Comment by the authors, May 23, 1995

We thank J. B. Shearer for pointing out that Theorem 3 of our paper is not new (see [1], [2], [3]), and that the argument given in our paper actually proves the result with c=1/64, which is weaker than the best known [3] value $c = 1/7$.

References

[1] B. Bollobás and P. Erdős, *Alternating Hamiltonian Cycles*, Israel Journal of Mathematics, 23 (1976), pp. 126-131, (c=1/69).

[2] C. C. Chen and D. E. Daykin, *Graphs with Hamiltonian Cycles Having Adjacent Lines Different Colors*, Journal of Combinatorial Theory B, 21 (1976), pp. 135-139, (c=1/17).

[3] J. B. Shearer, *A Property of the Complete Colored Graph*, Discrete Mathematics, 25 (1979), pp. 175-178, (c=1/7).

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There is a mistake in Lemma 2 of the paper, which requires halving the constants in Theorems 1,2,and 4. The constant in Theorems 1 and 4 should be changed from $c < 1/32$ to $c < 1/64$, and the constant in Theorem 2 should be changed from $c < 1/64$ to $c < 1/128$.

As stated in the paper, Lemma 2 reads as follows:

Let e, f be edges of K_n and $X \subseteq E(K_n)$ be such that no edge in X shares an endpoint with either e or f. Then we can find, for each Hamilton cycle C containing both e and f and no edges of X, a set $S(C)$ of $(n-6)(n-9)/2$ Hamilton cycles containing neither e, f or any edge in X, in such a way that if $C \neq C'$ then $S(C) \cap S(C') = \emptyset$.

We show with a counterexample that $S(C)$ can be no greater than $n^2/4(n-3)(n-4)$, and then give an algorithm which associates $(n-6)(n-7)/4$ distinct cycles without e, f, or any edges of X with each distinct cycle containing e, f, and no edges of X.

Counterexample:

We pick a set X such that the ratio of the number of Hamilton cycles not containing e, f, or any edges of X to the number of Hamilton cycles containing e and f and no edges of X is less than $n^2 : 4(n-3)(n-4)$.

Let e_0, e_1,and f_0, f_1 be the endpoints of e and f, respectively. Let S be a cycle on the $n-4$ nodes of $K_n \backslash \{e_0, e_1, f_0, f_1\}$. Let X contain all edges in K_n except those in S and those which share endpoints with e or f. Then the number of Hamilton cycles containing both e and f and no edges in X is $4(n-4)(n-3)$. Proof: In any such cycle, nodes other than endpoints of e and f may be adjacent only to their neighbors in S or to the endpoints of e and f. So all such cycles may be constructed by inserting e and f into S as follows.

First pick an edge (a, b) in S, and substitute one of the node sequences $< a, e_0, e_1, b >, < a, e_1, e_0, b >$ for the sequence $< a, b >$. Call the resulting cycle S'. Then pick an edge (c, d) in $S' \backslash \{e\}$, and insert f by substituting one of the node sequences $< c, f_0, f_1, d >, < c, f_1, f_0, d >$ for $< c, d >$. There are $(n-4)$ places to insert e in S, 2 ways to orient e in that position, $(n-3)$ places to insert f in $S' \backslash \{e\}$, and 2 ways to orient f, which makes $4(n-3)(n-4)$ cycles in all.

The number of cycles not containing e, f, or edges in f is less than n^4. Proof: The number of such cycles is just the number of ways to insert the four nodes e_0, e_1, f_0, f_1 into S without making e_0 and e_1 or f_0 and f_1 adjacent. Let $D = \{e_0, e_1, f_0, f_1\}$. There are $(n-4)(n-5)(n-6)(n-7)$ Hamilton cycles with none of the nodes in D adjacent to each other; if we add in the cycles with one or more pair of nodes in D adjacent, the number added is $O(n^3)$; the total number will still turn out to be less than n^4. Thus there are fewer than $n^4/4(n-3)(n-4)$ Hamilton cycles not containing e, f, or edges in X for each

Hamilton cycle containing e, f, and no edges of X.

Construction: Let C be a Hamilton cycle containing e and f, and no edges in X. We associate with C a set $T(C)$ of Hamilton cycles which don't contain e, f, or any edges of X, constructed as follows. The new cycles are created by taking one endpoint each of e and f, and moving them to new positions, leaving the order of all other nodes fixed.

1. Orient C so that the lower numbered of the two nodes adjacent to e_0 and e_1 follows e. Suppose without loss of generality that e_1 then follows e_0.

2. Pick any two edges of C other than the 6 edges incident with e_0, e_1, f_0, and f_1. There are $\binom{(n-6)}{2}$ ways to do this. Of the two chosen edges, let (a, b) be the first edge following e_0 in the given orientation of C; let (c, d) be the other edge. (If e_1 preceded e_0 in C, we would let (a, b) be the first edge preceding e.)

3. Remove e_1 from its original position and insert it between a and b.

4. Choose one of f_0 and f_1 to move between c and d, choosing so as to preserve the order in which e_0, f_0, and f_1 appear in the cycle.

Call the new cycle C'. Notice that it is possible to reconstruct the original positions of both e_0 and e_1 by looking at C': e_0 hasn't moved, and the original position of e_1 is given

by sliding it back around the cycle toward e_0, in whichever direction passes neither f_0 nor f_1. It is not possible, however, to reconstruct the original positions of f_0 and f_1; their relative positions are correct, but it is impossible to tell whether f_0 or f_1 has been moved. Thus each of the $(n-6)(n-7)/2$ cycles in $T(C)$ is associated with two cycles: our original cycle C, and a cycle identical to C except that the edge f is positioned between nodes c and d (in the same orientation as in C). Thus for each distinct cycle C containing e, f, and no edges of X, there are $(n-6)(n-7)/4$ distinct cycles not containing e, f, or any edges of X.