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Example A.1. *Consider the latin square,* $B_5 = \{(i, j, i + j\,(mod\;5)) \mid 0 \leq i, j \leq 4\}$, *of order* 5 *shown below. Next* (ζ_r, ζ_c)-3-*prolongate* B_5 *using the three transversals* $T_1 = \{(0,0,0),(1,1,2),(2,2,4),(3,3,1),(4,4,3)\}$, $T_2 = \{(0,1,1),(1,2,3),(2,3,0),(3,4,2),(4,0,4)\}$ *and* $T_3 = \{(0,2,2),(1,3,4),(2,4,1),(3,0,3),(4,1,0)\}$, *the permutations* $\zeta_r = id$, $\zeta_c = (1\ 2\ 3)$ *and the completing square* $\mathcal{P} = \{(i, j, i + j - 1\,(mod\;3) + 5) \mid 5 \leq i, j \leq 7\}$, *to achieve the latin square* $B_5(+3)$ *of order* 8.

B_5 :

0	1	2	3	4
1	2	3	4	0
2	3	4	0	1
3	4	0	1	2
4	0	1	2	3

$B_5(+3)$:

5	6	7	3	4	2	0	1
1	5	6	7	0	4	2	3
2	3	5	6	7	1	4	0
7	4	0	5	6	3	1	2
6	7	1	2	5	0	3	4
0	2	4	1	3	5	6	7
4	1	3	0	2	6	7	5
3	0	2	4	1	7	5	6

Example A.2. *Let* $B = \{(0,0,0),(0,1,2),(0,2,1),(1,0,2),(1,1,1),(1,2,0),(2,0,1),(2,1,0),(2,2,2)\}$ *and let* $k = 1$.

B :

0	2	1
2	1	0
1	0	2

Label 2 *disjoint transversals in* B *as follows* $T^0 = \{(0,0,0),(1,1,1),(2,2,2)\}$ *and* $T^1 = \{(0,1,2),(1,2,0),(2,0,1)\}$.

Now, construct $L_1 = A \times B$.

L_1 :

0	2	1	3	5	4
2	1	0	5	4	3
1	0	2	4	3	5
3	5	4	0	2	1
5	4	3	2	1	0
4	3	5	1	0	2

Then,

$$L_1 \setminus [\{(0,0,0)\} \times T^0 \;\cup\; \{(1,0,1)\} \times T^0 \;\cup\; \{(1,1,0)\} \times T^0] \cup$$

$$\gamma_c^6(\{(0,0,0)\} \times T^0) \;\cup\; \gamma_s^6(\{(0,0,0)\} \times T^0) \;\cup\; \gamma_r^6(\{(1,0,1)\} \times T^0) \;\cup$$
$$\gamma_c^6(\{(1,0,1)\} \times T^0) \;\cup\; \gamma_r^6(\{(1,1,0)\} \times T^0) \;\cup\; \gamma_s^6(\{(1,1,0)\} \times T^0) \;\cup$$
$$\{(3+i, i, i) \mid 0 \leq i \leq 2\} \text{ equals}$$

6	2	1	3	5	4	**0**
2	**6**	0	5	4	3	**1**
1	0	**6**	4	3	5	**2**
0	5	4	**6**	2	1	**3**
5	**1**	3	2	**6**	0	**4**
4	3	**2**	1	0	**6**	**5**
3	**4**	**5**	**0**	**1**	**2**	

So,

$$L_1 \setminus [\{(0,0,0)\} \times T^0 \,\cup\, \{(1,0,1)\} \times T^0 \,\cup\, \{(1,1,0)\} \times T^0 \,\cup\, (A \times T^1)] \cup$$

$$\gamma_c^6(\{(0,0,0)\} \times T^0) \,\cup\, \gamma_s^6(\{(0,0,0)\} \times T^0) \,\cup\, \gamma_r^6(\{(1,0,1)\} \times T^0) \,\cup$$
$$\gamma_c^6(\{(1,0,1)\} \times T^0) \,\cup\, \gamma_r^6(\{(1,1,0)\} \times T^0) \,\cup\, \gamma_s^6(\{(1,1,0)\} \times T^0) \,\cup$$
$$\{(3+i,i,i) \mid 0 \leq i \leq 2\} \cup$$

$$[\gamma_r^7(\{(0,0,0)\} \times T^1) \,\cup\, \gamma_c^7(\{(0,0,0)\} \times T^1) \,\cup\, \gamma_s^7(\{(0,0,0)\} \times T^1) \cup$$

$$\gamma_r^8(\{(1,0,1\} \times T^1) \,\cup\, \gamma_c^7(\{(1,0,1)\} \times T^1) \,\cup\, \gamma_s^8(\{(1,0,1)\} \times T^1) \cup$$

$$\gamma_r^8(\{(1,1,0)\} \times T^1) \,\cup\, \gamma_c^8(\{(1,1,0)\} \times T^1) \,\cup\, \gamma_s^7(\{(1,1,0)\} \times T^1) \cup$$

$$\gamma_r^7(\{(0,1,1)\} \times T^1) \,\cup\, \gamma_c^8(\{(0,1,1)\} \times T^1) \,\cup\, \gamma_s^8(\{(0,1,1)\} \times T^1)] \cup$$

$\{(6+j, 6+l, 6+(j+l\,(mod\;3))) \mid 0 \leq j,l \leq 2\}$ *equals*

6	**7**	1	3	**8**	4	0	**2**	**5**
2	6	**7**	5	4	**8**	1	**0**	**3**
7	0	6	**8**	3	5	2	**1**	**4**
0	**8**	4	6	**7**	1	3	**5**	**2**
5	1	**8**	2	6	**7**	4	**3**	**0**
8	3	2	**7**	0	6	5	**4**	**1**
3	4	5	0	1	2	**6**	**7**	**8**
1	**2**	**0**	**4**	**5**	**3**	**7**	**8**	**6**
4	**5**	**3**	**1**	**2**	**0**	**8**	**6**	**7**

Which is the latin square $L_1(+2.1+1) = L_1(+3)$*. Note, that the sets* $\{(0,0,6),(1,1,6),(2,2,6)\}$, $\{(3,6,3),(4,6,4),(5,6,5)\}$ *and* $\{(6,3,0),(6,4,1),(6,5,2)\}$ *form three disjoint* 3-*flowers in* $L_1(+3)$.

Example A.3. *Use the latin square* B *with transversals* T^0 *and* T^1 *from Example A.2 and again let* $k=1$ *to construct a latin square of order* $2m+3$.

First, $L_2 = A \times f_\alpha B$ *is constructed.*

L_2 :

1	0	2	4	3	5
0	2	1	3	5	4
2	1	0	5	4	3
4	3	5	1	0	2
3	5	4	0	2	1
5	4	3	2	1	0

Then,

$$L_2 \setminus ((\{(0,0,0)\} \times \phi_s^3 T^0 \cup \{(1,0,1)\} \times \sigma_c^3 T^0 \cup \{(1,1,0)\} \times \sigma_r^3 T^0)) \cup$$

$$\gamma_c^6(\{(0,0,0)\} \times \phi_s^3 T^0) \ \cup \ \gamma_s^6(\{(0,0,0)\} \times \phi_s^3 T^0) \ \cup \ \gamma_r^6(\{(1,0,1)\} \times \sigma_c^3 T^0) \ \cup$$
$$\gamma_c^6(\{(1,0,1)\} \times \sigma_c^3 T^0) \ \cup \ \gamma_r^6(\{(1,1,0)\} \times \sigma_r^3 T^0) \ \cup \ \gamma_s^6(\{(1,1,0)\} \times \sigma_r^3 T^0) \ \cup$$
$$\{(3+i, i+1\,(mod\ 3), i-1\,(mod\ 3)) \mid 0 \le i \le 2\} \ \textit{equals}$$

6	0	2	4	3	5	**1**
0	**6**	1	3	5	4	**2**
2	1	**6**	5	4	3	**0**
4	**2**	5	1	0	**6**	**3**
3	5	**0**	**6**	2	1	**4**
1	4	3	2	**6**	0	**5**
5	**3**	**4**	**0**	**1**	**2**	

.

