Some results on chromaticity of quasi-linear paths and cycles

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Abstract

Let $r \geqslant 1$ be an integer. An h-hypergraph H is said to be r-quasi-linear (linear for $r = 1$) if any two edges of H intersect in 0 or r vertices. In this paper it is shown that r-quasi-linear paths $P_m^{h,r}$ of length $m \geqslant 1$ and cycles $C_m^{h,r}$ of length $m \geqslant 3$ are chromatically unique in the set of h-uniform r-quasi-linear hypergraphs provided $r \geqslant 2$ and $h \geqslant 3r - 1$.

Keywords: quasi-linear hypergraph; sunflower hypergraph; quasi-linear path; quasi-linear cycle; chromatic polynomial; chromatic uniqueness; potential function

1 Notation and preliminary results

A simple hypergraph $H = (V, \mathcal{E})$, with order $n = |V|$ and size $m = |\mathcal{E}|$, consists of a vertex-set $V(H) = V$ and an edge-set $E(H) = \mathcal{E}$, where $E \subseteq V$ and $|E| \geqslant 2$ for each edge E in $\mathcal{E}$. H is h-uniform, or is an h-hypergraph, if $|E| = h$ for each E in $\mathcal{E}$ and H is linear if no two edges intersect in more than one vertex [1]. H is said to be antilinear if for every two edges E, F of H we have $|E \cap F| \neq 1$. Let $r \geqslant 1$ and $h \geqslant 2r + 1$. H is said to be r-quasi-linear (or shortly quasi-linear) [13] if any two edges intersect in 0 or r vertices. Examples of quasi-linear hypergraphs are t-stars [5, 8], also called sunflower hypergraphs [7, 11, 12]. We say that a hypergraph S is a t-star with kernel K where $K \subseteq V(S)$ and $t \geqslant 1$ if S has exactly t edges and $e \cap e' = K$ for all distinct edges e and e' of S. A system of t pairwise disjoint edges (matching) is a t-star with empty kernel. In [12] a sunflower hypergraph was denoted by $SH(n,p,h)$; it is an h-hypergraph having a kernel of cardinality $h-p$, n vertices and k edges, where $n = h + (k-1)p$ and $1 \leqslant p \leqslant h-1$. A hypergraph for which no edge is a subset of any other is called Sperner. Two vertices $u, v \in V(H)$ belong to the same component if there are vertices $x_0 = u, x_1, \ldots, x_k = v$ and edges $E_1, \ldots, E_k$ of H such that x_{i-1}, $x_i \in E_i$ for each i $(1 \leqslant i \leqslant k)$ [1]. H is said to be connected if it has only one component. An h-uniform hypertree is a connected linear h-hypergraph without cycles. We shall define two classes of quasi-linear uniform hypergraphs called quasi-linear elementary paths and quasi-linear elementary cycles and denoted by $P_m^{h,r}$ and $C_m^{h,r}$, respectively, as follows: $P_m^{h,r}$ consists of m edges $E_1, \ldots, E_m$ such that $|E_1| = \ldots = |E_m| = h, |E_k \cap E_l| = r$ if $\{k,l\} = \{i, i+1\}$ for any $1 \leqslant i \leqslant m-1$ and 0 otherwise. Cycles $C_m^{h,r}$ are defined analogously, by also imposing $|E_m \cap E_1| = r$.

If $\lambda \in \mathbb{N}$, a λ-coloring of a hypergraph H is a function $f : V(H) \to \{1, \ldots, \lambda\}$ such that for each edge E of H there exist x, y in E for which $f(x) \neq f(y)$. The number of λ-colorings of H is given by a polynomial $P(H,\lambda)$ of degree $|V(H)|$ in λ, called the chromatic polynomial of H. $P(H,\lambda)$ can be obtained applying inclusion-exclusion principle, in the same way as for graphs, getting the following formula:

$$P(H,\lambda) = \sum_{W \subseteq E(H)} (-1)^{|W|} \lambda^{c(W)}, \tag{1}$$

where $c(W)$ denotes the number of components of the spanning subhypergraph induced by the edges from W. By rearranging terms in (1) we obtain that if H has order n then $P(H,\lambda) = \lambda^n + a_{n-1}\lambda^{n-1} + \ldots + a_1\lambda$, where

$$a_i = \sum_{j \geqslant 0} (-1)^j N(i,j) \tag{2}$$

and $N(i,j)$ denotes the number of spanning subhypergraphs of H with n vertices, i components and j edges [10].

