Independent Dominating Sets
in Planar Triangulations

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Abstract

In 1996, Matheson and Tarjan proved that every near planar triangulation on n vertices contains a dominating set of size at most $n/3$, and conjectured that this upper bound can be reduced to $n/4$ for planar triangulations when n is sufficiently large. In this paper, we consider the analogous problem for independent dominating sets: What is the minimum ε for which every near planar triangulation on n vertices contains an independent dominating set of size at most εn? We prove that $2/7 \leqslant \varepsilon \leqslant 5/12$. Moreover, this upper bound can be improved to $3/8$ for planar triangulations, and to $1/3$ for planar triangulations with minimum degree 5.

Mathematics Subject Classifications: 05C10, 05C69

1   Introduction

Let S be a set of vertices of a graph G. We say that S is *dominating* if each vertex of G is in S or is a neighbor of some vertex in S; and that S is *independent* if no two vertices in S are adjacent. In particular, any maximal independent set in G is dominating. We denote by $\gamma(G)$ (resp. $\iota(G)$) the cardinality of a minimum dominating (resp. independent dominating) set of G. Note that $\gamma(G) \leqslant \iota(G)$. Such parameters are known as the *domination number* and the *independent domination number* of G, respectively, and their calculations are known to be NP-hard problems even on planar bipartite graphs with maximum degree 3 [14, Corollary 3]. Therefore, it is natural to explore such parameters in special classes of graphs, or to look for upper and lower bounds on them.

In this paper, we focus on *planar* graphs, i.e., graphs that can be drawn in the plane so that intersections of edges happen only at their ends. By a *plane* graph we mean a planar graph together with a fixed planar drawing of it. For general terminology on planar graphs we refer to the book of Diestel [3]. In particular, a *planar triangulation* is a

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plane graph in which each face is bounded by a triangle; and a *triangulated disk* or a *near (planar) triangulation* is a 2-connected plane graph in which each face, except possibly its outer face, is bounded by a triangle. In 1996, Matheson and Tarjan [9] proved that every near triangulation G on n vertices satisfies $\gamma(G) \leqslant n/3$, and posed the following conjecture.

Conjecture 1 (Matheson–Tarjan, 1996). For every sufficiently large planar triangulation G on n vertices, we have $\gamma(G) \leqslant n/4$.

Conjecture 1 is best possible since, for any planar triangulation G that consists of a triangulation of k vertex-disjoint copies of K_4 embedded so that each copy of K_4 is in the outer face of all other copies of K_4, we have $\gamma(G) = k = n/4$. In spite of the efforts of specialists [7, 8, 10, 11], the best general upper bound found so far for $\gamma(G)$ is due to Špacapan [13], who proved that $\gamma(G) \leqslant 17n/53$ for every planar triangulation G on $n \geqslant 6$ vertices. Observe also that the bound of Conjecture 1 is not satisfied for near triangulations [9, Figure 1]. Related results for maximal outerplanar graphs have been given in [2, 12].

We are interested in the analogous problem for the independent domination number: What is the minimum ε such that $\iota(G) \leqslant \varepsilon n$ for every near triangulation G on n vertices? In contrast to the domination number, this parameter has not received so much attention on near triangulations. It is known that $\iota(G) < 3n/4$ for any connected planar graph G on $n \geqslant 10$ vertices [5, Theorem 6]; and that $\iota(G) \leqslant n/2$ whenever G is planar and $\delta(G) \geqslant 2$ [5, Theorem 8]. Also, if G is a 2-connected outerplanar graph on $n \geqslant 5$ vertices, then $\iota(G) \leqslant \frac{2n+1}{5}$ [6, Theorem 1]. For an excellent survey on independent dominating sets, see [4].

Now, note that since every Eulerian planar triangulation has chromatic number 3, they contain three disjoint independent dominating sets. Goddard and Henning [5, Question 1] asked whether such three sets exist in any planar triangulation. In particular, this would imply that $\iota(G) \leqslant n/3$ for every n-vertex planar triangulation G. We state the later statement as a conjecture.

