Author's Response to the Reviewer Comments

We would like to thank the reviewer for his/her exceptionally thorough review of our paper. We have made all the changes that the reviewer has suggested.

In the following, we go through each of the reviewer's comments. The *brown* text is copy-pasted from the reviewer's report.

> *The introduction is extremely short and technicalities start before the reader can really feel convinced that the questions studied in the paper are interesting.*
>
> *The comparison to previous work is minimal.*

We have extended the introduction section to include a more thorough discussion of previous work. We also moved motivation text from the conclusions into the introduction (cf. item #31 below).

> *Since the rank of M(n,p) is n with high probability (Bourgain, Vu and Wood 2009) for a huge range of p(n), so will be the nonnegative rank of M(n,p). Thus I am not sure that the present random model is that interesting for that area.*

The reviewer misses the fact that, in Combinatorial Optimization, the matrix whose nonnegative rank one is interested in, is not the 01-matrix; it is a slack matrix, and generally has all kinds of numbers for entries. The rank of that matrix is typically very low ($\sim$ the dimension of the polytope), so lower bounding the nonnegative rank through the rank generally fails.
We have added that explanation to the text.

> *I found that a lot of proofs in the paper are tedious to read.*

Checking the technical estimates and calculations is tedious, and we would like to thank the referee for working through them diligently. As so often when analyzing random structures, the technical estimates and calculations just "need to be done" to determine the values of the interesting parameters – we don't think there is a way to avoid them.

> *I think that the paper should be accepted for publication in EJC, after addressing the comments below.*

Detailed comments:

1. *p1: log(rc(M))* $\surd$
2. *p2: what is H_M? Wheres the definition?*

   Reference added.
3. *p2: "the clique number of G(n,1/2) was shown [7] to be $\Theta(\log n)$": with high probability? (A similar remark applies many many times below.)* $\surd$
4. *p2: (footnote) There is a missing tilde in the url.* $\surd$
5. *p3: Before Lemma 2.2 you talk about generation of hypergraphs, in the proof you talk about separation of hypergraphs. Please make up your mind and propagate one of the two terminology to the whole paper!* $\surd$
6. *p4: (structure of G(n,p) for extremely small or extremely large p): you should not rush too mush and explain this.*

   Proposition w/ proof sketch added.
7. *p4: denoting the number of edges of G(n,p) by M is OK I guess, but it is a bit unfortunate that this introduces a notation clash with the binary matrix M.*

   Uh — true. Fixed!

8. *p4: $(n^2)_4$: is this a falling factorial? You should say so.* √
9. *p5: adding a reference for McDiarmids inequality would be a plus.* √
10. *p5: (sentence after Prop 3.1) Theres a lot of punctuation here. (Too much for my taste.)* √
11. *p5: Theorem 4.1 (a): is this exactly the statement in Park et al? If not, what is the difference? If yes, then why reprove it here?*

    Thanks for pointing this out. Park-Szpanowski don't deal with the case $p \to 0$ at all, and their proof does not work for non-constant p. We have added a corresponding note.
12. *p5: "The following makes the quantities a little clearer": I would change the formulation.* √
13. *p7: (first displayed eq on top): please state once here what upper bound you are using for binomial coefficients.* √
14. *p7: "the LB2": this is unclear.*

    Fixed.
15. *p8: (Lemma 4.7)*

    Fixed! We decided to replace the Chernoff bound in the following lemma (was 4.8 now 4.7; case (ii), subcase a=2,3) by one where no additional estimates are required.
16. *p8: "Now there exists a constant"* √
17. *p10: "if $n \geq \bar{p}n$": why say this? Isnt $\bar{p}$ a probability?* √
18. *p11: "The "1" on the RHS of the minimum": double-check the english.* √
19. *p11: "inside the parentheses"* √
20. *p11: "as $\delta \geq u - 1$" (no dot)* √
21. *p12: "cross-free matching": recall the definition here?* √
22. *p13: Corollary 5.3: how tight are these bounds? Could you comment?*

    We added a discussion of the ratio between upper and lower bounds.
23. *p16: "Item (a) of the lemma"* √
24. *p16: "cross-free"* √
25. *p16: "equation (1)": earlier you use "Eqn. (1)" → homogenize?* √
26. *p18: ". A convenient definition" (space missing)* √
27. *p19: The conditioning in (16) looks a bit funny. Cant you just condition on the (k,l)-entry to be 1?*

    We prefer this notation for this situation where there are two "sources of randomness": the random matrix $M^{n,p}$ and the random 1-rectangle R sampled according to the distribution π. Conditioning on $M^{n,p}$ makes clear that the probability is taken only over π, while at the same time not obscuring the fact that π depends on $M^{n,p}$ (which, we believe, would be the case with the alternative notation "$\mathbf{P}_\pi$").
28. *p19: "$Z := \max_\ell Z_\ell$"* √
29. *p20 (top): "Proof (of the lower bound in (a))"* √
30. *p21: "there is a large gap between our upper and lower bounds": where are the lower bounds?*

    Added reference.
31. *p22: Section 7: I think that a lot of the discussion here could easily be moved to the introduction, to motivate the results better.* √
32. *p22: "where the fractional chromatic number might be a weak lower bound": perhaps cite Stefan Weltges thesis here?* √