All h-uniform hypertrees have the same chromatic polynomial.

Lemma 1. [6]. *If T_k^h is any h-uniform hypertree with k edges, then*

$$P(T_k^h, \lambda) = \lambda(\lambda^{h-1} - 1)^k. \tag{3}$$

Two hypergraphs H and G are said to be chromatically equivalent or χ-equivalent, written $H \sim G$, if $P(H,\lambda) = P(G,\lambda)$. Let us restrict ourselves to the class of Sperner hypergraphs. A simple hypergraph H is said to be chromatically unique if H is isomorphic to H' for every simple hypergraph H' such that $H' \sim H$; that is, the structure of H is uniquely determined up to isomorphism by its chromatic polynomial. The notion of χ-unique graphs was first introduced and studied by Chao and Whitehead [4] (see also [9]). It is clear that all h-hypergraphs are Sperner. The notion of χ-uniqueness in the class of h-hypergraphs may be defined as follows: An h-hypergraph H is said to be h-chromatically unique if H is isomorphic to H' for every h-hypergraph H' such that $H' \sim H$.

Non-trivial chromatically unique hypergraphs are extremely rare. One example of a non-trivial chromatically unique hypergraph was proposed by Borowiecki and Lazuka; it is $SH(n,1,h)$.

Theorem 2. [3] *$SH(n,1,h)$ is chromatically unique.*

The proof of this result was completed in [11]. Note that for $p = h-1$, $SH(n,h-1,h)$ is an h-uniform hypertree. The chromaticity of $SH(n,p,h)$ may be stated as follows.

Theorem 3. [12] *Let $n = h + (k-1)p$, where $h \geqslant 3$, $k \geqslant 1$ and $1 \leqslant p \leqslant h-1$. Then SH(n,p,h) is h-chromatically unique for every $1 \leqslant p \leqslant h-2$; for $p = h-1$ $SH(n,h-1,h)$ is h-chromatically unique for $k = 1$ or $k = 2$ but it has not this property for $k \geqslant 3$. Moreover, $SH(n,p,h)$ is not chromatically unique for every $p, k \geqslant 2$.*

$SH(n,p,h)$ is quasi-linear with $r = h-p$ and it is a path for $k = 2$.
Since $P_2^{h,r}$ is a sunflower hypergraph $SH(n,p,h)$ with $p = h-r$ having $k = 2$ edges, from Theorem 1.3 it follows that $P_2^{h,r}$ is h-chromatically unique for every $1 \leqslant r \leqslant h-1$. Also $P_m^{h,1}$ is an h-uniform hypertree, hence for $m \geqslant 3$ it is not h-chromatically unique. We shall prove that $P_m^{h,r}$ for every $m \geqslant 1$ and $C_m^{h,r}$ for every $m \geqslant 3$ are h-chromatically unique hypergraphs in the set of quasi-linear hypergraphs provided $r \geqslant 2$ and $h \geqslant 3r-1$. In [10] it was shown that $C_m^{h,r}$ is h-chromatically unique for $r = 1$ and every $m, h \geqslant 3$, but it is not chromatically unique for $r = 1$ and $m, h \geqslant 3$ [2]. The chromaticity of non-uniform hypertrees was studied by Walter [15].

2 Main results

We need the following result about the first coefficients of the chromatic polynomial of a quasi-linear h-hypergraph with a particular structure relatively to subhypergraphs induced by 3 edges.

Lemma 4. *Let $r \geqslant 2$, $h \geqslant 2r+1$ and H be a quasi-linear h-hypergraph of order n and size m having the property that all subhypergraphs induced by 3 edges have one of the following patterns:*
a)$P_3^{h,r}$; b)$P_2^{h,r}$ and an isolated edge, or c) 3 isolated edges. Then

$$P(H,\lambda) = \lambda^n - m\lambda^{n-h+1} + \beta_1\lambda^{n-2h+r+1} + \beta_2\lambda^{n-2h+2} - \beta_3\lambda^{n-3h+2r+1} + R(\lambda), \tag{4}$$

where $R(\lambda)$ is a polynomial in λ of degree at most equal to $n-3h+2r$, β_2 is the number of pairwise disjoint edges of H and β_1 and β_3 are the numbers of induced subhypergraphs of H isomorphic to $P_2^{h,r}$ and $P_3^{h,r}$, respectively.