Conjecture 2. For every planar triangulation G on n vertices, we have $\iota(G) \leqslant n/3$.

Our main contribution is the following theorem.

Theorem 3. *For every near triangulation G on n vertices, we have $\iota(G) \leqslant 5n/12$. Moreover, if G is a planar triangulation, then $\iota(G) < 3n/8$, and if G is a planar triangulation with $\delta(G) = 5$, then $\iota(G) \leqslant n/3$.*

We also show that the first two upper bounds are not less than $2n/7$, by presenting an infinite family of planar triangulations G for which $\iota(G) \geqslant 2n/7$ (see Theorem 10). Note that this improves an observation of Goddard and Henning [5, Figure 6], who presented an infinite family of planar triangulations G for which $\iota(G) \geqslant 5n/19$.

As a starting example, we prove that $\iota(G) \leqslant 2n/5$ for every planar triangulation G on n vertices due to a relation between r-dynamic and acyclic colorings as follows. A *k-coloring* of a graph G is a partition of $V(G)$ into k independent sets. Each part in

such a partition is called a *color class*. A coloring of G is *r-dynamic* if each vertex v has neighbors in at least $\min\{r, d(v)\}$ color classes, where $d(v)$ denotes the degree of v in G; and a coloring of G is *acyclic* if the union of any two of its color classes induces a forest. We use the following result of Goddard and Henning [5, Lemma 4].

Lemma 4 (Goddard–Henning, 2020). *For every graph G on n vertices with $\delta(G) \geqslant r$ for which there is an r-dynamic k-coloring, we have $\iota(G) \leqslant (k-r)\, n/k$.*

Borodin [1] showed that every planar graph admits an acyclic 5-coloring. Let G be a planar triangulation and χ_a be an acyclic 5-coloring of G. If G is a triangle, clearly $\iota(G) \leqslant 2n/5$. Otherwise, $\delta(G) \geqslant 3$ and the neighborhood of each vertex contains a cycle; hence, because χ_a is an acyclic coloring, every vertex has neighbors in at least three color classes of χ_a. Therefore, χ_a is 3-dynamic and, by Lemma 4, we have $\iota(G) \leqslant 2n/5$.

2   An improved upper bound

In this section, we prove the main theorem of this paper, Theorem 3. For that, we introduce a concept and settle some notation. For a vertex v in G, denote by $N(v)$ the set of neighbors of v in G. For a vertex set S of G, denote by $N(S)$ the set of all neighbors of vertices in S (that may also include vertices of S), and denote by $N[S]$ the closed neighborhood of S, that is, $N[S] = N(S) \cup S$.

Recall that a *near triangulation* is a 2-connected planar graph in which any face is bounded by a triangle, except possibly for the outer face, and note that every planar triangulation is a near triangulation. We start with some basic properties of near triangulations. In the next propositions, we consider a near triangulation G on $n \geqslant 4$ vertices with a fixed planar embedding and let O be the boundary of the outer face of G.

Proposition 5. *Let $u \in V(G)$ and let $G' = G[N(u)]$. Then G' contains a spanning cycle unless $u \in V(O)$ and O is not a triangle, in which case G' contains a spanning path.*

For the next two propositions, let S be an independent set in G, and put $H = G - S$. Since the neighborhood of every vertex of S in G contains a spanning path, any path of G that joins two vertices of H can be modified to a path in G that avoids S. This yields the following result.

Proposition 6. *H is connected.*

Now, let Y be the vertices of S in O and put $X = S \setminus Y$.

Proposition 7. *Vertices of Y lie in the outer face of H. Vertices of X lie in inner faces of H. Moreover, each inner face of H contains at most one vertex of X.*

Now, given a plane graph H, we denote by $f_i(H)$ the number of faces of degree i in H, that is, faces with i vertices in their boundary.

