# Rainbow matchings in properly edge colored graphs

Guanghui Wang

School of Mathematics
Shandong University
Jinan, Shandong, 250100, P.R. China

ghwang@sdu.edu.cn

Submitted: Mar 14, 2011; Accepted: Jul 20, 2011; Published: Aug 5, 2011
Mathematics Subject Classifications: 05C15, 05C70

**Abstract**

Let $G$ be a properly edge colored graph. A rainbow matching of $G$ is a matching in which no two edges have the same color. Let $\delta$ denote the minimum degree of $G$. We show that if $|V(G)| \geq \frac{8\delta}{5}$, then $G$ has a rainbow matching of size at least $\lfloor \frac{3\delta}{5} \rfloor$. We also prove that if $G$ is a properly colored triangle-free graph, then $G$ has a rainbow matching of size at least $\lfloor \frac{2\delta}{3} \rfloor$.

**Keywords:** rainbow matchings, properly colored graphs, triangle-free graphs

## 1 Introduction and notation

We use [3] for terminology and notations not defined here and consider simple undirected graphs only. Let $G = (V, E)$ be a graph. A *proper edge-coloring* of $G$ is a function $c : E \to \mathbb{N}$ ($\mathbb{N}$ is the set of nonnegative integers) such that any two adjacent edges have distinct colors. If $G$ is assigned such a coloring $c$, then we say that $G$ is a *properly edge-colored graph*, or simply a *properly colored graph*. Let $c(e)$ denote the *color* of the edge $e \in E$. For a subgraph $H$ of $G$, let $c(H) = \{c(e) : e \in E(H)\}$. A subgraph $H$ of $G$ is called *rainbow* if its edges have distinct colors. Recently rainbow subgraphs have received much attention, see the survey paper [8]. Here we are interested in rainbow matchings. The study of rainbow matchings began with the following conjectures.

**Conjecture 1** (Ryser [5]) *Every Latin square of odd order has a Latin transversal.*

**Conjecture 2** (Brualdi-Stein [9, 11]) *Every latin square of order $n$ has a partial Latin transversal of size at least $n - 1$.*

An equivalent statement is that every proper $n$-edge-coloring of the complete bipartite graph $K_{n,n}$ contains a rainbow matching of size $n - 1$; Moreover, if $n$ is odd, there exists a rainbow perfect matching. Hatami and Shor [7] proved that there is always a partial Latin transversal (rainbow matching) of size at least $n - O(\log^2 n)$.

Another topic related to rainbow matchings is orthogonal matchings of graphs. Let $G$ be a graph on $n$ vertices which is an edge disjoint union of $m$ $k$-factors (i.e. $k$ regular spanning subgraphs). We ask if there is a matching $M$ of $m$ edges with exactly one edge from each $k$-factor? Such a matching is called orthogonal because of applications in design theory. A matching $M$ is suborthogonal if there is at most one edge from each $k$-factor. Alspach [1] posed the above problem in the case $k = 2$. Stong [10] proved that if $n \geq 3m - 2$, then there is a such orthogonal matching. For $k = 3$, the answer is yes, see [2]. In the same paper, Anstee and Caccetta proved the following theorem when $k = 1$.

**Theorem 2** [2] *Let $G$ be an $m$-regular graph on $n$ vertices. Then for any decomposition of $E(G)$ into $m$ 1-factors $F_1, F_2, \ldots, F_m$, there is a matching $M$ of $p$ edges, at most one edge from each 1-factor, with*

$$p > \min\left\{\frac{n}{2} - \frac{3}{2}(\frac{n}{2})^{\frac{2}{3}}, m - \frac{3}{2}m^{\frac{2}{3}}\right\}.$$