So,

$$L_2 \setminus [\{(0,0,0)\} \times \phi_s^3 T^0 \ \cup \ \{(1,0,1)\} \times \sigma_c^3 T^0 \ \cup \ \{(1,1,0)\} \times \sigma_r^3 T^0 \ \cup \ (A \times f_\alpha T^1)] \cup$$

$$\gamma_c^6(\{(0,0,0)\} \times \phi_s^3 T^0) \ \cup \ \gamma_s^6(\{(0,0,0)\} \times \phi_s^3 T^0) \ \cup \ \gamma_r^6(\{(1,0,1)\} \times \sigma_c^3 T^0) \ \cup$$
$$\gamma_c^6(\{(1,0,1)\} \times \sigma_c^3 T^0) \ \cup \ \gamma_r^6(\{(1,1,0)\} \times \sigma_r^3 T^0) \ \cup \ \gamma_s^6(\{(1,1,0)\} \times \sigma_r^3 T^0) \ \cup$$
$$\{(3+i, i+1\,(mod\ 3), i-1\,(mod\ 3)) \mid 0 \le i \le 2\} \cup$$

$$[\gamma_r^8(\{(0,0,0)\} \times \phi_s^3 T^1) \ \cup \ \gamma_c^8(\{(0,0,0)\} \times \phi_s^3 T^1) \ \cup \ \gamma_s^8(\{(0,0,0)\} \times \phi_s^3 T^1) \cup$$

$$\gamma_r^7(\{(1,0,1)\} \times \sigma_c^3 T^1) \ \cup \ \gamma_c^8(\{(1,0,1)\} \times \sigma_c^3 T^1) \ \cup \ \gamma_s^7(\{(1,0,1)\} \times \sigma_c^3 T^1) \cup$$

$$\gamma_r^7(\{(1,1,0)\} \times \sigma_r^3 T^1) \ \cup \ \gamma_c^7(\{(1,1,0)\} \times \sigma_r^3 T^1) \ \cup \ \gamma_s^8(\{(1,1,0)\} \times \sigma_r^3 T^1) \cup$$

$$\gamma_r^8(\{(0,1,1)\} \times \phi_s^3 T^1) \ \cup \ \gamma_c^7(\{(0,1,1)\} \times \phi_s^3 T^1) \ \cup \ \gamma_s^7(\{(0,1,1)\} \times \phi_s^3 T^1)] \cup$$

$$\{(6+j, 6+l, 6+(j+l+1\,(mod\ 3))) \mid 0 \le j,l \le 2\} \ \textit{equals}$$

6	**8**	2	4	**7**	5	1	**3**	**0**
0	6	**8**	3	5	**7**	2	**4**	**1**
8	1	6	**7**	4	3	0	**5**	**2**
4	2	**7**	**8**	0	6	3	**1**	**5**
7	5	0	6	**8**	1	4	**2**	**3**
1	**7**	3	2	6	**8**	5	**0**	**4**
5	3	4	0	1	2	**7**	**8**	**6**
3	**4**	**5**	**1**	**2**	**0**	**8**	**6**	**7**
2	**0**	**1**	**5**	**3**	**4**	**6**	**7**	**8**

.

Which is the latin square $L_2(+2.1+1) = L_2(+3)$. *Note, that the sets* $\{(0,0,6),(1,1,6),(2,2,6)\}$, $\{(3,6,3),(4,6,4),(5,6,5)\}$ *and* $\{(6,3,0),(6,4,1),(6,5,2)\}$ *form three disjoint* 3-*flowers in* $L_2(+3)$.

Example A.4. *The latin squares $L_1(+3)$ and $L_2(+3)$ from Example A.2 and Example A.3 intersect in the three disjoint m-flowers $\{(0,0,6),(1,1,6),(2,2,6)\}$, $\{(3,6,3),(4,6,4),(5,6,5)\}$ and $\{(6,3,0),(6,4,1),(6,5,2)\}$ (shown in bold).*

$L_1(+3)$:

6	7	1	3	8	4	0	2	5
2	**6**	7	5	4	8	1	0	3
7	0	**6**	8	3	5	2	1	4
0	8	4	6	7	1	**3**	5	2
5	1	8	2	6	7	**4**	3	0
8	3	2	7	0	6	**5**	4	1
3	4	5	**0**	**1**	**2**	6	7	8
1	2	0	4	5	3	7	8	6
4	5	3	1	2	0	8	6	7

$L_2(+3)$:

6	8	2	4	7	5	1	3	0
0	**6**	8	3	5	7	2	4	1
8	1	**6**	7	4	3	0	5	2
4	2	7	8	0	6	**3**	1	5
7	5	0	6	8	1	**4**	2	3
1	7	3	2	6	8	**5**	0	4
5	3	4	**0**	**1**	**2**	7	8	6
3	4	5	1	2	0	8	6	7
2	0	1	5	3	4	6	7	8

Example A.5. *The latin square B, shown below, is used to construct the latin square L_1.*

B :

0	2	3	1
3	1	0	2
1	3	2	0
2	0	1	3

$L_1 = A \times B$:

0	2	3	1	4	6	7	5
3	1	0	2	7	5	4	6
1	3	2	0	5	7	6	4
2	0	1	3	6	4	5	7
4	6	7	5	0	2	3	1
7	5	4	6	3	1	0	2
5	7	6	4	1	3	2	0
6	4	5	7	2	0	1	3

Hence, construct $L'_1(+2)$.

$L'_1(+2)$:

8	2	3	1	**9**	6	7	5	**0**	**4**
3	**8**	0	2	7	**9**	4	6	**1**	**5**
1	3	**8**	0	5	7	**9**	4	**2**	**6**
2	0	1	**8**	6	4	5	**9**	**3**	**7**
9	6	7	5	**8**	2	3	1	**4**	**0**
7	**9**	4	6	3	**8**	0	2	**5**	**1**
5	7	**9**	4	1	3	**8**	0	**6**	**2**
6	4	5	**9**	2	0	1	**8**	**7**	**3**
0	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**
4	**5**	**6**	**7**	**0**	**1**	**2**	**3**	**9**	**8**

Example A.6. *Consider the latin square $L'_1(+2)$ constructed in Example A.5. Now construct the latin square $L_1(+2) = (L'_1(+2) \setminus (R_1 \cup S_1)) \cup (R_2 \cup S_2)$. The triples in the sets $R_1 \cup S_1 \subset L'_1(+2)$ and $R_2 \cup S_2 \subset L_1(+2)$ are shown in bold.*

$L_1'(+2)$:

8	2	3	1	9	6	7	5	0	4
3	8	0	2	7	9	4	6	1	5
1	3	8	0	5	7	9	4	2	6
2	0	1	8	6	4	5	9	3	7
9	6	7	5	8	2	3	1	4	0
7	9	4	6	3	8	0	2	5	1
5	7	9	4	1	3	8	0	6	2
6	**4**	**5**	**9**	2	0	1	8	**7**	3
0	1	2	3	4	5	6	7	8	9
4	**5**	**6**	**7**	0	1	2	3	**9**	8

$L_1(+2)$:

8	2	3	1	9	6	7	5	0	4
3	8	0	2	7	9	4	6	1	5
1	3	8	0	5	7	9	4	2	6
2	0	1	8	6	4	5	9	3	7
9	6	7	5	8	2	3	1	4	0
7	9	4	6	3	8	0	2	5	1
5	7	9	4	1	3	8	0	6	2
4	**5**	**6**	**7**	2	0	1	8	**9**	3
0	1	2	3	4	5	6	7	8	9
6	**4**	**5**	**9**	0	1	2	3	**7**	8

Example A.7. *Let $m = 4$ and $k = 2$. A latin square of order $2m + 4 = 12$ will be constructed. Consider the latin squares B and L_1 from Example A.5, where $T^1 = \{(0,3,1),(1,2,0),(2,1,3),(3,0,2)\} \subset B$.*

Then, $L_1(+4) =$

$L_1 \setminus [T^0 \cup T^1]$

$\bigcup_{0\leq h\leq 1}$

$[\gamma_r^{8+h}(\{(0,0,0) \times T^h\}) \cup \gamma_c^{8+2h}(\{(0,0,0)\} \times T^h) \cup \gamma_s^{8+2h}(\{(0,0,0) \times T^h\}) \cup$

$\gamma_r^{8+h}(\{(0,1,1) \times T^h\}) \cup \gamma_c^{8+2h+1}(\{(0,1,1)\} \times T^h) \cup \gamma_s^{8+2h+1}(\{(0,1,1) \times T^h\}) \cup$

$\gamma_r^{8+2+h}(\{(1,0,1) \times T^h\}) \cup \gamma_c^{8+2h}(\{(1,0,1)\} \times T^h) \cup \gamma_s^{8+2h+1}(\{(1,0,1) \times T^h\}) \cup$

$\gamma_r^{8+3-h}(\{(1,1,0) \times T^h\}) \cup \gamma_c^{8+2h+1}(\{(1,1,0)\} \times T^h) \cup \gamma_s^{8+2h}(\{(1,1,0) \times T^h\})] \cup$

$\{(2m+j, 2m+l, 2m+(j+l\,(mod\ 4))) \mid 0 \leq j,l \leq 3\}$ *equals*

8	2	3	10	**9**	6	7	11	**0**	**4**	1	5
3	**8**	10	2	7	**9**	11	6	**1**	**5**	0	4
1	10	**8**	0	5	11	**9**	4	**2**	**6**	3	7
10	0	1	**8**	11	4	5	**9**	**3**	**7**	2	6
9	6	7	11	**8**	2	3	10	**4**	**0**	5	1
7	**9**	11	6	3	**8**	10	2	**5**	**1**	4	0
5	11	**9**	4	1	10	**8**	0	**6**	**2**	7	3
11	4	5	**9**	10	0	1	**8**	**7**	**3**	6	2
0	**1**	**2**	**3**	**4**	**5**	**6**	**7**	8	9	10	11
2	3	0	1	6	7	4	5	9	10	11	8
4	**5**	**6**	**7**	2	3	0	1	10	11	8	9
6	7	4	5	**0**	**1**	**2**	**3**	11	8	9	10

.

Example A.8. *Using the latin square B, from Example A.5, a latin square of order $2m + 4 = 12$ is constructed.*

First, the latin square L_2 is constructed.