Proof. By the hypothesis we have $n-h+1 > n-2h+r+1 > n-2h+2 > n-3h+2r+1$. If $W \subset E(H)$ in (1) consists of one edge we get $N(n-h+1, j) = m$ if $j = 1$ and $N(n-h+1, j) = 0$ otherwise.
If $|W| = 2$ then $N(n-2h+r+1, 2)$ and $N(n-2h+2, 2)$ count the number of unordered pairs $\{E, F\}$ of edges such that $|E \cap F| = r$ and $E \cap F = \emptyset$, respectively. In these two cases suppose that there exists an edge $G \in \mathcal{E}$, $G \neq E, F$, such that $G \subset E \cup F$. Denote $i = |G \cap E \cap F|$. It follows that $0 \leqslant i \leqslant r$, $|G \cap (E \backslash F)| = r-i$, $|G \cap (F \backslash E)| = r-i$, thus yielding $h = |G| = 2r - i \leqslant 2r$, which contradicts the hypothesis. It follows that

$N(n-2h+r+1,j) = N(n-2h+2,j) = 0$ for every $j \neq 2$ and β_1, β_2 represent the numbers of induced subhypergraphs of H consisting of $P_2^{h,r}$ and of an unordered pair of disjoint edges, respectively.
Similarly, if $|W| = 3$, by the hypoyhesis 3 edges can induce only subhypergraphs of types a), b) or c). We obtain that $N(n-3h+2r+1), N(n-3h+r+2,3)$ and $N(n-3h+3,3)$ count the subhypergraphs of H induced by an unordered triple of edges $\{D,E,F\}$ such that these subhypergraphs are isomorphic to $P_3^{h,r}$, $P_2^{h,r}$ and an isolated edge and 3 isolated edges, respectively. We also have $n-3h+2r+1 > n-3h+r+2 > n-3h+3$ since $r \geqslant 2$ and $N(n-3h+2r+1,j) = 0$ for every $0 \leqslant j \leqslant 2$.
If the edges D,E,F of H induce a $P_3^{h,r}$, where $D \cap F = \emptyset$, suppose that there exists an edge $G \neq D,E,F$ such that $G \subset D \cup E \cup F$. We have proved that $G \not\subset E \cup F$, thus implying $G \cap D \neq \emptyset$; similarly $G \not\subset D \cup F$ implies $G \cap E \neq \emptyset$. We have found 3 edges D,E,G such that $D \cap E \neq \emptyset, D \cap G \neq \emptyset$ and $E \cap G \neq \emptyset$, which contradicts the hypothesis. This implies that $N(n-3h+2r+1,j) = 0$ for every $j \geqslant 4$. Since by adding new edges to W the number of components $c(W)$ decreases, it follows that $P(H,\lambda)$ is given by (4), where β_3 is the number of induced subhypergraphs of H isomorphic to $P_3^{h,r}$. □

Theorem 5. *Let H be an antilinear h-hypergraph such that $P(H,\lambda) = P(G,\lambda)$, where G is $P_m^{h,r} (m \geqslant 1)$ or $C_m^{h,r} (m \geqslant 3)$. If $r \geqslant 2$ and $h \geqslant 3r-1$ then H is isomorphic to G.*

Proof. It is trivial to see that $P_m^{h,r}$ for $1 \leqslant m \leqslant 3$ and $C_3^{h,r}$ are h-chromatically unique. Let $m \geqslant 4$. We shall consider two subcases:
I. $G = P_m^{h,r}$ and II. $G = C_m^{h,r}$.
I. Let H be an antilinear h-hypergraph such that $P(H,\lambda) = P(P_m^{h,r},\lambda)$.
The order of an hypergraph is being determined by the leading term of the chromatic polynomial, it follows that H has order $n = h + (m-1)(h-r)$. From (2) one deduces that

$$P(P_m^{h,r},\lambda) = \lambda^n - m\lambda^{n-h+1} + \alpha_1\lambda^{n-2h+r+1} + \alpha_2\lambda^{n-2h+2} - \alpha_3\lambda^{n-3h+2r+1} + Q(\lambda), \quad (5)$$