In any decomposition of $E(G)$ into $m$ $k$-factors, we can construct an edge-colored graph by giving each $k$-factor a color. Then a rainbow matching of $G$ corresponds to a suborthogonal matching of $G$. In particular, when $k = 1$, the edge-colored graph obtained above is properly colored. So we can pose a more general problem: Let $G$ be a properly colored graph of minimum degree $\delta(G)$. Is there a rainbow matching of size $\delta(G)$? Unfortunately, the answer is negative: Let $C_4^2$ denote a properly 2-edge-colored cycle with four vertices and $K_4^3$ be a properly 3-edge-colored complete graph with four vertices. Let $K_4^3 - e$ denote the graph obtained from $K_4^3$ by deleting an edge. Then there is no rainbow matchings of size two in $C_4^2, K_4^3$, or $K_4^3 - e$. Moreover, if $G$ is a properly colored complete graph, then $G$ has no rainbow matching of size more than $\lceil\frac{\delta(G)}{2}\rceil$. In addition, the following theorem was shown in [6].

**Theorem 3** [6] *Let $G$ be a properly colored graph, $G \neq K_4$, and $|V(G)| \neq \delta(G) + 2$. Then $G$ contains a rainbow matching of size $\lceil\frac{\delta(G)}{2}\rceil$.*

However, we believe that if the order of a properly colored graph $G$ is much larger than its minimum degree $\delta(G)$, there should be a rainbow matching of size $\delta(G)$. So we propose the following problem.

**Problem 4** *Is there a function $f(n)$ such that for each properly colored graph $G$ with $|V(G)| \geq f(\delta(G))$, $G$ must contain a rainbow matching of size $\delta(G)$?*

Since when $n$ is even, there exists an $n \times n$ Latin square that has no Latin transversal (perfect rainbow matching) (see [4, 11]), if the function $f(n)$ exists, $f(n)$ should be greater than $2n$. Motivated by this problem, we prove the following results.

**Theorem 5** *Let $G$ be a properly colored graph and $|V(G)| \geq \frac{8\delta(G)}{5}$. Then $G$ has a rainbow matching of size at least $\lfloor \frac{3\delta(G)}{5} \rfloor$.*

**Theorem 6** *Let $G$ be a properly colored triangle-free graph. Then $G$ has a rainbow matching of size at least $\lfloor \frac{2\delta(G)}{3} \rfloor$.*

## 2 Proof of Theorem 5

For simplicity, let $\delta = \delta(G)$. If $\delta \leq 3$, it is easy to check that our theorem holds. If $4 \leq \delta \leq 9$, by Theorem 3, $G$ contains a rainbow matching of size $\lceil \frac{\delta}{2} \rceil$. Since $\lceil \frac{\delta}{2} \rceil \geq \lfloor \frac{3\delta}{5} \rfloor$, when $4 \leq \delta \leq 9$, our conclusion holds too. So now we assume that $\delta \geq 10$. We will prove it by contradiction. Suppose our conclusion is not true. We choose a maximum rainbow matching $M$. Let $t = |E(M)|$. Then $t \leq \lfloor \frac{3\delta}{5} \rfloor - 1$. Suppose that $E(M) = \{e_1, e_2, \dots, e_t\}$ and $e_i = x_i y_i$. Moreover, without loss of generality, we assume that $c(e_i) = i$, for $1 \leq i \leq t$. Put $V_1 = V - V(M)$. We call a color a *new* color if it is not in $c(M)$ and call an edge $uv$ *special* if $v \in V(M)$, $u \in V_1$ and $c(uv)$ is a new color. For $v \in V(M)$, let $d_s(v)$ denote the number of the special edges incident with $v$. Let $V_2$ denote the vertices $v \in V(M)$ with $d_s(v) \geq 4$. We have the following claim.