L_2 :

1	*3*	*0*	*2*	*6*	*4*	*5*	*7*
0	*2*	*1*	*3*	*4*	*6*	*7*	*5*
2	*0*	*3*	*1*	*7*	*5*	*4*	*6*
3	*1*	*2*	*0*	*5*	*7*	*6*	*4*
6	*4*	*5*	*7*	*1*	*0*	*2*	*3*
4	*6*	*7*	*5*	*2*	*3*	*1*	*0*
7	*5*	*4*	*6*	*0*	*1*	*3*	*2*
5	*7*	*6*	*4*	*3*	*2*	*0*	*1*

Then, $L_2(+4) =$

$L_2 \setminus [\bigcup_{0\leq i\leq 1} A \times \{f_\alpha T^i\}_{\alpha\in A}] \cup$

$\gamma_r^8(\{(0,0,0)\} \times \sigma_s^m T^0) \;\cup\; \gamma_c^8(\{(0,0,0)\} \times \sigma_s^m T^0) \;\cup\; \gamma_s^8(\{(0,0,0)\} \times \sigma_s^m T^0) \cup$

$\gamma_r^8(\{(0,1,1)\} \times \sigma_r^m T^0) \;\cup\; \gamma_c^9(\{(0,1,1)\} \times \sigma_r^m T^0) \;\cup\; \gamma_s^9(\{(0,1,1)\} \times \sigma_r^m T^0) \cup$

$\gamma_r^9(\{(1,0,1)\} \times \sigma_r^m T^0) \;\cup\; \gamma_c^8(\{(1,0,1)\} \times \sigma_r^m T^0) \;\cup\; \gamma_s^9(\{(1,0,1)\} \times \sigma_r^m T^0) \cup$

$\gamma_r^9(\{(1,1,0)\} \times \sigma_c^m T^0) \;\cup\; \gamma_c^9(\{(1,1,0)\} \times \sigma_c^m T^0) \;\cup\; \gamma_s^8(\{(1,1,0)\} \times \sigma_c^m T^0) \cup$

$\gamma_r^{11}(\{(0,0,0)\} \times \sigma_s^m T^1) \;\cup\; \gamma_c^{11}(\{(0,0,0)\} \times \sigma_s^m T^1) \;\cup\; \gamma_s^{11}(\{(0,0,0)\} \times \sigma_s^m T^1) \cup$

$\gamma_r^{11}(\{(0,1,1)\} \times \sigma_r^m T^1) \;\cup\; \gamma_c^{10}(\{(0,1,1)\} \times \sigma_r^m T^1) \;\cup\; \gamma_s^{10}(\{(0,1,1)\} \times \sigma_r^m T^1) \cup$

$\gamma_r^{10}(\{(1,0,1)\} \times \sigma_r^m T^1) \;\cup\; \gamma_c^{11}(\{(1,0,1)\} \times \sigma_r^m T^1) \;\cup\; \gamma_s^{10}(\{(1,0,1)\} \times \sigma_r^m T^1) \cup$

$\gamma_r^{10}(\{(1,1,0)\} \times \sigma_c^m T^1) \;\cup\; \gamma_c^{10}(\{(1,1,0)\} \times \sigma_c^m T^1) \;\cup\; \gamma_s^{11}(\{(1,1,0)\} \times \sigma_c^m T^1) \cup$

$\{(2m+j, 2m+l, 2m+(j+l+1 \,(mod\; 4))) \mid 0 \leq j,l \leq 3\}$ *equals*

8	*3*	*0*	11	10	*4*	*5*	**9**	**1**	**7**	6	2
0	**8**	11	*3*	**9**	*6*	*7*	10	**2**	**4**	5	1
2	11	**8**	*1*	*7*	**9**	10	*6*	**3**	**5**	4	0
11	*1*	*2*	**8**	*5*	10	**9**	*4*	**0**	**6**	7	3
10	*4*	*5*	**9**	11	**8**	*2*	*3*	**7**	**0**	1	6
9	*6*	*7*	10	*2*	*3*	**8**	11	**4**	**1**	0	5
7	**9**	10	*6*	*0*	*1*	11	**8**	**5**	**2**	3	4
5	10	**9**	*4*	**8**	11	*0*	*1*	**6**	**3**	2	7
1	**2**	**3**	**0**	**4**	**5**	**6**	**7**	*9*	*10*	*11*	*8*
4	**5**	**6**	**7**	**3**	**0**	**1**	**2**	*10*	*11*	*8*	*9*
6	7	4	5	1	2	3	0	*11*	*8*	*9*	*10*
3	0	1	2	6	7	4	5	*8*	*9*	*10*	*11*

.

Example A.9. *Using the latin square B from Example A.5, the latin squares* $L_1(+2)$ *and* $L_2(+2)$ *of order ten are constructed. Their intersection is composed precisely of the three disjoint 4-flowers* $\{(i,i,8) \mid 0 \leq i \leq 3\}$, $\{(8, 4+i, 4+i) \mid 0 \leq i \leq 3\}$ *and* $\{(4+i, 9, i) \mid 0 \leq i \leq 3\}$ *(shown in bold).*

$L_1(+2)$:

8	*2*	*3*	*1*	*9*	*6*	*7*	*5*	*0*	*4*
3	**8**	*0*	*2*	*7*	*9*	*4*	*6*	*1*	*5*
1	*3*	**8**	*0*	*5*	*7*	*9*	*4*	*2*	*6*
2	*0*	*1*	**8**	*6*	*4*	*5*	*9*	*3*	*7*
9	*6*	*7*	*5*	*8*	*2*	*3*	*1*	*4*	**0**
7	*9*	*4*	*6*	*3*	*8*	*0*	*2*	*5*	**1**
5	*7*	*9*	*4*	*1*	*3*	*8*	*0*	*6*	**2**
4	*5*	*6*	*7*	*2*	*0*	*1*	*8*	*9*	**3**
0	*1*	*2*	*3*	**4**	**5**	**6**	**7**	*8*	*9*
6	*4*	*5*	*9*	*0*	*1*	*2*	*3*	*7*	*8*

$L_2(+2)$:

8	*3*	*0*	*2*	*6*	*4*	*5*	*9*	*1*	*7*
0	**8**	*1*	*3*	*9*	*6*	*7*	*5*	*2*	*4*
2	*0*	**8**	*1*	*7*	*9*	*4*	*6*	*3*	*5*
3	*1*	*2*	**8**	*5*	*7*	*9*	*4*	*0*	*6*
6	*4*	*5*	*9*	*1*	*8*	*2*	*3*	*7*	**0**
9	*6*	*7*	*5*	*2*	*3*	*8*	*0*	*4*	**1**
7	*9*	*4*	*6*	*0*	*1*	*3*	*8*	*5*	**2**
5	*7*	*9*	*4*	*8*	*2*	*0*	*1*	*6*	**3**
1	*2*	*3*	*0*	**4**	**5**	**6**	**7**	*9*	*8*
4	*5*	*6*	*7*	*3*	*0*	*1*	*2*	*8*	*9*

Example A.10. *Consider the latin squares* $L_1(+4)$*, from Example A.7, and* $L_2(+4)$*, from Example A.8. The intersection of these latin squares is composed precisely of the three disjoint* 4*-flowers* $\{(i,i,8) \mid 0 \leq i \leq 3\}$, $\{(8,4+i,4+i) \mid 0 \leq i \leq 3\}$ *and* $\{(4+i,9,i) \mid 0 \leq i \leq 3\}$ *(shown in bold).*

$L_1(+4)$:

8	*2*	*3*	*10*	*9*	*6*	*7*	*11*	*0*	*4*	*1*	*5*
3	**8**	*10*	*2*	*7*	*9*	*11*	*6*	*1*	*5*	*0*	*4*
1	*10*	**8**	*0*	*5*	*11*	*9*	*4*	*2*	*6*	*3*	*7*
10	*0*	*1*	**8**	*11*	*4*	*5*	*9*	*3*	*7*	*2*	*6*
9	*6*	*7*	*11*	*8*	*2*	*3*	*10*	*4*	**0**	*5*	*1*
7	*9*	*11*	*6*	*3*	*8*	*10*	*2*	*5*	**1**	*4*	*0*
5	*11*	*9*	*4*	*1*	*10*	*8*	*0*	*6*	**2**	*7*	*3*
11	*4*	*5*	*9*	*10*	*0*	*1*	*8*	*7*	**3**	*6*	*2*
0	*1*	*2*	*3*	**4**	**5**	**6**	**7**	*8*	*9*	*10*	*11*
2	*3*	*0*	*1*	*6*	*7*	*4*	*5*	*9*	*10*	*11*	*8*
4	*5*	*6*	*7*	*2*	*3*	*0*	*1*	*10*	*11*	*8*	*9*
6	*7*	*4*	*5*	*0*	*1*	*2*	*3*	*11*	*8*	*9*	*10*

$L_2(+4)$:

8	*3*	*0*	*11*	*10*	*4*	*5*	*9*	*1*	*7*	*6*	*2*
0	**8**	*11*	*3*	*9*	*6*	*7*	*10*	*2*	*4*	*5*	*1*
2	*11*	**8**	*1*	*7*	*9*	*10*	*6*	*3*	*5*	*4*	*0*
11	*1*	*2*	**8**	*5*	*10*	*9*	*4*	*0*	*6*	*7*	*3*
10	*4*	*5*	*9*	*11*	*8*	*2*	*3*	*7*	**0**	*1*	*6*
9	*6*	*7*	*10*	*2*	*3*	*8*	*11*	*4*	**1**	*0*	*5*
7	*9*	*10*	*6*	*0*	*1*	*11*	*8*	*5*	**2**	*3*	*4*
5	*10*	*9*	*4*	*8*	*11*	*0*	*1*	*6*	**3**	*2*	*7*
1	*2*	*3*	*0*	**4**	**5**	**6**	**7**	*9*	*10*	*11*	*8*
4	*5*	*6*	*7*	*3*	*0*	*1*	*2*	*10*	*11*	*8*	*9*
6	*7*	*4*	*5*	*1*	*2*	*3*	*0*	*11*	*8*	*9*	*10*
3	*0*	*1*	*2*	*6*	*7*	*4*	*5*	*8*	*9*	*10*	*11*

Example A.11. *Consider*

B :

0	*2*	*1*
2	*1*	*0*
1	*0*	*2*

.