Lemma 8. *For every connected plane graph H, $f_4(H) + 2\sum_{i \geqslant 6} f_i(H) \leqslant |V(H)| - 2$.*

Proof. For every i, let $f_i = f_i(H)$. Note that, $\sum_{i \geqslant 3} i f_i = 2|E(H)|$ and the number of faces of H is $\sum_{i \geqslant 3} f_i$. Thus, by Euler's formula (e.g., [3, Theorem 4.2.9]), we have

$$\sum_{i \geqslant 3} f_i + |V(H)| = \frac{1}{2} \sum_{i \geqslant 3} i f_i + 2.$$

Hence

$$\begin{aligned}
|V(H)| - 2 &= \frac{1}{2} \sum_{i \geqslant 3} i f_i - \sum_{i \geqslant 3} f_i \\
&= \sum_{i \geqslant 3} \frac{i-2}{2} f_i \\
&\geqslant \sum_{i \geqslant 3} \frac{i-2}{2} f_i - \frac{1}{2} f_3 - \frac{3}{2} f_5 \\
&\geqslant f_4 + 2 \sum_{i \geqslant 6} f_i. \qquad \square
\end{aligned}$$

Although not needed here, we observe that Lemma 8 can be strengthen to the statement $f_4(H) + 2 \sum_{i \geqslant 6} f_i(H) \leqslant |V(H)| - 2 - \frac{f_3(H) + 3 f_5(H)}{2}$.

Proof of Theorem 3. Let G be a near triangulation on n vertices. The celebrated Four-Color Theorem assures that there exists a 4-coloring for G (e.g., [3, Theorem 5.1.1]). Let C_1, C_2, C_3, C_4 be the color classes in such a coloring. Each C_i is an independent set. If some C_i is empty, then each of the other color classes is non-empty and dominating, because each vertex is in a triangle. Therefore, the smallest of the three non-empty color classes is an independent dominating set of size at most $n/3 < 3n/8 \leqslant 5n/12$.

So suppose each C_i is non-empty and note that $n \geqslant 4$. For each i, let U_i be the set of vertices that are not dominated by C_i. Note that $U_1, U_2, U_3,$ and U_4 are pairwise disjoint, as the neighborhood of any vertex u is colored with at least two colors, distinct from the color used in u. We start by proving a stronger statement on $U_1, U_2, U_3,$ and U_4, namely that

$$N[U_i] \cap U_j = \emptyset \qquad \text{if } i \neq j. \tag{1}$$

Indeed, by contradiction, say $v \in N(U_1) \cap U_2$. Then $v \in C_3 \cup C_4$. Let u be a neighbor of v in U_1. As $v \in U_2$, we conclude that $u \in N(U_2) \cap U_1$, and hence $u \in C_3 \cup C_4$ also. Because G is a near triangulation, it is 2-connected. As u and v are adjacent, u and v have a common neighbor, say w. Since either $v \in C_3$ and $u \in C_4$, or $v \in C_4$ and $u \in C_3$, w has either color 1 or 2, contradicting the fact that $v \in U_2$ and $u \in U_1$.

Let S_i be an independent dominating set of $G[U_i]$ and let $S = S_1 \cup S_2 \cup S_3 \cup S_4$. Because each S_i is a subset of U_i, by (1), set S is independent in G. In order to use Propositions 5, 6, and 7, let O be the boundary of the outer face of G, let Y be the vertices of S in O, and let $X = S \setminus Y$. Finally, let $H = G - S$, $n' = |V(H)|$, and consider the plane embedding of H induced by G, which is connected by Proposition 6.

Claim 9. $2|X| \leqslant n' - 2 + f_4(H)$.