**Claim 1.** *For each edge $x_i y_i \in E(M)$, if $d_s(x_i) + d_s(y_i) \geq 5$, then either $d_s(x_i) = 0$ or $d_s(y_i) = 0$.*

*Proof.* Otherwise, it holds that $d_s(x_i) + d_s(y_i) \geq 5$ and $d_s(x_i), d_s(y_i) \geq 1$. Then one of $d_s(x_i)$, $d_s(y_i)$ is at least 3. Suppose that $d_s(x_i) \geq 3$. Since $d_s(y_i) \geq 1$, we choose a special edge $y_i u$. As $d_s(x_i) \geq 3$, we can also choose a special edge $x_i w$ such that $c(x_i w) \neq c(y_i u)$ and $w \neq u$. Now $M \cup \{x_i w, y_i u\} \backslash x_i y_i$ is a rainbow matching of size $t+1$, a contradiction. □

**Claim 2.** $|V_2| \geq \lceil \frac{2\delta}{5} \rceil$.

*Proof.* Let $x \in V_1$. If there is an edge $xy$ such that $c(xy) \notin c(M)$, then $y \in V(M)$. Otherwise, there is a rainbow matching $M \cup xy$ of size $t+1$, which is a contradiction. Let $E_s$ denote the set formed by all special edges. Since each vertex in $V_1$ has degree at least $\delta$, $|E_s| \geq (\delta - t)|V_1| \geq (\lceil \frac{2\delta}{5} \rceil + 1)|V_1|$. By Claim 1, for each edge $x_i y_i \in E(M)$, if $d_s(x_i) + d_s(y_i) \geq 5$, then $d_s(x_i) = 0$ or $d_s(y_i) = 0$, so $d_s(x_i) + d_s(y_i) \leq |V_1|$; If $d_s(x_i) + d_s(y_i) \leq 4$, recall that $|V_1| = |V(G)| - |V(M)| \geq \frac{8\delta}{5} - 2(\lfloor \frac{3\delta}{5} \rfloor - 1) \geq \frac{2\delta}{5} + 2 \geq 5$, thus $d_s(x_i) + d_s(y_i) \leq |V_1|$. Hence $|E_s| \leq |V_2||V_1| + 4(|E(M)| - |V_2|)$. This implies $(\lceil \frac{2\delta}{5} \rceil + 1)|V_1| \leq |V_2||V_1| + 4(|E(M)| - |V_2|)$. Hence

$$|V_2| \geq \frac{(\lceil \frac{2\delta}{5} \rceil + 1)|V_1| - 4|E(M)|}{|V_1| - 4} \geq \frac{(\lceil \frac{2\delta}{5} \rceil + 1)|V_1| - 4(\lfloor \frac{3\delta}{5} \rfloor - 1)}{|V_1| - 4}$$
$$= \left\lceil \frac{2\delta}{5} \right\rceil + 1 - \frac{4\lfloor \frac{3\delta}{5} \rfloor - 4\lceil \frac{2\delta}{5} \rceil - 8}{|V_1| - 4}.$$

Since $|V_1| \geq \frac{2\delta}{5} + 2$, $|V_2| \geq \lceil \frac{2\delta}{5} \rceil$. $\square$

By Claim 1, there cannot be an edge in $M$ such that both end vertices of this edge are in $V_2$. Then, without loss of generality, we assume that $V_2 = \{x_1, x_2, \ldots, x_p\}$, where $p = |V_2| \geq \lceil \frac{2k}{5} \rceil$. Let $G'$ denote the subgraph induced by $\{y_1, y_2, \ldots, y_p\}$.

**Claim 3.** *No color in $c(E(G'))$ is a new color.*

*Proof.* Suppose, to the contrary, there exists an edge, say $y_1y_2$ such that $c(y_1y_2)$ is a new color. Then we can find two independent edges $x_1w_1$ and $x_2w_2$ such that $w_1, w_2 \in V_1$, $c(x_1w_1), c(x_2w_2) \notin c(M) \cup \{c(y_1y_2)\}$ and $c(x_1w_1) \neq c(x_2w_2)$. We can do this, since each vertex in $V_2$ is incident with four special edges. Now we obtain a rainbow matching $M \cup \{x_1w_1, x_2w_2, y_1y_2\} \backslash \{x_1y_1, x_2y_2\}$ of size $t+1$, which is a contradiction. $\square$

**Claim 4.** $c(E(G')) \cap \{1, 2, \ldots, p\} = \emptyset$.

*Proof.* Otherwise, there is an edge, say $y_1y_2$ such that $c(y_1y_2) \in \{1, \ldots, p\}$. We assume that $c(y_1y_2) = j$. We know that $G$ is properly colored, so $j \neq 1, 2$. For convenience, assume that $j = 3$. We will show the following fact.

