by Case A (see [3]). Since $\kappa(uv) = c$, the edge containing $u$ and $v$ no longer exists in the hypergraph. By triangle-freeness, there are no vertices $w$ which share an edge with both $u$ and $v$. Therefore the events $c \notin L(u)$ and $c \notin L(v)$ are independent. Also, if $c \notin L(u)$, then $\gamma_w(c) = 0$ for all $w \in N_G(u)$, so in particular, $\gamma_v(c) = 0$. Consequently,

$$\mathsf{P}[\bar{R}_c | c \notin L(u) \cup L(v)] = \mathsf{P}[\bar{R}_c | c \notin L(u)] = \mathsf{P}[\gamma_v(c) = 0 \cup c \in L(v) | c \notin L(u)] = 1.$$

Therefore, by the independence of colors,

$$\begin{aligned}
\mathsf{P}[v \in U' | c \notin L(u) \cup L(v)] &= \mathsf{P}[\cap_{d \notin B(v)} \bar{R}_d | c \notin L(u) \cup L(v)] \\
&= \mathsf{P}[\cap_{d \notin B(v) \cup \{c\}} \bar{R}_d] \mathsf{P}[\bar{R}_c | c \notin L(u) \cup L(v)] \\
&= \mathsf{P}[\cap_{d \notin B(v) \cup \{c\}} \bar{R}_d] \\
&= \mathsf{P}[\cap_{d \notin B(v) \cup \{c\}} \bar{R}_d] \mathsf{P}[\bar{R}_c] / \mathsf{P}[\bar{R}_c] \\
&= \mathsf{P}[\cap_{d \notin B(v)} \bar{R}_d] / \mathsf{P}[\bar{R}_c] \\
&\leq \mathsf{P}[v \in U'] / (1 - \theta\hat{p}) \\
&\leq \mathsf{P}[v \in U'](1 + 2\theta\hat{p}).
\end{aligned}$$

Note that this also implies $\mathsf{P}[v \in U' | c \notin L(u)] \leq \mathsf{P}[v \in U'](1 + 2\theta\hat{p})$. If $c \in L(v) \cup L(u)$, then $p'_u(c)p'_v(c) = 0$, so by the claim,

$$\begin{aligned}
\mathbf{E}[p'_u(c)p'_v(c)1_{v\in U'}] &= \mathbf{E}[p'_u(c)p'_v(c)|v \in U']\mathsf{P}[v \in U'] \\
&\le \frac{p_u(c)}{q_u(c)}\frac{p_v(c)}{q_v(c)}\mathsf{P}[c \notin L(u) \cup L(v)|v \in U']\mathsf{P}[v \in U'] \\
&= \frac{p_u(c)}{q_u(c)}\frac{p_v(c)}{q_v(c)}\mathsf{P}[v \in U'|c \notin L(u) \cup L(v)]\mathsf{P}[c \notin L(u) \cup L(v)] \\
&= \frac{p_u(c)}{q_u(c)}\frac{p_v(c)}{q_v(c)}\mathsf{P}[v \in U'|c \notin L(u) \cup L(v)]\mathsf{P}[c \notin L(u)]\mathsf{P}[c \notin L(v)] \\
&= p_u(c)p_v(c)\mathsf{P}[v \in U'|c \notin L(u) \cup L(v)] \\
&\le p_u(c)p_v(c)\mathsf{P}[v \in U'](1 + 2\theta\hat{p}) \\
&\le p_u(c)p_v(c)(1 - \theta(1 - 6\epsilon)).
\end{aligned}$$

Suppose $p'_u(c)$ is determined by Case A, and $p'_v(c)$ is determined by Case B. Recall that the previous case showed that $\mathsf{P}[v \in U'|c \notin L(u)] \le \mathsf{P}[v \in U'](1+2\theta\hat{p})$. If $c \in L(u)$, then $p'_u(c)p'_v(c) = 0$, so

$$\begin{aligned}
\mathbf{E}[p'_u(c)p'_v(c)1_{v\in U'}] &= p_v(c)\,\mathbf{E}[p'_u(c)|v \in U']\mathsf{P}[v \in U'] \\
&\le p_v(c)\frac{p_u(c)}{q_u(c)}\mathsf{P}[c \notin L(u)|v \in U']\mathsf{P}[v \in U'] \\
&= p_v(c)\frac{p_u(c)}{q_u(c)}\mathsf{P}[v \in U'|c \notin L(u)]\mathsf{P}[c \notin L(u)] \\
&= p_u(c)p_v(c)\mathsf{P}[v \in U'|c \notin L(u)] \\
&\le p_u(c)p_v(c)(1 - \theta(1 - 6\epsilon)).
\end{aligned}$$