Also, denote $\{(0,1,2),(1,2,0),(2,0,1)\} \subset B$*, by* T^1*.*

Now, construct L_1*.*

L_1 :

0	2	1	6	8	7	3	5	4
2	1	0	8	7	6	5	4	3
1	0	2	7	6	8	4	3	5
6	8	7	3	5	4	0	2	1
8	7	6	5	4	3	2	1	0
7	6	8	4	3	5	1	0	2
3	5	4	0	2	1	6	8	7
5	4	3	2	1	0	8	7	6
4	3	5	1	0	2	7	6	8

Next 2-prolongate L_1 *along the transversals* $T_0 = \{(t,t,t) \mid 0 \le t \le 2\} \times T^0 \subset L_1$ *and* $T_1 = \{(0,1,2),(1,2,0),(2,0,1)\} \times T^1$*, and use the completing square* $\{(3m+h, 3m+l, 3m+(h+l\,(mod\ 2))) \mid 0 \le h,l \le 1\}$*.*

$L_1(+2)$:

9	2	1	6	10	7	3	5	4	**0**	8
2	**9**	0	8	7	10	5	4	3	**1**	6
1	0	**9**	10	6	8	4	3	5	**2**	7
6	8	7	**9**	5	4	0	10	1	**3**	2
8	7	6	5	**9**	3	2	1	10	**4**	0
7	6	8	4	3	**9**	10	0	2	**5**	1
3	10	4	0	2	1	**9**	8	7	**6**	5
5	4	10	2	1	0	8	**9**	6	**7**	3
10	3	5	1	0	2	7	6	**9**	**8**	4
0	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	9	10
4	5	3	7	8	6	1	2	0	10	9

Example A.12. *Using the latin square* B *and transversal* T^1 *from Example A.11 a latin square of order* $3m+2 = 11$ *is constructed.*

First, construct the latin square L_2*.*

L_2 :

1	0	2	7	6	8	4	3	5
0	2	1	6	8	7	3	5	4
2	1	0	8	7	6	5	4	3
7	6	8	4	3	5	1	0	2
6	8	7	3	5	4	0	2	1
8	7	6	5	4	3	2	1	0
4	3	5	1	0	2	7	6	8
3	5	4	0	2	1	6	8	7
5	4	3	2	1	0	8	7	6

Next, 2-prolongate L_2 *along the transversals* $\{(t,t,t) \mid 0 \le t \le 2\} \times f_\alpha T^i$*, where* $0 \le i \le 1$*, and use the completing square* $\{(3m+i, 3m+j, 3m+(i+j+1\,(mod\ 2))) \mid 0 \le i,j \le 1\}$ *to yield* $L_2(+2)$*.*

$L_2(+2)$:

9	10	2	7	6	8	4	3	5	**1**	0
0	**9**	10	6	8	7	3	5	4	**2**	1
10	1	**9**	8	7	6	5	4	3	**0**	2
7	6	8	4	**9**	10	1	0	2	**3**	5
6	8	7	10	5	**9**	0	2	1	**4**	3
8	7	6	**9**	10	3	2	1	0	**5**	4
4	3	5	1	0	2	10	6	**9**	**8**	7
3	5	4	0	2	1	**9**	10	7	**6**	8
5	4	3	2	1	0	8	**9**	10	**7**	6
1	**2**	**0**	**5**	**3**	**4**	**6**	**7**	**8**	10	9
2	0	1	3	4	5	7	8	6	9	10

Example A.13. *Using the latin square B and transversal T^1 from Example A.11 construct the latin squares $L_1(+2)$ and $L_2(+2)$ (from Examples A.11 and A.12) of order eleven whose intersection is composed precisely of three disjoint 3-flowers, $\{(i,i,i) \mid 0 \leq i \leq 2\}$, $\{(3+i,9,3+i) \mid 0 \leq i \leq 2\}$ and $\{(9,6+i,6+i) \mid 0 \leq i \leq 2\}$ (shown in bold).*

$L_1(+2)$:

9	2	1	6	10	7	3	5	4	0	8
2	**9**	0	8	7	10	5	4	3	1	6
1	0	**9**	10	6	8	4	3	5	2	7
6	8	7	9	5	4	0	10	1	**3**	2
8	7	6	5	9	3	2	1	10	**4**	0
7	6	8	4	3	9	10	0	2	**5**	1
3	10	4	0	2	1	9	8	7	6	5
5	4	10	2	1	0	8	9	6	7	3
10	3	5	1	0	2	7	6	9	8	4
0	1	2	3	4	5	**6**	**7**	**8**	9	10
4	5	3	7	8	6	1	2	0	10	9

$L_2(+2)$:

9	10	2	7	6	8	4	3	5	1	0
0	**9**	10	6	8	7	3	5	4	2	1
10	1	**9**	8	7	6	5	4	3	0	2
7	6	8	4	9	10	1	0	2	**3**	5
6	8	7	10	5	9	0	2	1	**4**	3
8	7	6	9	10	3	2	1	0	**5**	4
4	3	5	1	0	2	10	6	9	8	7
3	5	4	0	2	1	9	10	7	6	8
5	4	3	2	1	0	8	9	10	7	6
1	2	0	5	3	4	**6**	**7**	**8**	10	9
2	0	1	3	4	5	7	8	6	9	10

Example A.14. *Using the latin square B from Example A.11 the latin squares $L_1(+1)$ and $L_3(+1)$ of order ten are constructed.*

$L_1(+1)$:

9	2	1	**6**	8	7	**3**	5	4	**0**
2	9	0	8	7	6	5	4	3	1
1	0	9	7	6	8	4	3	5	2
6	8	7	**9**	5	4	**0**	2	1	3
8	7	6	5	9	3	2	1	0	4
7	6	8	4	3	9	1	0	2	5
3	5	4	0	2	1	**9**	8	7	**6**
5	4	3	2	1	0	8	9	6	7
4	3	5	1	0	2	7	6	9	8
0	1	2	**3**	4	5	6	7	8	**9**

$L_3(+1)$:

9	2	1	**3**	8	7	**0**	5	4	**6**
2	9	0	8	7	6	5	4	3	1
1	0	9	7	6	8	4	3	5	2
0	8	7	**6**	5	4	**9**	2	1	3
8	7	6	5	9	3	2	1	0	4
7	6	8	4	3	9	1	0	2	5
6	5	4	0	2	1	**3**	8	7	**9**
5	4	3	2	1	0	8	9	6	7
4	3	5	1	0	2	7	6	9	8
3	1	2	**9**	4	5	6	7	8	**0**

Example A.15. *Let*

B :

0	2	1
2	1	0
1	0	2

.

Now, construct $L_1(+1)$.