Proof. By Proposition 7, any vertex of X is in an inner face of H, and each such face has at most one vertex of X. Now, let $u \in X$, and let F be the face of H that contains u. Since $u \in U_i$ for some i, only two colors can appear in the vertices in the boundary of F, and hence the number of vertices in the boundary of F is even. Thus,

$$2|X| \leqslant 2\sum_{i \text{ is even}} f_i(H) \leqslant 2f_4(H) + 2\sum_{i \geqslant 6} f_i(H) \leqslant n' - 2 + f_4(H),$$

where the last inequality follows from Lemma 8. $\qquad\square$

Now, because $|S| = |X| + |Y|$, by Claim 9, we have

$$3|X| + |Y| = 2|X| + |S| \leqslant n' - 2 + f_4(H) + |S| = n - 2 + f_4(H). \qquad (2)$$

In what follows, we use $|O|$ to denote the number of vertices in O. Because Y is an independent set and O is a cycle, we have $|Y| \leqslant |O|/2$. Thus, from (2), we deduce that

$$3|S| = 3|X| + 3|Y| \leqslant n - 2 + f_4(H) + |O|. \qquad (3)$$

By the definition of S_i, each set $C_i \cup S_i$ is an independent dominating set. Moreover,

$$\sum_{i=1}^{4}(|C_i| + |S_i|) = n + |S| \leqslant \frac{4n - 2 + f_4(H) + |O|}{3}.$$

Therefore, the smallest of these four independent dominating sets has size at most $n/3 + (f_4(H) + |O| - 2)/12$. Let f_4' be the number of inner faces of degree 4 of H, and let X_4 be the set of vertices of S in such inner faces. Note that $f_4(H) \leqslant f_4' + 1$ because the outer face may have degree 4. Since G is a near triangulation, there is exactly one vertex of X_4 in each such inner face, so $f_4' = |X_4|$. By Proposition 7, the vertices in X_4 are in X, and hence not in O. So $|X_4| + |O| \leqslant n$ and $f_4(H) + |O| \leqslant f_4' + 1 + |O| = |X_4| + 1 + |O| \leqslant n + 1$. Thus, $\iota(G) \leqslant n/3 + (f_4(H) + |O| - 2)/12 < 5n/12$.

Now, if G is a planar triangulation, then $|O| = 3$ and $|Y| \leqslant 1$. Thus, from (2), we deduce that

$$3|S| \leqslant 3|X| + |Y| + 2 \leqslant n + f_4(H). \qquad (4)$$

The number of faces in G is $2n - 4$, and there are $4f_4(H)$ faces of G incident to the vertices of degree 4 in S. Therefore, $f_4(H) \leqslant (2n - 4)/4 < n/2$ and hence, by (4), we have $|S| < n/2$. By the same argument as before, we conclude that $\iota(G) \leqslant (n + |S|)/4 < 3n/8$. Furthermore, if G has minimum degree 5, then $f_4(H) = 0$ and hence, by (4), we have $|S| \leqslant n/3$. Hence $\iota(G) \leqslant (n + |S|)/4 \leqslant n/3$. $\qquad\square$

3 A lower bound

As far as we know, Theorem 3 might not be tight: we do not know a family of planar triangulations G on n vertices with $\iota(G)$ approaching $3n/8$. We improve the previous lower bound on ε given by Goddard and Henning [5, Figure 6] in the next result.

Theorem 10. *There is an infinite family $\mathcal{F}$ of planar triangulations such that $\iota(G) = 2n/7$ for every $G \in \mathcal{F}$, where n is the number of vertices in G.*