where $Q(\lambda)$ is a polynomial of degree at most equal to $n-3h+r+2$, $\alpha_1 = m-1$ is the number of subpaths $P_2^{h,r}$ of length two, $\alpha_2 = \binom{m}{2} - m + 1$ is the number of pairs of pairwise disjoint edges and $\alpha_3 = m-2$ is the number of subpaths $P_3^{h,r}$ of length three in $P_m^{h,r}$. Also since any spanning subhypergraph of $P_m^{h,r}$ induced by less than m edges is not connected, it follows that in (5) the coefficient of λ is $(-1)^m$, which implies that H is also connected [15].
Since H has all edges of cardinality h, it follows that the number of components of a spanning subhypergraph of H may be $n, n-h+1$ or a smaller number. Any spanning subhypergraph of H with n vertices and $n-h+1$ components must contain only one edge. From (2) we deduce that $a_{n-h+1} = -N(n-h+1,1) = -|E(H)|$, hence H has exactly m edges. Every spanning subhypergraph of H with n vertices has two kinds of components: isolated vertices and components including at least h vertices. The components including at least h vertices will be called major components [10].
If such a spanning subhypergraph has at least two major components then it contains at most $n-2h+2$ components and this bound is reached when the major components

are two disjoint edges and minor components are $n-2h$ isolated vertices. It follows that all coefficients $a_{n-h+1},\ldots,a_{n-2h+r+1}$ given by (2) correspond to the case when all spanning subhypergraphs of H of order n contain only one major component. In this way $N(n-h,j)$ counts the spanning subhypergraphs of H consisting of a subset Y of vertices (the major component) and $n-h-1$ isolated vertices, where $Y\subset V(H), |Y|=h+1$. Denote by $\varphi(Y)$ the number of edges included in Y. Because Y induces a component having $h+1$ vertices, it follows that $\varphi(Y)\geqslant 2$ and for each $i\geqslant 2$ the union of any i edges included in Y equals Y. Since $N(n-h,0)=N(n-h,1)=0$, by (2) we get

$$a_{n-h}=\sum_{j\geqslant 2}(-1)^j N(n-h,j)=\sum_{j\geqslant 2}(-1)^j\sum_{|Y|=h+1,\,\varphi(Y)\geqslant 2}\binom{\varphi(Y)}{j}$$

$$=\sum_{|Y|=h+1,\,\varphi(Y)\geqslant 2}\ \sum_{j\geqslant 2}(-1)^j\binom{\varphi(Y)}{j}=\sum_{|Y|=h+1,\,\varphi(Y)\geqslant 2}(\varphi(Y)-1).$$

Since $a_{n-h}=0$ it follows that no such Y can exist, or equivalently, for any two distinct edges E,F we have $|E\cup F|\geqslant h+2$. If $Y\subset V(H), |Y|=h+2$ and $E,F\in E(H), E\neq F$ and $E,F\subset Y$ we get $E\cup F=Y$ since $|E\cup F|\geqslant h+2$. Since $a_{n-h-1}=0$ we deduce in the same way that $|E\cup F|\geqslant h+3$ and by induction we obtain that for any two distinct edges $E,F\in E(H)$ we have $|E\cup F|\geqslant 2h-r$, or $|E\cap F|\leqslant r$.
Let now $Y\subset V(H), |Y|=2h-r$ be a major component of a spanning subhypergraph of H such that Y contains exactly $j\geqslant 2$ edges. We shall prove that $j=2$. For this let $E,F\subset Y$ be two distinct edges such that $E\cup F=Y$. Suppose that there exists an edge $G, G\neq E,F$ such that $G\subset Y$. By denoting $a=|(E\backslash F)\cap G|$ and $b=|(F\backslash E)\cap G|$ we get $a+b\leqslant h$. Since $|G\cup E|=h+b\geqslant 2h-r$, $|G\cup F|=h+a\geqslant 2h-r$, it follows $a,b\geqslant h-r$, hence $a+b\geqslant 2h-2r$, which implies $h\geqslant 2h-2r$. But this contradicts the hypotheses $h\geqslant 3r-1$ and $r\geqslant 2$. For hypergraph H we can write