$L_1(+1)$:

12	2	1	9	11	10	3	5	4	6	8	7	**0**
2	**12**	0	11	10	9	5	4	3	8	7	6	**1**
1	0	**12**	10	9	11	4	3	5	7	6	8	**2**
6	8	7	**12**	5	4	9	11	10	0	2	1	**3**
8	7	6	5	**12**	3	11	10	9	2	1	0	**4**
7	6	8	4	3	**12**	10	9	11	1	0	2	**5**
9	11	10	0	2	1	**12**	8	7	3	5	4	**6**
11	10	9	2	1	0	8	**12**	6	5	4	3	**7**
10	9	11	1	0	2	7	6	**12**	4	3	5	**8**
3	5	4	6	8	7	0	2	1	**12**	11	10	**9**
5	4	3	8	7	6	2	1	0	11	**12**	9	**10**
4	3	5	7	6	8	1	0	2	10	9	**12**	**11**
0	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**

Example A.16. *Using the latin square* B *from Example A.15, the latin squares* $L_2'(+1)$ *and* $L_2(+1)$ *are constructed.*

$L_2'(+1)$:

12	0	2	10	9	11	4	3	5	7	6	8	**1**
0	**12**	1	9	11	10	3	5	4	6	8	7	**2**
2	1	**12**	11	10	9	5	4	3	8	6	7	**0**
7	6	8	4	3	**12**	10	9	11	1	0	2	**5**
6	8	7	**12**	5	4	9	11	10	0	2	1	**3**
8	7	6	5	**12**	3	11	10	9	2	1	0	**4**
10	9	11	1	0	2	7	**12**	8	4	3	5	**6**
9	11	10	0	2	1	6	8	**12**	3	5	4	**7**
11	10	9	2	1	0	**12**	7	6	5	4	3	**8**
4	3	5	7	6	8	1	0	2	**12**	11	10	**9**
3	5	4	6	8	7	0	2	1	11	**12**	9	**10**
5	4	3	8	7	6	2	1	0	10	9	**12**	**11**
1	**2**	**0**	**3**	**4**	**5**	**8**	**6**	**7**	**9**	**10**	**11**	**12**

$L_2(+1)$:

12	0	2	10	9	11	4	3	5	7	6	8	1
0	12	1	9	11	10	3	5	4	6	8	7	2
2	1	12	11	10	9	5	4	3	8	6	7	0
7	6	8	4	3	12	10	9	11	1	0	2	5
6	8	7	12	5	4	9	11	10	0	2	1	3
8	7	6	5	12	3	11	10	9	2	1	0	4
10	9	11	1	0	2	7	12	8	4	3	5	6
9	11	10	0	2	1	6	8	12	3	5	4	7
11	10	9	2	1	0	12	7	6	5	4	3	8
4	3	5	7	6	8	1	0	2	9	12	11	10
3	5	4	6	8	7	0	2	1	12	9	10	11
5	4	3	8	7	6	2	1	0	11	10	9	12
1	2	0	3	4	5	8	6	7	10	11	12	9

Example A.17. *Consider the following latin square B of order* 3.

B :

0	2	1
2	1	0
1	0	2

Label two disjoint transversals in B as follows $T^0 = \{(0,0,0),(1,1,1),(2,2,2)\}$ and $T^1 = \{(0,1,2),(1,2,0),(2,0,1)\}$. Let $k = 3$; a latin square $L_1'(+3)$ of order 15 *will be constructed. Let $k_1 = 1$ (so, $k_2 = k_3 = k_4 = 0$). First, construct L_1.*

$L_1 = A_1 \times B$:

0	2	1	3	5	4	6	8	7	9	11	10
2	1	0	5	4	3	8	7	6	11	10	9
1	0	2	4	3	5	7	6	8	10	9	11
3	5	4	0	2	1	9	11	10	6	8	7
5	4	3	2	1	0	11	10	9	8	7	6
4	3	5	1	0	2	10	9	11	7	6	8
6	8	7	9	11	10	0	2	1	3	5	4
8	7	6	11	10	9	2	1	0	5	4	3
7	6	8	10	9	11	1	0	2	4	3	5
9	11	10	6	8	7	3	5	4	0	2	1
11	10	9	8	7	6	5	4	3	2	1	0
10	9	11	7	6	8	4	3	5	1	0	2

Then,

$$L_1 \setminus [((A_1 \setminus \{(0,1,1),(0,3,3),(2,1,3),(2,3,1)\}) \times T^0)] \cup$$

$$\begin{aligned}
&\gamma_r^{12}(\{(1,0,1),(1,1,0),(1,2,3),(1,3,2)\} \times T^0) \cup \\
&\gamma_r^{13}(\{(3,0,3),(3,1,2),(3,2,1),(3,3,0)\} \times T^0) \cup \\
&\gamma_c^{12}(\{(3,0,3),(2,0,2),(1,0,1),(0,0,0)\} \times T^0) \cup \\
&\gamma_c^{13}(\{(3,2,1),(2,2,0),(1,2,3),(0,2,2)\} \times T^0) \cup \\
&\gamma_s^{12}(\{(0,0,0),(1,1,0),(2,2,0),(3,3,0)\} \times T^0) \cup \\
&\gamma_s^{13}(\{(0,2,2),(1,3,2),(2,0,2),(3,1,2)\} \times T^0) \cup
\end{aligned}$$

$\{(3+i,i,i),(3+i,6+i,6+i),(9+i,i,6+i),(9+i,6+i,i) \mid 0 \leq i \leq 2\}$ *equals*

12	2	1	3	5	4	**13**	8	7	9	11	10	**0**	**6**
2	**12**	0	5	4	3	8	**13**	6	11	10	9	**1**	**7**
1	0	**12**	4	3	5	7	6	**13**	10	9	11	**2**	**8**
0	5	4	**12**	2	1	**6**	11	10	**13**	8	7	**3**	**9**
5	**1**	3	2	**12**	0	11	**7**	9	8	**13**	6	**4**	**10**
4	3	**2**	1	0	**12**	10	9	**8**	7	6	**13**	**5**	**11**
13	8	7	9	11	10	**12**	2	1	3	5	4	**6**	**0**
8	**13**	6	11	10	9	2	**12**	0	5	4	3	**7**	**1**
7	6	**13**	10	9	11	1	0	**12**	4	3	5	**8**	**2**
6	11	10	**13**	8	7	**0**	5	4	**12**	2	1	**9**	**3**
11	**7**	9	8	**13**	6	5	**1**	3	2	**12**	0	**10**	**4**
10	9	**8**	7	6	**13**	4	3	**2**	1	0	**12**	**11**	**5**
3	**4**	**5**	**0**	**1**	**2**	**9**	**10**	**11**	**6**	**7**	**8**		
9	**10**	**11**	**6**	**7**	**8**	**3**	**4**	**5**	**0**	**1**	**2**		

.

So, $L'_1(+3) =$

$$L_1 \setminus [((A_1 \setminus \{(0,1,1),(0,3,3),(2,1,3),(2,3,1)\}) \times T^0) \,\cup\, U_1 \times T^1] \cup$$

$$\gamma_r^{12}(\{(1,0,1),(1,1,0),(1,2,3),(1,3,2)\} \times T^0) \cup$$
$$\gamma_r^{13}(\{(3,0,3),(3,1,2),(3,2,1),(3,3,0)\} \times T^0) \cup$$
$$\gamma_c^{12}(\{(3,0,3),(2,0,2),(1,0,1),(0,0,0)\} \times T^0) \cup$$
$$\gamma_c^{13}(\{(3,2,1),(2,2,0),(1,2,3),(0,2,2)\} \times T^0) \cup$$
$$\gamma_s^{12}(\{(0,0,0),(1,1,0),(2,2,0),(3,3,0)\} \times T^0) \cup$$
$$\gamma_s^{13}(\{(0,2,2),(1,3,2),(2,0,2),(3,1,2)\} \times T^0) \cup$$

$$\{(3+i,i,i),(3+i,6+i,6+i),(9+i,i,2m+i),(9+i,6+i,i) \mid 0 \le i \le 2\} \cup$$

$$\gamma_r^{14}(U_1 \times T^1) \,\cup\, \gamma_c^{14}(U_1 \times T^1) \,\cup\, \gamma_s^{14}(U_1 \times T^1)) \cup$$

$$\{(12+h, 12+j, 12+(h+j\ (mod\ 3))) \mid 0 \le h,j \le 3\}\ \textit{equals}$$

12	**14**	1	3	5	4	13	8	7	9	11	10	0	6	**2**
2	12	**14**	5	4	3	8	13	6	11	10	9	1	7	**0**
14	0	12	4	3	5	7	6	13	10	9	11	2	8	**1**
0	5	4	12	2	1	6	**14**	10	13	8	7	3	9	**11**
5	1	3	2	12	0	11	7	**14**	8	13	6	4	10	**9**
4	3	2	1	0	12	**14**	9	8	7	6	13	5	11	**10**
13	8	7	9	11	10	12	2	1	3	**14**	4	6	0	**5**
8	13	6	11	10	9	2	12	0	5	4	**14**	7	1	**3**
7	6	13	10	9	11	1	0	12	**14**	3	5	8	2	**4**
6	11	10	13	**14**	7	0	5	4	12	2	1	9	3	**8**
11	7	9	8	13	**14**	5	1	3	2	12	0	10	4	**6**
10	9	8	**14**	6	13	4	3	2	1	0	12	11	5	**7**
3	4	5	0	1	2	9	10	11	6	7	8	12	13	14
9	10	11	6	7	8	3	4	5	0	1	2	13	14	12
1	**2**	**0**	**7**	**8**	**6**	**10**	**11**	**9**	**4**	**5**	**3**	14	12	13

.

Example A.18. *The latin square B with transversals labelled T^0 and T^1 from Example A.17 is used to construct $L_2(+3)$.*

First, construct L_2.