**Fact**. *There exists a rainbow matching formed by three special edges $\{x_1u_1, x_2u_2, x_3u_3\}$.*

*Proof of the Fact.* We prove it by contradiction. We choose three special edges incident with $x_1, x_2, x_3$ to form a matching $M_1$ such that $|c(M_1)|$ is as large as possible. Since each $x_i$ is incident with four special edges and by our assumption, we can assume that $|c(M_1)| = 2$. Without loss of generality, assume that $M_1 = \{x_1u, x_2v, x_3w\}$ and $c(x_1u) = a_1, c(x_2v) = a_2, c(x_3w) = a_1$. As $x_3$ is incident with four special edges, there are two special edges $x_3v_1, x_3v_2$ such that $v_1, v_2 \in V_1$ and $c(x_3v_1), c(x_3v_2) \notin c(M) \cup \{a_1, a_2\}$. We claim that $\{v_1, v_2\} = \{u, v\}$, otherwise we will get a rainbow matching satisfying our condition. Now we assume that $c(x_3u) = a_3, c(x_3v) = a_4$. Similarly, we assume that $c(x_1v) = b_1$ and $c(x_1w) = b_2$, where $b_1, b_2 \notin c(M) \cup \{a_1, a_2\}$. Then $b_2 = a_3$, otherwise $\{x_1w, x_3u, x_2v\}$ forms a rainbow matching, which is a contradiction. Moreover, $b_1 \neq a_4$, since $G$ is properly colored.

Now consider the vertex $x_2$. Since $x_2$ is incident with four special edges, there is an edge, say $x_2z$ such that $c(x_2z) \notin c(M) \cup \{a_2\}$ and $z \notin \{u, v, w\}$. Then $c(x_2z) = a_3$, otherwise either $\{x_2z, x_1v, x_3u\}$ or $\{x_2z, x_1w, x_3v\}$ would be a rainbow matching, and we are done. Hence $\{x_2z, x_1v, x_3w\}$ is a rainbow matching with colors $\{a_3, a_1, b_1\}$, which is a contradiction. This completes the proof of the fact.

By the above fact, $M \cup \{x_1u_1, x_2u_2, x_3u_3, y_1y_2\} \backslash \{e_1, e_2, e_3\}$ is a rainbow matching of size $t+1$. This contradiction completes the proof of Claim 4. $\square$

**Claim 5.** *If there is an edge $y_ju$, where $y_j \in V(G')$ and $u \in V_1$, then $c(y_ju) \in c(M)$ and $c(y_ju) \cap \{1, 2, \ldots, p\} = \emptyset$.*

*Proof.* Otherwise, suppose that $c(y_ju)$ is a new color. Then $d_s(y_j) \geq 1$. Since $d_s(x_j) \geq 4$, $d_s(x_j) + d_s(y_j) \geq 5$, which contradicts with Claim 1. So $c(y_ju) \in c(M)$. Suppose $c(y_ju) = k$, where $1 \leq k \leq p$. Since $G$ is properly colored, $k \neq j$. Since $x_j, x_k \in V_2$, we can find a special edge $x_jw_1$ such that $w_1 \neq u$. Next, there is a special edge $x_kw_2$ such that $w_2 \notin \{u, w_1\}$ and $c(x_kw_2) \neq c(x_jw_1)$. Hence we have a rainbow matching $M \cup \{x_jw_1, x_kw_2, y_ju\} \backslash \{x_jy_j, x_ky_k\}$, which is a contradiction. Thus Claim 5 holds. □

