

**TO REVIEWER B: REVISIONS OF  
NEW COMBINATORIAL FORMULAS FOR CLUSTER MONOMIALS OF TYPE  $A$   
QUIVERS.**

We made the following revisions suggested by the referee.

Page 1, "the coefficients in any cluster variable": The authors should clarify that they mean the coefficients in the Laurent expansion with respect to a fixed cluster.

— Done.

Page 6, "Two pseudo-diagonals are said to be crossing if they intersect in the interior of the polygon": Do the authors mean that the corresponding diagonals do not cross in the interior of the polygon. (The given definition is not invariant under isotopy.)

— Done. We defined a "pseudo-diagonal" to be a curve instead of an isotopy class of curves; so two pseudo-diagonals are crossing if they intersect in the usual sense. We added the following statement to clarify the situation:

"If two pseudo-diagonals are not crossing, then the corresponding diagonals either coincide or do not cross in the interior of the polygon. "

Page 7, "identified with a finite set": Do the authors mean a finite multiset? (The same pseudo-diagonal can appear more than once.)

— Done. We assume the pseudo-diagonals to be pairwise distinct; it can always be done by perturbing. We revised by saying:

Using this bijection, a cluster monomial of  $\mathcal{A}(Q)$  can be identified with a finite set of pairwise non-crossing (and non-identical) pseudo-diagonals, or equivalently with

$$\{(D_1, d_1), \dots, (D_m, d_m)\},$$

where  $m$  is a non-negative integer,  $D_1, \dots, D_m$  are pairwise non-crossing diagonals, and  $d_1, \dots, d_m$  are positive integers. It is easy to see that given  $\{(D_1, d_1), \dots, (D_m, d_m)\}$ , we can always find a finite set of pairwise non-crossing (and non-identical) pseudo-diagonals, such that  $d_i$  of these pseudo-diagonals are isotopic to  $D_i$  for  $1 \leq i \leq m$ .

Page 7, "It is easy to see that the d-vector [...]": The referee does not see the argument. It would be helpful to provide a short proof of Property A.

— Done. A detailed proof is added.

Page 8, "It is straightforward to prove [...]": It would be helpful to provide a short proof of Proposition 3.2 (especially the case of negative entries in the d-vector.)

— Done. A detailed proof is added. The case of negative entries in the d-vector is handled in Step 1 of the proof of Proposition 3.2.

Page 20, "3-cycle  $v_i \rightarrow v_{i+1} \rightarrow v_{i,i+1}$ ": This is a path of length 2 and not a 3-cycle.

— Done. Changed to  $v_i \rightarrow v_{i+1} \rightarrow v_{i,i+1} \rightarrow v_i$ .

Page 21, the symbol "not in" in the last paragraph of the proof of Lemma 7.1 should read "not equal".

— Done.

Page 22, "Sing(D)": It would be helpful to include a definition.

— Done.

Page "22", "A consistent scattering diagram D is a scattering diagram if" should read "A scattering diagram D is a consistent scattering diagram if".

— Done. Changed to "A *consistent scattering diagram*  $\mathfrak{D}$  is a scattering diagram such that  $\theta_{\gamma, \mathfrak{D}}$  only depends on the endpoints of  $\gamma$ ."

Page 23, Remark 7.4: There are more opening brackets than closing brackets.

— Done.

Page 25, " $\ll$ ": It would be helpful to include a definition.

— Done. We added "(Here  $x \ll y$  means  $0 < x/y \leq \varepsilon$ , where  $\varepsilon > 0$  is a fixed real number satisfying  $(1 + \varepsilon)^n < 2$ .)" to Theorem 7.10, and rewrite the proof of Theorem 7.10 accordingly. We changed the statement of Theorem 7.13 similarly.

Page 25, Theorem 7.10 (iv): Do the authors mean "zmLi"?

— Done. It should be  $z^{m_i}$  instead of  $z^{m_L}$ .

Page 25: The authors might check the sentence structure in the first paragraph of the proof of Theorem 7.10.

— Done.

Page 27, "a adjustable" should read "an adjustable".

— Done.

Page 27, Theorem 7.10 (iv): Do the authors mean "z? Li"?

— Done. It should be  $z^{\tilde{m}_i}$  instead of  $z^{\tilde{m}_L}$ .

Page 34, "domain of linearity" should read "domains of linearity".

— Done.

Page 34, "The monomial attached to" should read "The monomials attached to".

— Done.

Page 34, The reader already knows the second paragraph of the Appendix from Section 3.

— Done. We removed the second paragraph.

Page 35, "The last two set is" should read "The last two sets are".

— Done.

Page 35, "The process of constructing ... ": The authors might want to rewrite this paragraph. Several things remain unclear. It is not clear what is meant by "placing the diagonal". When diagonals are rotated, it is not clear around which point they are rotated.

— Done. We rewrote the paragraph.

Page 35-36, "natural graph homomorphism": The authors specify the homomorphism only on edges. How is it defined on vertices?

— Done. We explained how the images of the vertices are determined.

Page 37, the map " $\psi_{P,G}$ " is not self-inverse.

— The referee may misread as  $\psi_{P,G} = \psi_{P,G}^{-1}$ . To avoid confusion, we rewrite "=" as "⋮=" so it is now  $\psi_{P,G} \coloneqq \psi_{G,P}^{-1}$ .

Page 40, "Schiffer" should read "Schiffler" (five occurrences).

— Done.