$L_2 = A_2 \times f_\alpha B$:

1	0	2	4	3	5	10	9	11	6	8	7
0	2	1	3	5	4	9	11	10	8	7	6
2	1	0	5	4	3	11	10	9	7	6	8
4	3	5	1	0	2	6	8	7	9	11	10
3	5	4	0	2	1	8	7	6	11	10	9
5	4	3	2	1	0	7	6	8	10	9	11
9	11	10	7	6	8	1	0	2	4	3	5
11	10	9	6	8	7	0	2	1	3	5	4
10	9	11	8	7	6	2	1	0	5	4	3
7	6	8	9	11	10	4	3	5	1	0	2
6	8	7	11	10	9	3	5	4	0	2	1
8	7	6	10	9	11	5	4	3	2	1	0

Then,

$L_2 \setminus [((A \setminus \{(0,1,1),(1,3,3),(2,0,3),(2,3,1)\}) \times f_\alpha T^0)] \cup$

$\gamma_r^{12}(\{(1,0,1),(1,1,0),(0,2,3),(0,3,2)\} \times f_\alpha T^0) \cup$
$\gamma_r^{13}(\{(3,0,2),(3,1,3),(3,2,1),(3,3,0)\} \times f_\alpha T^0) \cup$
$\gamma_c^{12}(\{(0,0,0),(1,0,1),(2,1,2),(3,1,3)\} \times f_\alpha T^0) \cup$
$\gamma_c^{13}(\{(0,2,3),(1,2,2),(2,2,0),(3,2,1)\} \times f_\alpha T^0) \cup$
$\gamma_s^{12}(\{(0,0,0),(1,1,0),(2,2,0),(3,3,0)\} \times f_\alpha T^0) \cup$
$\gamma_s^{13}(\{(0,3,2),(1,2,2),(2,1,2),(3,0,2)\} \times f_\alpha T^0) \cup$

$\{(i, 6+i, 6+i), (3+i, i, (i+1\,(mod\;3))), (9+i, 3+i, 6+(i+1\,(mod\;3))), (9+(i+1\,(mod\;3)), 6+i, (i-1\,(mod\;3))) \mid 0 \le i \le 2\}$ *equals*

12	0	2	4	3	5	**6**	9	11	**13**	8	7	**1**	**10**
0	**12**	1	3	5	4	9	**7**	10	8	**13**	6	**2**	**11**
2	1	**12**	5	4	3	11	10	**8**	7	6	**13**	**0**	**9**
1	3	5	**12**	0	2	**13**	8	7	9	11	10	**4**	**6**
3	**2**	4	0	**12**	1	8	**13**	6	11	10	9	**5**	**7**
5	4	**0**	2	1	**12**	7	6	**13**	10	9	11	**3**	**8**
9	11	10	**13**	6	8	1	**12**	2	4	3	5	**7**	**0**
11	10	9	6	**13**	7	0	2	**12**	3	5	4	**8**	**1**
10	9	11	8	7	**13**	**12**	1	0	5	4	3	**6**	**2**
13	6	8	**7**	11	10	4	3	**1**	**12**	0	2	**9**	**5**
6	**13**	7	11	**8**	9	**2**	5	4	0	**12**	1	**10**	**3**
8	7	**13**	10	9	**6**	5	**0**	3	2	1	**12**	**11**	**4**
4	**5**	**3**	**1**	**2**	**0**	**10**	**11**	**9**	**6**	**7**	**8**		
7	**8**	**6**	**9**	**10**	**11**	**3**	**4**	**5**	**1**	**2**	**0**		

.

So, $L_2(+3) =$

$L_2 \setminus [((A \setminus \{(0,1,1),(1,3,3),(2,0,3),(2,3,1)\}) \times f_\alpha T^0) \cup U_1' \times f_\alpha T^1] \cup$

$\gamma_r^{12}(\{(1,0,1),(1,1,0),(0,2,3),(0,3,2)\} \times f_\alpha T^0) \cup$
$\gamma_r^{13}(\{(3,0,2),(3,1,3),(3,2,1),(3,3,0)\} \times f_\alpha T^0) \cup$
$\gamma_c^{12}(\{(0,0,0),(1,0,1),(2,1,2),(3,1,3)\} \times f_\alpha T^0) \cup$
$\gamma_c^{13}(\{(0,2,3),(1,2,2),(2,2,0),(3,2,1)\} \times f_\alpha T^0) \cup$
$\gamma_s^{12}(\{(0,0,0),(1,1,0),(2,2,0),(3,3,0)\} \times f_\alpha T^0) \cup$
$\gamma_s^{13}(\{(0,3,2),(1,2,2),(2,1,2),(3,0,2)\} \times f_\alpha T^0) \cup$

$\{(i,6+i,6+i),(3+i,i,(i+1\,(mod\,3))),(9+i,3+i,6+(i+1\,(mod\,3))),(9+(i+1\,(mod\,3)),6+i,(i-1\,(mod\,3)))\mid 0\le i\le 2\}\cup$

$(\gamma_r^{14}(U_1'\times f_\alpha T^1)\cup\gamma_c^{14}(U_1'\times f_\alpha T^1)\cup\gamma_s^{14}(U_1'\times f_\alpha T^1))\cup$

$\{(12+j,12+l,12+(j+l+1\,(mod\,3)))\mid 0\le j,l\le 2\}$ *equals*

12	0	2	4	3	5	6	9	11	13	**14**	7	1	10	**8**
0	12	1	3	5	4	9	7	10	8	13	**14**	2	11	**6**
2	1	12	5	4	3	11	10	8	**14**	6	13	0	9	**7**
1	3	5	12	**14**	2	13	8	7	9	11	10	4	6	**0**
3	2	4	0	12	**14**	8	13	6	11	10	9	5	7	**1**
5	4	0	**14**	1	12	7	6	13	10	9	11	3	8	**2**
9	**14**	10	13	6	8	1	12	2	4	3	5	7	0	**11**
11	10	**14**	6	13	7	0	2	12	3	5	4	8	1	**9**
14	9	11	8	7	13	12	1	0	5	4	3	6	2	**10**
13	6	8	7	11	10	**14**	3	1	12	0	2	9	5	**4**
6	13	7	11	8	9	2	**14**	4	0	12	1	10	3	**5**
8	7	13	10	9	6	5	0	**14**	2	1	12	11	4	**3**
4	5	3	1	2	0	10	11	9	6	7	8	13	14	12
7	8	6	9	10	11	3	4	5	1	2	0	14	12	13
10	**11**	**9**	**2**	**0**	**1**	**4**	**5**	**3**	**7**	**8**	**6**	12	13	14

.

Example A.19. *Construct the latin square $L_1(+3)$ from the latin square $L_1'(+3)$ constructed in Example A.17. This latin square intersects the latin square $L_2(+3)$, constructed in Example A.18, in the five disjoint 3-flowers $\{(i,i,12)\mid 0\le i\le 2\}$, $\{(6+i,13,i)\mid 0\le i\le 2\}$, $\{(9+i,12,9+i)\mid 0\le i\le 2\}$, $\{(12,9+i,6+i)\mid 0\le i\le 2\}$ and $\{(13,6+i,3+i)\mid 0\le i\le 2\}$ (shown in bold).*

$L_1(+3) =$

12	14	1	3	5	4	13	8	7	9	11	10	0	6	2
2	**12**	14	5	4	3	8	13	6	11	10	9	1	7	0
14	0	**12**	4	3	5	7	6	13	10	9	11	2	8	1
0	5	4	13	2	1	6	14	10	12	8	7	3	9	11
5	1	3	2	13	0	11	7	14	8	12	6	4	10	9
4	3	2	1	0	13	14	9	8	7	6	12	5	11	10
13	8	7	9	11	10	12	2	1	3	14	4	6	**0**	5
8	13	6	11	10	9	2	12	0	5	4	14	7	**1**	3
7	6	13	10	9	11	1	0	12	14	3	5	8	**2**	4
6	11	10	12	14	7	0	5	4	13	2	1	**9**	3	8
11	7	9	8	12	14	5	1	3	2	13	0	**10**	4	6
10	9	8	14	6	12	4	3	2	1	0	13	**11**	5	7
3	4	5	0	1	2	9	10	11	**6**	**7**	**8**	12	13	14
9	10	11	6	7	8	**3**	**4**	**5**	0	1	2	13	14	12
1	2	0	7	8	6	10	11	9	4	5	3	14	12	13

$L_2(+3) =$

12	0	2	4	3	5	6	9	11	13	14	7	1	10	8
0	**12**	1	3	5	4	9	7	10	8	13	14	2	11	6
2	1	**12**	5	4	3	11	10	8	14	6	13	0	9	7
1	3	5	12	14	2	13	8	7	9	11	10	4	6	0
3	2	4	0	12	14	8	13	6	11	10	9	5	7	1
5	4	0	14	1	12	7	6	13	10	9	11	3	8	2
9	14	10	13	6	8	1	12	2	4	3	5	7	**0**	11
11	10	14	6	13	7	0	2	12	3	5	4	8	**1**	9
14	9	11	8	7	13	12	1	0	5	4	3	6	**2**	10
13	6	8	7	11	10	14	3	1	12	0	2	**9**	5	4
6	13	7	11	8	9	2	14	4	0	12	1	**10**	3	5
8	7	13	10	9	6	5	0	14	2	1	12	**11**	4	3
4	5	3	1	2	0	10	11	9	**6**	**7**	**8**	13	14	12
7	8	6	9	10	11	**3**	**4**	**5**	1	2	0	14	12	13
10	11	9	2	0	1	4	5	3	7	8	6	12	13	14

.