Now consider a vertex $y_j$, where $1 \leq j \leq p$. By Claims 3,4, and 5, we know that if $y_j$ has a neighbor $u \in V_1 \cup \{y_1, \ldots, y_p\}$, then $p < c(y_ju) \leq t$. Thus $|V(M)| - |V(G')| \geq d(y_j) - (t-p)$. It follows that $2t - p \geq \delta - (t-p)$. Hence $t \geq \frac{\delta+2p}{3} \geq \frac{2\lceil\frac{2\delta}{5}\rceil+\delta}{3} \geq \lfloor\frac{3\delta}{5}\rfloor$, which is a contradiction. This completes the whole proof of Theorem 5.

# 3 Proof of Theorem 6

Let $\delta = \delta(G)$. If $\delta \leq 3$, it is easy to check that our theorem holds. So now we assume that $\delta \geq 4$. Suppose our conclusion is not true. Let $M$ be a maximum rainbow matching of size $t$. Then $t \leq \lfloor\frac{2\delta}{3}\rfloor - 1$. Suppose that $E(M) = \{e_1, e_2, \ldots, e_t\}$ and $e_i = x_iy_i$. Moreover, without loss of generality, we assume that $c(e_i) = i$. Put $V_1 = V - V(M)$. A color is called a *new* color if it is not in $c(M)$ and we call an edge $uv$ *special* if $v \in V(M)$, $u \in V_1$ and $c(uv)$ is a new color. For $v \in V(M)$, let $d_s(v)$ denote the number of the special edges incident with $v$. Let $V_2 = \{v | v \in V(M),\, d_s(v) \geq 3\}$. We have the following claim.

**Claim 1.** *For each edge $x_iy_i \in E(M)$, if $d_s(x_i) + d_s(y_i) \geq 3$, then either $d_s(x_i) = 0$ or $d_s(y_i) = 0$.*

*Proof.* Otherwise, suppose that $d_s(x_i) + d_s(y_i) \geq 3$ and $d_s(x_i), d_s(y_i) \geq 1$. Then either $d_s(x_i) \geq 2$ or $d_s(y_i) \geq 2$. Assume that $d_s(x_i) \geq 2$. As $d_s(y_i) \geq 1$, we choose a special edge $y_iu$. By $d_s(x_i) \geq 2$, there is a special edge $x_iw$ such that $c(x_iw) \neq c(y_iu)$. Clearly, $u \neq w$, because $G$ is triangle-free. Now $M \cup \{x_iw, y_iu\} \backslash x_iy_i$ is a rainbow matching of size $t+1$, a contradiction. □

**Claim 2.** $|V_2| \geq \lceil\frac{\delta}{3}\rceil$.

*Proof.* Let $x \in V_1$. If there is an edge $xy$ such that $c(xy) \notin c(M)$, then $y \in V(M)$. Otherwise, there is a rainbow matching $M \cup xy$ of size $t+1$, which is a contradiction. Let $E_s$ denote the set of all the special edges. Since each vertex in $V_1$ has degree at least $\delta$, $|E_s| \geq (\delta - t)|V_1| \geq (\lceil\frac{\delta}{3}\rceil + 1)|V_1|$. Note that $|V_1| = |V(G)| - |V(M)| \geq 2\delta - 2(\lfloor\frac{2\delta}{3}\rfloor - 1) \geq \frac{2\delta}{3} + 2 \geq 3$ (recall that if $G$ is triangle-free, then $|V(G)| \geq 2\delta$). On the other hand, by Claim 1, for each edge $x_iy_i \in E(M)$, if $d_s(x_i) + d_s(y_i) \geq 3$, then $d_s(x_i) = 0$ or $d_s(y_i) = 0$.

