We begin of course by expressing our appreciation to the anonymous referee for reviewing our paper "An Algebraic Exploration of Dominating Sets and Vizing's Conjecture". All corrections have been made, except in one minor point where other readers had expressed confusion. For simplicity, we have included the review here, with short annotations indicating that all comments were handled.

On behalf of all the authors, very best regards,
Susan Margulies

- Abstract, Line 2: etc.. ⟶ etc...

  Fixed.

- Abstract, Lines 5 and 8: "We then" is used to begin the two consecutive sentences.

  Fixed.

- Abstract, Line 9: "Vizing's" ⟶ "this".

  Fixed.

- Introduction, Lines 3,4,5, ...: The references are not numerically sorted. The same in some other parts of the manuscript.

  Fixed.

- Introduction, Line 6: It is stated that "techniques from computer algebra are certainly not widely used by combinatorists and graph theorists". That sentence is not completely true, since there are many papers that analyze the use of toric algebraic tools for solving combinatorial optimization problems. However, which is true is that the encoding of the graph problems proposed in this paper is not very usual. You should add the references of Conti-Trverso, 1991, Thomas, 1995; and Blanco-Puerto, 2009 to show that your approach is different of those proposed there.

  Done, and thank you.

- Introduction, Second paragraph, Line 2: v is adjacent to u in G.

  Fixed.

- Page 2, After Conjecture 1.1.: The authors concentrate on the importance of Vizing's conjecture because the researchers that have been studying the conjecture. However, they do not mention why is the conjecture important by the point of view of its consequences on Graph Theory.

  Done: Added a short comment on the relevance of Vizing's conjecture.

- Page 3, Proof of Lemma 2.2: The proof is easy. I would left it to the interested reader since it is technical and it does not contribute to the novelty of the ideas in the paper.

  True enough, but we are optimistically attempting to appeal to graph theorists who may be interested in Vizing's conjecture but have no background in algebraic geometry at all. We have left this very short proof in the paper at this time.

- Page 4, (definition of Gröbner basis): The definition of Gröbner basis is due to Bruno Buchberger, in his paper in 1965. The paper should cite such a paper in the definition of the basis. The same when the authors define the Buchberger's S-pair criterion.

  Done and thank you!

- Page 4, (definition of Universal GB): The reader that is not familiar with these structures may think that the Universal Gröbner basis is not finite or that there are finitely many monomial orderings. None of those assertions is true. I would fix it adding: *Although there are infinitely many monomial orderings, the Universal Gröbner basis is finite and unique. (see [8] for further details).*

  Done.

- Page 4 ($\overline{f}^F$): The division of a polynomial by a set of polynomials with respect to a monomial ordering is widely used through the paper but never detailed. Some other algebraic details are given but not this one which is crucial for the understanding of many of the given proofs (all those related with the Universal GB).

  Thank you. I added a reference to the thorough discussion in Cox, Little, O'Shea "Ideals, Varieties and Algorithms" (Chap. 2, Sec. 3), and also added a short example.

- Page 9, Parragraph 2: The authors write that e_{ij} refers to the edge between vertex i and vertex j, but e_{ij} is a binary decision variable taking value 1 when the edge (i, j) is in the graph and zero otherwise. This should be corrected.

  Thank you. Added additional clarification.

- Page 9, Example 4.2: The example illustrates very good the approach. However, it would be even better if they add the solutions of the system in graph from, i.e., with pictures similar to those in Examples 4.5, 4.7 or 4.11. In that way, the reader will be able to see the equivalence of Theorem 4.1.

  Done. Added picture and solutions to example.

- Page 11: The three polynomial equations in the theorem should be tagged. They are used as P_G^k, P_H^l and $P_{G\square H}^r$ but never defined.

  Done.

- Page 12, Line 2, $G' = G + (u, v)$ is understood as the graph with vertices $V(G)$ and edges $E(G') = E(G) \cup \{(u, v)\}$ but never defined.

  Quite right, but we do feel this is clear from context, and this is a very common notation in graph theory. We have left it as is at this time.

- Page 12, Line 13, $cmN_G(S) := \bigcap_{u\in S} N_G(u)$ must be $cmN_G(S) := \bigcap_{u\in S} N_G(\{u\})$.

  Done.

- Page 16, Line 13: I would move the paragraph "We note that if $k = n$, ..." after the proof of Thm. 4.16.

  Regretfully, this point caused confusion in other readers, and so we wished to address it at once, even before the proof.

- Page 21, Line -17: It would be nice to see in an example the structure of the systems concerning this paragraph.

  They are too big! And the paper is getting a bit long...

- A conclusion section must be added by summarizing the results in the paper as well as the future plans on this topic. It would be nice to add some comments about which is the computational advantage of this approach.

  Done.