Algebraic properties of chromatic roots

Revisions

January 17, 2017

Reviewer p2 The sentence is a complex number α which assigns the name α to the root which is never used

Response Removed the "α".

Reviewer p2 Suggest replacing which we now outline with as we now outline

Response Done

Reviewer p4 Is the Golden Ratio really $(\sqrt{5}-1)/2$? It seems more common to use this term to denote $(1+\sqrt{5})/2$.

Response Replaced with "The complex number $\alpha = (\sqrt{5}-1)/2$ is an algebraic integer, since it satisfies $\alpha^2+\alpha-1=0$. It is not a chromatic root, as it lies in $(0,1)$."

Reviewer p4 A recent paper on arxiv has given a graph with a chromatic root at $\alpha+3$ which is presumably easy to verify, so these paragraphs need to be brought up to date (I guess this appeared between submission and reviewing)

Response This result was found after submission of the paper. We have updated the paragraph as follows: "Recently, Royle showed that $\alpha+3$ is a chromatic root [18] (the smallest graphs having chromatic root $\alpha+3$ have 11 vertices) and hence so is $\alpha+n$ for any natural number $n \geq 3$. We will see that the smallest graph having chromatic root $\alpha+4$ has eight vertices. "

We have also removed the last sentence in the Remark, namely, "Perhaps resolving

Reviewer I think some sort of end-of-proof symbol would be useful to the reader, as on several occasions the proof flows seamlessly into the subsequent text, making it hard to skip a proof and generally navigate the paper.

Response Done.

Reviewer I think that the graphs given as indices into McKays list should be given explicitly either as a graph picture or (second best) as a binary string representing the adjacency matrix. There are only a few graphs to draw, and there are no space restrictions here. If the reader can easily reproduce the graphs, it makes the paper self-contained, whereas the long term availability of the list of graphs in precisely that order is unclear.

Response We have added pictures of the graphs.

Reviewer p7 Should say "The intersection of all the sets F_i is empty" for otherwise the reader wonders why the word "all" has been included in one of the conditions, but not the other.

Response Fixed

Reviewer p8 Woodall was the first to find the chromatic polynomials of the complete bipartite graphs (and thereby demonstrate that real chromatic roots are not bounded as a function of chromatic number) so his work should probably be mentioned, or at least cited.

Response We have added "Woodall [23] showed that the real chromatic roots of complete bipartite graphs can be arbitrarily large. " in the section on real roots.

Reviewer p11 consecutive path of lengths should probably be paths of consecutive lengths.

Response Done

Reviewer p11 the definition says where $a \geq 2$ but there is no a occurring anywhere in the definition

Response Replaced a with s.

Reviewer p11 Is Conjecture 5.1 a companion to the $n\alpha$ conjecture (that $n\alpha$ is a chromatic root for all n) or is it more a companion to the $n + \alpha$ conjecture (that $n + \alpha$ is a chromatic root for some n)?

Response This conjecture has the flavour of parts of each of these conjectures, so replaced this sentence with: "This leads to the following question, a companion to the earlier α conjectures: "

Reviewer p12 the sentences on maxmaxflow seem a bit out of place – the term is not defined (or rather, the definition is incomplete) and the conclusions are really about the location of chromatic roots rather than their algebraic properties. I think that it makes more sense to either address this properly or to omit that portion

Response Added definition of maxmaxflow, namely, "Let $\lambda(u, v)$ be the number of edge-disjoint paths between vertices u and v in graph G. The *maxmaxflow* of G is $\Lambda = \max_{u,v \in V(G), u \neq v} \lambda_G(u, v)$. "

Reviewer p12 The section on Galois groups seems a little under-developed. I would have liked the authors to discuss (even speculate on) aspects such as whether there is likely to be a link between graph structure and the Galois groups? Or is it just a case that chromatic polynomials have no particular structure and finding all transitive groups as Galois groups of chromatic polynomials is just a matter of finding enough families of graphs. For example, what are the graphs with dihedral groups?

Response The section on Galois groups covers the current results including links to work on Galois groups of theta graphs and all graphs of small order.

Reviewer p12 In the section on Galois groups, the authors mention that they are limited to degree 15, but earlier they indicated testing the polynomials $F_{2,n}$ associated with the complete bipartite graphs for $n \leq 100$.

Response Changed "We note in passing that the Galois group of each of these irreducible polynomials that we tested is the symmetric group." to "We note in passing that the Galois group of each of these irreducible polynomials that we were able to test is the symmetric group." for clarity.

Reviewer p14 Dong and Koh have an interesting conjecture that a 4-chromatic graph with linear factors is chordal, and they have proved this for planar graphs; this seems to fit with the theme of Section 6 and I think would be of interest to readers, especially as their paper is not easy to obtain.

Response Added " However, Dong and Koh [9] showed that every planar graph that has only integer chromatic roots is chordal; they conjectured that the same conclusion holds for every graph with chromatic number at most four that has only integer roots."