So $d_s(x_i) + d_s(y_i) \le |V_1|$. Thus by Claim 1, $|E_s| \le |V_2||V_1| + 2(|E(M)| - |V_2|)$. So we have the following inequality: $(\lceil \frac{\delta}{3} \rceil + 1)|V_1| \le |V_2||V_1| + 2(|E(M)| - |V_2|)$. Hence

$$\begin{aligned} |V_2| \ge \frac{(\lceil \frac{\delta}{3} \rceil + 1)|V_1| - 2|E(M)|}{|V_1| - 2} &\ge \frac{(\lceil \frac{\delta}{3} \rceil + 1)|V_1| - 2(\lfloor \frac{2\delta}{3} \rfloor - 1)}{|V_1| - 2} \\ &= \left\lceil \frac{\delta}{3} \right\rceil + 1 - \frac{2\lfloor \frac{2\delta}{3} \rfloor - 2\lceil \frac{\delta}{3} \rceil - 4}{|V_1| - 2} \\ &\ge \left\lceil \frac{\delta}{3} \right\rceil. \end{aligned}$$

□

For each edge $e$ of $M$, at most one end vertex of $e$ is in $V_2$. Thus, without loss of generality, we assume that $V_2 = \{x_1, x_2, \ldots, x_p\}$, where $p = |V_2| \ge \lceil \frac{\delta}{3} \rceil$. Let $G'$ denote the subgraph induced by $\{y_1, y_2, \ldots, y_p\}$.

**Claim 3.** *There is a vertex $v \in V_2$ such that $d_s(v) \ge 5$.*

*Proof.* Otherwise, we have that each vertex $v \in V(M)$ has $d_s(v) \le 4$. By Claim 1, it holds that for each edge $x_iy_i \in E(M)$, $d_s(x_i) + d_s(y_i) \le 4$. Then $|E_s| \le 4(\lfloor \frac{2\delta}{3} \rfloor - 1)$. On the other hand, $|E_s| \ge |V_1|(\lceil \frac{\delta}{3} \rceil + 1) \ge (\lceil \frac{2\delta}{3} \rceil + 2)(\lceil \frac{\delta}{3} \rceil + 1)$. It follows that $4(\lfloor \frac{2\delta}{3} \rfloor - 1) \ge (\lceil \frac{2\delta}{3} \rceil + 2)(\lceil \frac{\delta}{3} \rceil + 1)$. Hence $2\delta^2 - 12\delta + 54 \le 0$, which is a contradiction. □

Without loss of generality, we assume that $d_s(x_1) \ge 5$. By Claim 1, $d_s(y_1) = 0$.

**Claim 4.** *If $y_1$ has a neighbor $y \in V(G') \cup V_1$, then $c(y_1y) \in c(M)$ and $c(y_1y) \notin \{1, 2, \ldots, p\}$.*

*Proof.* We distinguish the following two cases:

**Case 1.** Assume that $y_1$ has a neighbor, say $y = y_2 \in V(G')$. We prove it by contradiction. Firstly, suppose that $c(y_1y_2)$ is a new color. Then we can find two independent special edges $x_1w_1$ and $x_2w_2$ such that $c(x_1w_1), c(x_2w_2) \notin c(M) \cup \{c(y_1y_2)\}$ and $c(x_1w_1) \ne c(x_2w_2)$. We can do this, because $d_s(x_1) \ge 5$ and $d_s(x_2) \ge 3$. Now we obtain a rainbow matching $M \cup \{x_1w_1, x_2w_2, y_1y_2\} \backslash \{x_1y_1, x_2y_2\}$ of size $t + 1$, which is a contradiction.

Next, suppose that $c(y_1y_2) \cap \{1, 2, \ldots, p\} \ne \emptyset$. Since $G$ is properly colored, $c(y_1y_2) \ne 1, 2$. Without loss of generality, we assume that $c(y_1y_2) = 3$. As $d_s(x_3), d_s(x_2) \ge 3$ and $d_s(x_1) \ge 5$, we can easily find three special edges $x_1w_1, x_2w_2, x_3w_3$ to form a rainbow matching. Hence $M \cup \{x_1w_1, x_2w_2, x_3w_3, y_1y_2\} \backslash \{e_1, e_2, e_3\}$ is a rainbow matching of size $t + 1$.