Suppose $p'_u(c)$ is determined by Case B, and $p'_v(c)$ is determined by Case A. If $c \in L(v)$, then $p'_u(c)p'_v(c) = 0$, so by the claim,

$$\begin{aligned}
\mathbf{E}[p'_u(c)p'_v(c)1_{v\in U'}] &= p_u(c)\,\mathbf{E}[p'_v(c)|v \in U']\mathsf{P}[v \in U'] \\
&\le p_u(c)\frac{p_v(c)}{q_v(c)}\mathsf{P}[c \notin L(v)|v \in U']\mathsf{P}[v \in U'] \\
&= p_u(c)\frac{p_v(c)}{q_v(c)}\mathsf{P}[v \in U'|c \notin L(v)]\mathsf{P}[c \notin L(v)] \\
&= p_u(c)p_v(c)\mathsf{P}[v \in U'|c \notin L(v)] \\
&\le p_u(c)p_v(c)(1 - \theta(1 - 6\epsilon)).
\end{aligned}$$

If both $p'_u(c)$ and $p'_v(c)$ are determined by Case B, then $p'_u(c)$ and $p'_v(c)$ are independent of each other and of $v \in U'$. Hence

$$\begin{aligned}\mathbf{E}[p'_u(c)p'_v(c)1_{v\in U'}] &= \mathbf{E}[p'_u(c)]\,\mathbf{E}[p'_v(c)]\,\mathbf{E}[1_{v\in U'}]\\ &= p_u(c)p_v(c)\mathsf{P}[v\in U']\\ &\le p_u(c)p_v(c)(1-\theta(1-6\epsilon)).\end{aligned}$$

## 1.2 $S_2$

By (2) below,

$$\begin{aligned}\mathbf{E}[S_2] &= \sum_c\sum_{uvw}(\mathbf{E}[p'_u(c)p'_v(c)1_{\gamma_w(c)=1}] + \mathbf{E}[p'_u(c)p'_w(c)1_{\gamma_v(c)=1}])\\ &= \sum_c\sum_{uvw}\mathbf{E}[p'_u(c)p'_v(c)|\gamma_w(c)=1]\mathsf{P}[\gamma_w(c)=1]\\ &+ \sum_c\sum_{uvw}\mathbf{E}[p'_u(c)p'_w(c)|\gamma_v(c)=1]\mathsf{P}[\gamma_v(c)=1]\\ &\le \sum_c\sum_{uvw}(p_u(c)p_v(c)\mathsf{P}[\gamma_w(c)=1] + p_u(c)p_w(c)\mathsf{P}[\gamma_v(c)=1])\\ &= \sum_c\sum_{uvw}(p_u(c)p_v(c)\theta p_w(c) + p_u(c)p_w(c)\theta p_v(c))\\ &= 2\theta e_u.\end{aligned}$$

Let

$$S_{2,c} = \sum_{uvw}(p'_u(c)p'_v(c)1_{\gamma_w(c)=1} + p'_u(c)p'_w(c)1_{\gamma_v(c)=1}),$$

and

$$\hat{S}_2 = \sum_c \min\{S_{2,c}, 2\Delta\hat{p}^3\}.$$

Then $\hat{S}_2$ is the sum of $q$ independent random variables, each bounded by $2\Delta\hat{p}^3$. By [23],

$$\mathsf{P}[\hat{S}_2 \ge \mathbf{E}[\hat{S}_2] + \Delta^{-1/10}] \le e^{-\frac{\Delta^{-1/5}}{4q\Delta^2\hat{p}^6}} \le e^{-\Delta^{-1/5-1/2-2+66/24}/4} = e^{-\Delta^{1/20}/4}.$$

Observe that if $S_2 \ne \hat{S}_2$, then $S_{2,c} > 2\Delta\hat{p}^3$ for some color c. This would imply that $\gamma_w(c) = 1$ for at least $\Delta\hat{p}$ neighbors $w$ of $u$. Therefore,

$$\mathsf{P}[S_2 \ne S_{2,c}] \le q\binom{2\Delta}{\Delta\hat{p}}(\hat{p}\theta)^{\Delta\hat{p}} \le q\left(\frac{2e}{\hat{p}}\right)^{\Delta\hat{p}}(\hat{p}\theta)^{\Delta\hat{p}} = q(2e\theta)^{\Delta^{13/24}}.$$

Since $\mathbf{E}[S_2] > \mathbf{E}[\hat{S}_2]$, this implies

$$\begin{aligned}\mathsf{P}[S_2 > \mathbf{E}[S_2] + \Delta^{-1/10}] &\leq \mathsf{P}[S_2 > \mathbf{E}[\hat{S}_2] + \Delta^{-1/10}] \\ &\leq \mathsf{P}[S_2 \neq \hat{S}_2] + \mathsf{P}[\hat{S}_2 > \mathbf{E}[\hat{S}_2] + \Delta^{-1/10}] \\ &\leq q(2e\theta)^{\Delta^{13/24}} + e^{-\Delta^{1/20}/4} \\ &< e^{-\Delta^{1/21}}.\end{aligned}$$

Therefore, with probability at least $1 - e^{-\Delta^{1/21}} - e^{-\Delta^{1/25}}$,

$$\begin{aligned}f'_u &\leq f_u(1 - \theta(1 - 7\epsilon)) + \Delta^{-1/12} + 2\theta e_u + \Delta^{-1/10} \\ &\leq f_u(1 - \theta(1 - 7\epsilon)) + 2\theta e_u + \Delta^{-1/21},\end{aligned}$$

which is [15].