$$\sum_{|Y|=2h-r,\,\varphi(Y)=2}1=a_{n-2h+r+1}=m-1,$$

which implies that H contains exactly $m-1$ pairs of edges $\{E,F\}$ such that $|E\cap F|=r$, or $|E\cup F|=2h-r$.
Let p be such that $n-2h+2<p<n-2h+r+1$. If $Y\subset V(H), |Y|=n+1-p$ is a vertex subset inducing a major component of a spanning subhypergraph of H it follows that $2h-r<|Y|<2h-1$. For every three distinct edges E,F,G of H we have

$$|E\cup F\cup G|\geqslant |E|+|F|+|G|-|E\cap F|-|E\cap G|-|F\cap G|\geqslant 3h-3r,$$

since every two edges have at most r elements in common. But $3h-3r\geqslant 2h-1$ since $h\geqslant 3r-1$, which contradicts the property $|Y|<2h-1$. Hence one has $\varphi(Y)=2$. This yields

$$\sum_{|Y|=n+1-p,\,\varphi(Y)=2}1=a_p=0.$$

It follows that no such Y can exist, or for any two distinct edges E, F we cannot have $2h-r < |E\cup F| < 2h-1$, or $1 < |E\cap F| < r$. But H is antilinear, hence $|E\cap F| \neq 1$ and we have seen that $|E\cap F| \leqslant r$. It follows that $|E\cap F| = 0$ or r, i.e., H is also quasi-linear. Since H has m edges, is quasi-linear and connected, it may be obtained from $P_2^{h,r}$ by succesively adding $m-2$ distinct edges such that every new edge has r vertices in common with at least one existing edge.

We will define two potential functions, α and β, for any h-uniform hypergraph K of size m: $\alpha(K) = \alpha_1(K) - m$ and $\beta(K) = \alpha_3(K) - m$, where $\alpha_1(K)$ and $\alpha_3(K)$ are the numbers of induced subhypergraphs of K isomorphic to $P_2^{h,r}$ and to $P_3^{h,r}$, respectively. We have deduced that for every $m \geqslant 1$ $\alpha(P_m^{h,r}) = \alpha(H) = -1$. If K is an h-uniform quasi-linear hypergraph, then by adding a new edge $E \subset V(K)$, $E \notin E(K)$ which intersects at least an edge from $E(K)$, we get a new hypergraph K_1 and $\alpha(K_1) \geqslant \alpha(K)$. Equality holds if and only if E intersects exactly one edge from $E(K)$. Since $\alpha(H) = \alpha(P_2^{h,r}) = -1$, it follows that H is obtained from $P_2^{h,r}$ by adding $m-2$ distinct edges such that every new edge has r vertices in common with exactly one existing edge. This implies that every subhypergraph of H induced by three edges has one of types a), b) or c). Since $P(H,\lambda) = P(P_m^{h,r},\lambda)$, by Lemma 2.1 we obtain that $\alpha_3(H) = m-2 = \alpha_3(P_m^{h,r})$, hence $\beta(H) = \beta(P_m^{h,r}) = -2$. With the same notation as above we deduce $\beta(K_1) \geqslant \beta(K)$ and equality holds if and only if E intersects exactly one edge which belongs to exactly one path $P_2^{h,r}$, which is an induced subhypergraph of K, unless K_1 is a sunflower hypergraph. Now the proof follows by induction: if we add a new edge to $P_2^{h,r}$ (having $\alpha(P_2^{h,r}) = -1$) such that potential function α remains unchanged, we get $P_3^{h,r}$. Let $i \geqslant 3$; if we add a new edge to $P_i^{h,r}$ such that both potential functions α and β remain unchanged, this new edge must have r vertices in common only with a terminal edge of $P_i^{h,r}$ and one obtains in this way the hypergraph $P_{i+1}^{h,r}$.