**Case 2.** $y_1$ has a neighbor $y \in V_1$. We prove it by contradiction. Firstly, suppose that $c(y_1y)$ is a new color. Then there is a special edges $x_1w_1$ such that $c(x_1w_1) \ne c(y_1y)$, because $d_s(x_1) \ge 5$. Now we obtain a rainbow matching $M \cup \{x_1w_1, y_1y\} \backslash \{x_1y_1\}$ of size $t + 1$, which is a contradiction.

Next, suppose that $c(y_1y) \cap \{1, 2, \ldots, p\} \neq \emptyset$. Since $G$ is properly colored, $c(y_1y) \neq 1$. Without loss of generality, we assume that $c(y_1y) = 2$. As $d_s(x_2) \geq 3$ and $d_s(x_1) \geq 5$, we can easily find two independent special edges $x_1w_1, x_2w_2$ such that $w_2 \neq y$ to form a rainbow matching. Hence we can obtain a rainbow matching $M \cup \{x_1w_1, x_2w_2, y_1y\} \backslash \{e_1, e_2\}$ of size $t + 1$. This contradiction completes the proof of Claim 4. $\square$

Now consider the vertex $y_1$. By Claims 3,4 and $d_s(y_1) = 0$, we know that if $y_1$ has a neighbor $u \in V_1 \cup \{y_1, \ldots, y_p\}$, then $c(y_1u) \in c(M)$ and $c(y_1u) \notin \{1, 2, \ldots, p\}$. Thus $|\{x_1, \ldots, x_p\}| + |\{e_{p+1}, \ldots, e_t\}| \geq d(y_1) - (t - p)$. It follows that $t \geq \delta - (t - p)$. Hence $t \geq \frac{\delta+p}{2} \geq \frac{\lceil \frac{\delta}{3} \rceil + \delta}{2} \geq \lfloor \frac{2\delta}{3} \rfloor$, which is a contradiction. This completes the whole proof.


## Acknowledgement

I would like to thank the referee for the careful review and the valuable comments. This research was supported by NSFC Grants (61070230,11026184,10901097), IIFSDU (2009hw001), RFDP(20100131120017) and SRF for ROCS.


# References


[1] B. Alspach, Problem 89, *Discrete Math.* **69** (1988) 106.

[2] R. P. Anstee and L. Caccetta, Orthogonal matchings, *Discrete Math.* **179** (1998) 37–47.

[3] J. A. Bondy and U. S. R. Murty, *Graph Theory with Applications*, Macmillan Press, New York, 1976.

[4] R. A. Brualdi and H. J. Ryser, *Combinatorial Matrix Theory*, Cambridge University Press, Cambridge, UK, 1991.

[5] H. J. Ryser, Neuere probleme der kombinatorik, *Vorträge über Kombinatorik Oberwolfach*, Mathematisches Forschungsinstitut Oberwolfach, July 1967.

[6] T. D. LeSaulnier, C. Stocker, P.S. Wenger and D. B. West, Rainbow Matching in Edge-Colored Graphs, *Electron. J. Combin.* **17** (2010), #N26.

[7] P. Hatami and P. W. Shor, A lower bound for the length of a partial transversal in a Latin square, *J. Combin. Theory Ser. A* **115** (2008), 1103-1113.

[8] M. Kano and X. Li, Monochromatic and heterochromatic subgraphs in edge-colored graphs – a survey, *Graphs Combin.* **24** (2008), 237–263.

[9] S.K. Stein, Transversals of Latin squares and their generalizations, *Pacific J. Math.* **59** (1975), 567–575.

[10] R. Stong, Orthogonal Matchings, *Discrete Math.* **256** (2002), 515–518.

[11] I. M. Wanless, Transversals in Latin squares: A survey, in R. Chapman (ed.), *Surveys in Combinatorics 2011*, London Math. Soc. Lecture Note Series **392**, Cambridge University Press, 2011, pp403–437.