### 1.2.1 Proof of (2)

We prove that

$$\mathbf{E}[p'_u(c)p'_v(c)|\gamma_w(c) = 1] \leq p_u(c)p_v(c). \tag{2}$$

We assume first that both $p'_u(c)$ and $p'_v(c)$ are determined by Case A. If $c \in L(u)$ or $c \in L(v)$, then $p'_u(c)p'_v(c) = 0$, so

$$\mathbf{E}[p'_u(c)p'_v(c)|\gamma_w(c) = 1] \leq \frac{p_u(c)}{q_u(c)}\frac{p_v(c)}{q_v(c)}\mathsf{P}[c \notin L(u) \cup L(v)|\gamma_w(c) = 1].$$

Since

$$\mathsf{P}[c \notin L(u)] = \prod_{uxy \in H}(1 - \mathsf{P}[\gamma_x(c) = 1, \gamma_y(c) = 1]) \prod_{\kappa(ux)=c}(1 - \mathsf{P}[\gamma_x(c) = 1] = q_u(c),$$

we see that

$$\begin{aligned}&\mathsf{P}[c \notin L(u)|c \notin L(v), \gamma_w(c) = 1] \\ &= (1 - \mathsf{P}[\gamma_v(c) = 1]) \prod_{uxy \in H - uvw}(1 - \mathsf{P}[\gamma_x(c) = 1, \gamma_y(c) = 1]) \prod_{\kappa(ux)=c}(1 - \mathsf{P}[\gamma_x(c) = 1]) \\ &= \frac{1 - \mathsf{P}[\gamma_v(c) = 1]}{1 - \mathsf{P}[\gamma_v(c) = 1, \gamma_w(c) = 1]}q_u(c) \\ &= \frac{1 - \theta p_v(c)}{1 - \theta^2 p_v(c)p_w(c)}q_u(c).\end{aligned}$$

Similiarly,

$$\mathsf{P}[c \notin L(v)|\gamma_w(c) = 1] = \frac{1 - \theta p_u(c)}{1 - \theta^2 p_u(c)p_w(c)}q_v(c).$$

Therefore, using $\theta p_w(c) \leq 1$,

$$\begin{aligned}
\mathsf{P}[c \notin L(u) \cup L(v) | \gamma_w(c) = 1] &= \mathsf{P}[c \notin L(u) | c \notin L(v), \gamma_w(c) = 1] \mathsf{P}[c \notin L(v) | \gamma_w(c) = 1] \\
&= \frac{q_u(c)(1 - \theta p_v(c))}{1 - \theta^2 p_v(c) p_w(c)} \frac{q_v(c)(1 - \theta p_u(c))}{1 - \theta^2 p_u(c) p_w(c)} \\
&\leq q_u(c) q_v(c),
\end{aligned}$$

and (2) follows.

If $p'_u(c)$ or $p'_v(c)$ is determined by Case B, then these values are independent, and (2) follows in a similar way.

## 2 Correction to the proof of property [8]

There was a trivial error in the calculation justifying [8]. We correct here for completeness. Replace the last sentence with: So, using $f_u \leq 3(1 - \theta/4)^t \omega$,

$$\begin{aligned}
f'_u &\leq 3(1 - \theta(1 - 7\epsilon))(1 - \theta/4)^t \omega + 2\theta\omega(1 - \theta/3)^t + \theta\Delta^{-1/22} \\
&= 3(1 - \theta/4)^{t+1}\omega + \omega(1 - \theta/4)^t \left( -\theta\,(9/4 - 21\epsilon) + 2\theta \left( \frac{1 - \theta/3}{1 - \theta/4} \right)^t \right) + \theta\Delta^{-1/22} \\
&\leq 3(1 - \theta/4)^{t+1}\omega + \omega(1 - \theta/4)^t(-\theta(9/4 - 21\epsilon) + 2\theta) + \theta\Delta^{-1/22} \\
&\leq 3(1 - \theta/4)^{t+1}\omega - \omega\theta(1/4 - 21\epsilon)(\log\Delta)^{-O(1)} + \theta\Delta^{-1/22} \\
&\leq 3(1 - \theta/4)^{t+1}\omega.
\end{aligned}$$

## References

[1] A.M. Frieze and D. Mubayi, On the chromatic number of simple triangle-free triple systems, Electronic Journal of Combinatorics 15 (2008), no. 1, Research Paper 121, 27 pp.