II. In the case of cycles $C_m^{h,r}$ with $m \geqslant 4$ we deduce as above that polynomial $P(C_m^{h,r},\lambda)$ has $\alpha_1 = m, \alpha_2 = \binom{m}{2} - m, \alpha_3 = m$. If H is antilinear and chromatically equivalent to $C_m^{h,r}$ then H has order $m(h-r)$ and size m and it is connected. As in the case of paths $P_m^{h,r}$ we deduce that H has exactly m unordered pairs of edges $\{E, F\}$ such that $|E\cap F| = r$ and H is quasi-linear too. Also H may be built from $P_2^{h,r}$ in $m-2$ steps, each step consisting in addition of a new edge E, having r vertices in common with $t \geqslant 1$ existing edges $F_1, \ldots, F_t$, i.e., $|E\cap F_1| = \ldots = |E\cap F_t| = r$.

We have $\alpha(C_m^{h,r}) = \alpha(H) = 0$, but $\alpha(P_2^{h,r}) = -1$. Since at each step potential function α increases or remains constant, it follows that in one step α increases by 1 and in $m-3$ steps it remains constant (equal to 0 or -1). It increases by 1 when the new edge E intersects exactly two existing edges and remains constant when E intersects exactly one existing edge. Suppose that E intersects exactly two existing edges, F_1 and F_2, i.e., $|E\cap F_1| = |E\cap F_2| = r$ and $|F_1\cap F_2| = r$. We shall prove that this case is not possible, i.e., we must have $F_1\cap F_2 = \emptyset$. Suppose that $|F_1\cap F_2| = r$ and denote $i = |E\cap F_1\cap F_2|$. It follows that $0 \leqslant i \leqslant r$, $|E\cap(F_1\backslash F_2)| = |E\cap(F_2\backslash F_1)| = r-i$. In this case E contributes $h-2r+i = |E\backslash(F_1\cup F_2)|$ new vertices. Since $P_2^{h,r}$ and H have $2h-r$ and $m(h-r)$ vertices respectively, and whenever α remains unchanged the new edge contributes $h-r$ new vertices ($m-3$ times), we obtain that $i = 0$, which means that E, F_1, F_2 induce a

subhypergraph isomorphic to $C_3^{h,r}$.
In this case H has the property that all subhypergraphs induced by 3 edges have the types a), b), c) and exactly one subhypergraph is isomorphic to $C_3^{h,r}$. A result similar to Lemma 2.1 also holds and the contribution of the spanning subhypergraph of H consisting of $C_3^{h,r}$ and $n-3h+3r$ isolated vertices is $-\lambda^{n-3h+3r+1}$, which must be added to the polynomial given by (4). We have $n-3h+3r+1 > n-3h+2r+1$ and $n-3h+3r+1 < n-2h+2$, unless $h = 3r-1$. If $n-3h+3r+1 < n-2h+2$ the monomial $-\lambda^{n-3h+3r+1}$ does not appear in $P(C_m^{h,r}, \lambda)$; if $h = 3r-1$ the coefficient of λ^{n-2h+2} equals $\alpha_2 - 1 = \binom{m}{2} - m - 1$, a contradiction.
Consequently, E intersects two existing edges F_1, F_2 such that $F_1 \cap F_2 = \emptyset$, which implies that H contains an induced subhypergraph H_1 which is isomorphic to a cycle $C_s^{h,r}$ with $4 \leqslant s \leqslant m$. If $s = m$ then H is isomorphic to $C_m^{h,r}$ and we are done. Otherwise, H may be obtained from $C_s^{h,r}$ by succesively adding $m-s$ distinct edges such that every new edge has r vertices in common with exactly one existing edge. We have $\beta(C_s^{h,r}) = 0$; at the first step we get $\beta(C_s^{h,r} + E) = 1$ if E is such an edge. Since β is increasing, we deduce $\beta(H) \geqslant 1$. But in this case every 3 edges of H induce a subhypergraph of type a), b) or c), which implies that $\beta(H) = \beta_3 - m = \alpha_3 - m = 0$, a contradiction.

□

Note that $P_m^{h,r}$ is not chromatically unique for any $m \geqslant 3, r \geqslant 1$ and $h \geqslant 2r+1$, since any hypergraph containing a pendant path of length at least two is not chromatically unique [14].

Since every quasi-linear hypergraph is antilinear for every $r \geqslant 2$ we get:

Corollary 6. *Let $r \geqslant 2, h \geqslant 3r-1, m \geqslant 3$ and H be a quasi-linear hypergraph such that $P(H, \lambda) = P(G, \lambda)$, where G is $P_m^{h,r}$ or $C_m^{h,r}$. Then H is isomorphic to G.*

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