Example A.20. *Using the latin square B and transversal T^1 from Example A.17 the latin square N_1 is constructed.*

N_1 :

0	2	1	3	5	4	6	8	7	9	11	10
2	1	0	5	4	3	8	7	6	11	10	9
1	0	2	4	3	5	7	6	8	10	9	11
3	5	4	0	2	1	9	11	10	6	8	7
5	4	3	2	1	0	11	10	9	8	7	6
4	3	5	1	0	2	10	9	11	7	6	8
6	8	7	9	11	10	0	2	1	3	5	4
8	7	6	11	10	9	2	1	0	5	4	3
7	6	8	10	9	11	1	0	2	4	3	5
9	11	10	6	8	7	3	5	4	0	2	1
11	10	9	8	7	6	5	4	3	2	1	0
10	9	11	7	6	8	4	3	5	1	0	2

Then, the latin square $N_1(+3)$, where $k_1 = 1$ and $k_2 = k_3 = k_4 = 0$ is constructed.

$N_1(+3)$:

12	**14**	1	3	5	4	**13**	8	7	9	11	10	**0**	**6**	**2**
2	**12**	**14**	5	4	3	8	**13**	6	11	10	9	**1**	**7**	**0**
14	0	**12**	4	3	5	7	6	**13**	10	9	11	**2**	**8**	**1**
13	5	4	0	2	1	**12**	**14**	10	6	8	7	**9**	**3**	**11**
5	**13**	3	2	1	0	11	**12**	**14**	8	7	6	**10**	**4**	**9**
4	3	**13**	1	0	2	**14**	9	**12**	7	6	8	**11**	**5**	**10**
6	8	7	**13**	11	10	0	2	1	**12**	**14**	4	**3**	**9**	**5**
8	7	6	11	**13**	9	2	1	0	5	**12**	**14**	**4**	**10**	**3**
7	6	8	10	9	**13**	1	0	2	**14**	3	**12**	**5**	**11**	**4**
9	11	10	**12**	**14**	7	3	5	4	**13**	2	1	**6**	**0**	**8**
11	10	9	8	**12**	**14**	5	4	3	2	**13**	0	**7**	**1**	**6**
10	9	11	**14**	6	**12**	4	3	5	1	0	**13**	**8**	**2**	**7**
0	**1**	**2**	**6**	**7**	**8**	**9**	**10**	**11**	**3**	**4**	**5**	12	13	14
3	**4**	**5**	**9**	**10**	**11**	**6**	**7**	**8**	**0**	**1**	**2**	13	14	12
1	**2**	**0**	**7**	**8**	**6**	**10**	**11**	**9**	**4**	**5**	**3**	14	12	13

Example A.21. *Using the latin square B and transversal T^1 from Example A.17 the latin square N_2 is constructed.*

N_2 :

1	0	2	4	3	5	7	6	8	10	9	11
0	2	1	3	5	4	6	8	7	9	11	10
2	1	0	5	4	3	8	7	6	11	10	9
4	3	5	1	0	2	10	9	11	7	6	8
3	5	4	0	2	1	9	11	10	6	8	7
5	4	3	2	1	0	11	10	9	8	7	6
7	6	8	10	9	11	1	0	2	4	3	5
6	8	7	9	11	10	0	2	1	3	5	4
8	7	6	11	10	9	2	1	0	5	4	3
10	9	11	7	6	8	4	3	5	1	0	2
9	11	10	6	8	7	3	5	4	0	2	1
11	10	9	7	8	6	5	4	3	2	1	0

Then, the latin square $N_2(+3)$ is constructed.

$N_2(+3)$:

13	0	2	4	**14**	5	**12**	6	8	10	9	11	**1**	**7**	**3**
0	**13**	1	3	5	**14**	6	**12**	7	9	11	10	**2**	**8**	**4**
2	1	**13**	**14**	4	3	8	7	**12**	11	10	9	**0**	**6**	**5**
4	**12**	5	1	0	2	10	9	**13**	7	**14**	8	**11**	**3**	**6**
3	5	**12**	0	2	1	**13**	11	10	6	8	**14**	**9**	**4**	**7**
12	4	3	2	1	0	11	**13**	9	**14**	7	6	**10**	**5**	**8**
7	6	8	**12**	9	11	1	**14**	2	**13**	3	5	**4**	**10**	**0**
6	8	7	9	**12**	10	0	2	**14**	3	**13**	4	**5**	**11**	**1**
8	7	6	11	10	**12**	**14**	1	0	5	4	**13**	**3**	**9**	**2**
10	**14**	11	7	**13**	8	4	3	5	1	0	**12**	**6**	**2**	**9**
9	11	**14**	6	8	**13**	3	5	4	**12**	2	1	**7**	**0**	**10**
14	10	9	**13**	7	6	5	4	3	2	**12**	0	**8**	**1**	**11**
1	**2**	**0**	**8**	**6**	**7**	**9**	**10**	**11**	**4**	**5**	**3**	13	14	12
5	**3**	**4**	**10**	**11**	**9**	**7**	**8**	**6**	**0**	**1**	**2**	14	12	13
11	**9**	**10**	**5**	**3**	**4**	**2**	**0**	**1**	**8**	**6**	**7**	12	13	14

Example A.22. *Using the latin square B and transversal T^1 from Example A.17 latin squares $N_1(+3)$ and $N_2(+3)$ of order fifteen are constructed. The intersection of these latin squares is composed precisely of four disjoint 3-flowers $\{(3+i, 13, m+i) \mid 0 \leq i \leq 2\}$, $\{(9+i, 12, 6+i) \mid 0 \leq i \leq 2\}$, $\{(12, 6+i, 9+i) \mid 0 \leq i \leq 2\}$ and $\{(13, 9+i, i) \mid 0 \leq i \leq 2\}$ (shown in bold).*

$N_1(+3)$:

12	14	1	3	5	4	13	8	7	9	11	10	0	6	2
2	12	14	5	4	3	8	13	6	11	10	9	1	7	0
14	0	12	4	3	5	7	6	13	10	9	11	2	8	1
13	5	4	0	2	1	12	14	10	6	8	7	9	**3**	11
5	13	3	2	1	0	11	12	14	8	7	6	10	**4**	9
4	3	13	1	0	2	14	9	12	7	6	8	11	**5**	10
6	8	7	13	11	10	0	2	1	12	14	4	3	9	5
8	7	6	11	13	9	2	1	0	5	12	14	4	10	3
7	6	8	10	9	13	1	0	2	14	3	12	5	11	4
9	11	10	12	14	7	3	5	4	13	2	1	**6**	0	8
11	10	9	8	12	14	5	4	3	2	13	0	**7**	1	6
10	9	11	14	6	12	4	3	5	1	0	13	**8**	2	7
0	1	2	6	7	8	**9**	**10**	**11**	3	4	5	12	13	14
3	4	5	9	10	11	6	7	8	**0**	**1**	**2**	13	14	12
1	2	0	7	8	6	10	11	9	4	5	3	14	12	13

$N_2(+3)$:

13	0	2	4	14	5	12	6	8	10	9	11	1	7	3
0	13	1	3	5	14	6	12	7	9	11	10	2	8	4
2	1	13	14	4	3	8	7	12	11	10	9	0	6	5
4	12	5	1	0	2	10	9	13	7	14	8	11	**3**	6
3	5	12	0	2	1	13	11	10	6	8	14	9	**4**	7
12	4	3	2	1	0	11	13	9	14	7	6	10	**5**	8
7	6	8	12	9	11	1	14	2	13	3	5	4	10	0
6	8	7	9	12	10	0	2	14	3	13	4	5	11	1
8	7	6	11	10	12	14	1	0	5	4	13	3	9	2
10	14	11	7	13	8	4	3	5	1	0	12	**6**	2	9
9	11	14	6	8	13	3	5	4	12	2	1	**7**	0	10
14	10	9	13	7	6	5	4	3	2	12	0	**8**	1	11
1	2	0	8	6	7	**9**	**10**	**11**	4	5	3	13	14	12
5	3	4	10	11	9	7	8	6	**0**	**1**	**2**	14	12	13
11	9	10	5	3	4	2	0	1	8	6	7	12	13	14

Example A.23. *Let $m = 2$ and $l = 3$. This example constructs the latin square $L_1^9(+1)$. Consider the following latin square, B_1, of order five.*

B_1 :

0	1	2	3	4
1	2	3	4	0
2	3	4	0	1
3	4	0	1	2
4	0	1	2	3

Note, B_1 *contains the transversal* $V_1^1 = \{(0,1,1),(1,3,4),(2,0,2),(3,2,0),(4,4,3)\}$.

By Lemma 4.20 there exists a pair of latin squares of order five that intersect in 3 *disjoint* 2*-flowers (shown in bold) and one other triple (shown in italics).*

C_1 :

0	**3**	**2**	4	1
4	2	0	1	3
1	0	3	2	4
3	1	4	**0**	2
2	4	1	3	**0**

C_2 :

0	**3**	**2**	1	4
4	0	1	3	2
1	2	0	4	3
2	4	3	**0**	1
3	1	4	2	**0**

Let A *be the latin square* $\{(0,0,0),(0,1,2),(0,2,1),(1,0,2),(1,1,1),(1,2,0),(2,0,1),(2,1,0),(2,2,2)\}$ *of order three. Note that* A *contains three disjoint transversals* $U_0 = \{(0,0,0),(1,1,1),(2,2,2)\}$, $U_1 = \{(0,1,2),(1,2,0),(2,0,1)\}$ *and* $U_2 = \{(0,2,1),(1,0,2),(2,1,0)\}$.

