

Mesh patterns and the expansion of permutation statistics as sums of permutation patterns

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In Theorem 7 we show that $\pi \in \mathfrak{S}_n$ is an André permutation of the first kind if and only if it avoids

$$\mathbf{andr\acute{e}} = \begin{array}{c} \bullet \\ | \\ \hline \bullet \\ | \\ \hline \bullet \\ | \\ \hline \bullet \end{array} \quad \text{and} \quad \begin{array}{c} \bullet \\ | \\ \hline \bullet \\ | \\ \hline \bullet \end{array} .$$

Here, the second pattern only serves to require that $\pi(n) = n$. It is the first pattern, $\mathbf{andr\acute{e}}$, that plays the important role. In particular, $|\mathfrak{S}_n(\mathbf{andr\acute{e}})| = E_{n+1}$ (Corollary 8).

Gábor [H. Gábor, André permutations and barred (generalized) patterns (2009), paper in preparation] has independently characterized the André permutations using a barred generalized pattern: $\mathbf{gábor} = 4\text{-}\bar{1}\text{-}32$. In terms of mesh patterns,

$$\mathbf{gábor} = \begin{array}{c} \bullet \\ | \\ \hline \bullet \\ | \\ \hline \bullet \end{array}$$

and it is easy to show that a permutation avoids $\mathbf{andr\acute{e}}$ if and only if it avoids $\mathbf{gábor}$.