A :

0	2	1
2	1	0
1	0	2

Let $c = 3$ *and* $b = d = e = 0$.

Hence, construct $L_1^9(+0)'$.

$L_1^9(+0)'$:

0	3	2	4	1	10	11	12	13	14	5	6	7	8	9
4	2	0	1	3	11	12	13	14	10	6	7	8	9	5
1	0	3	2	4	12	13	14	10	11	7	8	9	5	6
3	1	4	0	2	13	14	10	11	12	8	9	5	6	7
2	4	1	3	0	14	10	11	12	13	9	5	6	7	8
10	11	12	13	14	5	8	7	9	6	0	1	2	3	4
11	12	13	14	10	9	7	5	6	8	1	2	3	4	0
12	13	14	10	11	6	5	8	7	9	2	3	4	0	1
13	14	10	11	12	8	6	9	5	7	3	4	0	1	2
14	10	11	12	13	7	9	6	8	5	4	0	1	2	3
5	6	7	8	9	0	1	2	3	4	10	13	12	14	11
6	7	8	9	5	1	2	3	4	0	14	12	10	11	13
7	8	9	5	6	2	3	4	0	1	11	10	13	12	14
8	9	5	6	7	3	4	0	1	2	13	11	14	10	12
9	5	6	7	8	4	0	1	2	3	12	14	11	13	10

Next, construct $L_1^9(+0)$.

$L_1^9(+0)$:

5	3	2	4	1	**0**	11	12	13	14	**10**	6	7	8	9
4	2	0	1	3	11	12	13	14	10	6	7	8	9	5
1	0	3	2	4	12	13	14	10	11	7	8	9	5	6
3	1	4	0	2	13	14	10	11	12	8	9	5	6	7
2	4	1	3	0	14	10	11	12	13	9	5	6	7	8
0	11	12	13	14	**10**	8	7	9	6	**5**	1	2	3	4
11	12	13	14	10	9	7	5	6	8	1	2	3	4	0
12	13	14	10	11	6	5	8	7	9	2	3	4	0	1
13	14	10	11	12	8	6	9	5	7	3	4	0	1	2
14	10	11	12	13	7	9	6	8	5	4	0	1	2	3
10	6	7	8	9	**5**	1	2	3	4	**0**	13	12	14	11
6	7	8	9	5	1	2	3	4	0	14	12	10	11	13
7	8	9	5	6	2	3	4	0	1	11	10	13	12	14
8	9	5	6	7	3	4	0	1	2	13	11	14	10	12
9	5	6	7	8	4	0	1	2	3	12	14	11	13	10

Finally, construct $L_1^9(+1)$.

$L_1^9(+1)$:

5	3	2	4	1	0	**15**	12	13	14	10	6	7	8	9	**11**
4	2	0	1	3	11	12	13	**15**	10	6	7	8	9	5	**14**
1	0	3	2	4	**15**	13	14	10	11	7	8	9	5	6	**12**
3	1	4	0	2	13	14	**15**	11	12	8	9	5	6	7	**10**
2	4	1	3	0	14	10	11	12	**15**	9	5	6	7	8	**13**
0	11	12	13	14	10	8	7	9	6	5	**15**	2	3	4	**1**
11	12	13	14	10	9	7	5	6	8	1	2	3	**15**	0	**4**
12	13	14	10	11	6	5	8	7	9	**15**	3	4	0	1	**2**
13	14	10	11	12	8	6	9	5	7	3	4	**15**	1	2	**0**
14	10	11	12	13	7	9	6	8	5	4	0	1	2	**15**	**3**
10	**15**	7	8	9	5	1	2	3	4	0	13	12	14	11	**6**
6	7	8	**15**	5	1	2	3	4	0	14	12	10	11	13	**9**
15	8	9	5	6	2	3	4	0	1	11	10	13	12	14	**7**
8	9	**15**	6	7	3	4	0	1	2	13	11	14	10	12	**5**
9	5	6	7	**15**	4	0	1	2	3	12	14	11	13	10	**8**
7	**6**	**5**	**9**	**8**	**12**	**11**	**10**	**14**	**13**	**2**	**1**	**0**	**4**	**3**	**15**

Example A.24. *The latin squares* A, B_1 *and* C_2 *from Example A.23 and* $B_2 = \sigma_s^{2m+1} B_1$ *are used to construct the latin square* $L_2^9(+1)$. *As in Example A.23, let* $c = 3$ *and* $b = d = e = 0$.

$L_2^9(+1)$:

0	3	2	1	4	11	12	13	14	10	6	**15**	8	9	5	**7**
4	0	1	3	2	12	13	14	10	11	7	8	9	**15**	6	**5**
1	2	0	4	3	13	14	10	11	12	**15**	9	5	6	7	**8**
2	4	3	0	1	14	10	11	12	13	9	5	**15**	7	8	**6**
3	1	4	2	0	10	11	12	13	14	5	6	7	8	**15**	**9**
11	**15**	13	14	10	5	8	7	6	9	1	2	3	4	0	**12**
12	13	14	**15**	11	9	5	6	8	7	2	3	4	0	1	**10**
15	14	10	11	12	6	7	5	9	8	3	4	0	1	2	**13**
14	10	**15**	12	13	7	9	8	5	6	4	0	1	2	3	**11**
10	11	12	13	**15**	8	6	9	7	5	0	1	2	3	4	**14**
6	7	8	9	5	1	**15**	3	4	0	10	13	12	11	14	**2**
7	8	9	5	6	2	3	4	**15**	1	14	10	11	13	12	**0**
8	9	5	6	7	**15**	4	0	1	2	11	12	10	14	13	**3**
9	5	6	7	8	4	0	**15**	2	3	12	14	13	10	11	**1**
5	6	7	8	9	0	1	2	3	**15**	13	11	14	12	10	**4**
13	**12**	**11**	**10**	**14**	**3**	**2**	**1**	**0**	**4**	**8**	**7**	**6**	**5**	**9**	**15**

Example A.25. *Consider the intersection of the latin squares $L_1^9(+1)$ and $L_2^9(+1)$ constructed in Examples A.23 and A.24. The intersection of these two latin squares is composed of nine disjoint 2-flowers and one other triple (shown in bold).*

$L_1^9(+1)$:

5	**3**	**2**	4	1	0	15	12	13	14	10	6	7	8	9	11
4	2	0	1	3	11	12	13	15	10	6	7	8	9	5	14
1	0	3	2	4	15	13	14	10	11	7	8	9	5	6	12
3	1	4	**0**	2	13	14	15	11	12	8	9	5	6	7	10
2	4	1	3	**0**	14	10	11	12	15	9	5	6	7	8	13
0	11	12	13	14	10	**8**	**7**	9	6	5	15	2	3	4	1
11	12	13	14	10	**9**	7	5	6	8	1	2	3	15	0	4
12	13	14	10	11	**6**	5	8	7	9	15	3	4	0	1	2
13	14	10	11	12	8	6	9	**5**	7	3	4	15	1	2	0
14	10	11	12	13	7	9	6	8	**5**	4	0	1	2	15	3
10	15	7	8	9	5	1	2	3	4	0	**13**	**12**	14	11	6
6	7	8	15	5	1	2	3	4	0	**14**	12	10	11	13	9
15	8	9	5	6	2	3	4	0	1	**11**	10	13	12	14	7
8	9	15	6	7	3	4	0	1	2	13	11	14	**10**	12	5
9	5	6	7	15	4	0	1	2	3	12	14	11	13	**10**	8
7	6	5	9	8	12	11	10	14	13	2	1	0	4	3	**15**

$L_2^9(+1)$:

0	**3**	**2**	1	4	11	12	13	14	10	6	15	8	9	5	7
4	0	1	3	2	12	13	14	10	11	7	8	9	15	6	5
1	2	0	4	3	13	14	10	11	12	15	9	5	6	7	8
2	4	3	**0**	1	14	10	11	12	13	9	5	15	7	8	6
3	1	4	2	**0**	10	11	12	13	14	5	6	7	8	15	9
11	15	13	14	10	5	**8**	**7**	6	9	1	2	3	4	0	12
12	13	14	15	11	**9**	5	6	8	7	2	3	4	0	1	10
15	14	10	11	12	**6**	7	5	9	8	3	4	0	1	2	13
14	10	15	12	13	7	9	8	**5**	6	4	0	1	2	3	11
10	11	12	13	15	8	6	9	7	**5**	0	1	2	3	4	14
6	7	8	9	5	1	15	3	4	0	10	**13**	**12**	11	14	2
7	8	9	5	6	2	3	4	15	1	**14**	10	11	13	12	0
8	9	5	6	7	15	4	0	1	2	**11**	12	10	14	13	3
9	5	6	7	8	4	0	15	2	3	12	14	13	**10**	11	1
5	6	7	8	9	0	1	2	3	15	13	11	14	12	**10**	4
13	12	11	10	14	3	2	1	0	4	8	7	6	5	9	**15**

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