Proof. Consider the diamond graph depicted in Figure 1(a). Let us describe a family $\mathcal{F}$ of planar triangulations using this graph. Each planar triangulation in $\mathcal{F}$ consists of a circular chain of such diamond graphs, as depicted in Figure 1(b), with edges added to result in a planar triangulation. The planar triangulation G_k obtained in this way with k diamond graphs has $n = 7k$ vertices. The squared vertices in Figure 1(b) show an independent dominating set with $2k = 2n/7$ vertices. Note that any independent dominating set in such a planar triangulation G_k must contain at least two vertices in each diamond graph, therefore $\iota(G_k) = 2k = 2n/7$. $\square$
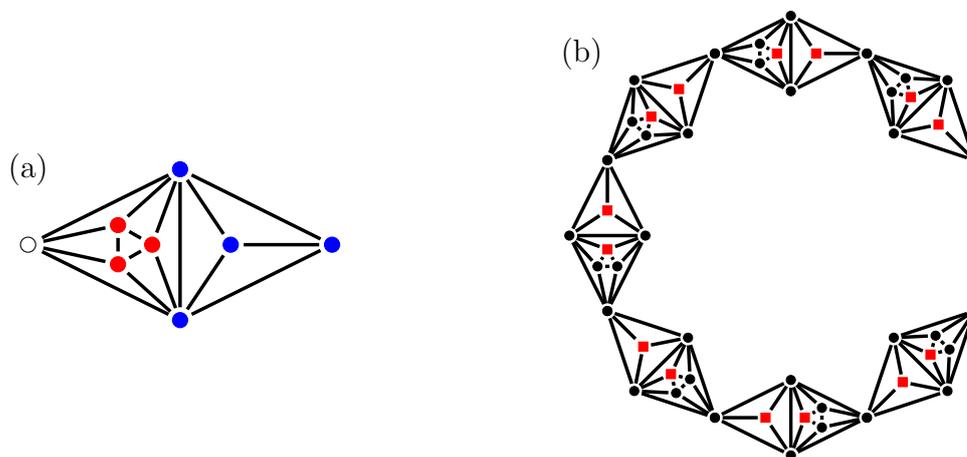

Figure 1: (a) A gadget consisting of seven vertices: the white vertex is not part of the gadget. Any independent dominating set has one of the red vertices, otherwise, being independent, it cannot dominate the three red vertices. Analogously, any independent dominating set has one of the blue vertices, otherwise it does not dominate the middle blue vertex. (b) A graph consisting of a circular chain of gadgets. In each copy of the gadget, two of its vertices are needed in any independent dominating set. The red squared vertices form an independent set with exactly two vertices in each gadget.

4 Further results and concluding remarks

In this section, we explore a few families of planar triangulations for which we can obtain better bounds on their independent domination number. A planar 3-tree is a planar

triangulation that can be obtained from a triangle by repeatedly choosing one of its faces and adding a new vertex inside of it while joining this new vertex to the three vertices of the face. It is not hard to prove that any planar 3-tree admits a 4-coloring in which each of its color classes is dominating. Thus $\iota(G) \leqslant n/4$ for every planar 3-tree on n vertices.

As we observed before, Conjecture 2 holds for any Eulerian planar triangulation. We can prove a better bound for a particular class of Eulerian triangulations, which we call *recursive Eulerian triangulations*, and define as follows. A recursive Eulerian triangulation is either a triangle, or a graph obtained from a recursive Eulerian triangulation by selecting one of its faces, drawing a triangle inside of it, and joining both ends of each edge of the new triangle with a different vertex of the selected face, so that the subgraph induced by the chosen face and the new triangle is isomorphic to the octahedron. It is not hard to check that recursive Eulerian triangulations have the following property.

Proposition 11. *In a recursive Eulerian triangulation of order at least 9, the degree-4 vertices induce a graph whose components have at most three vertices.*

Now, given a graph G, we denote by $\chi_r(G)$ the minimum number of colors in an r-dynamic coloring of G. Also, for a fixed coloring of G, we denote by $L_G(u)$ the set of colors that u does not see, i.e., that do not appear in the closed neighborhood of u.

Lemma 12. *Let G be a recursive Eulerian triangulation. There is a 5-dynamic 6-coloring of G such that, if u and v are adjacent degree-4 vertices, then $L_G(u) \neq L_G(v)$. In particular $\chi_5(G) \leqslant 6$.*

Proof. Let G be a recursive Eulerian triangulation on n vertices. We proceed by induction on n. If $n \leqslant 6$ then G is a triangle or the octahedron and the result is immediate. So, suppose that $n \geqslant 9$. Let abc be the last triangle added in the recursive construction of G.

Let $G' = G - abc$, and let xyz be the face of G' where abc lies. We may assume that $\{a, x\}, \{b, y\}$ and $\{c, z\}$ are independent sets. By induction hypothesis, there exists a 5-dynamic 6-coloring $\chi' = \{C_1', C_2', C_3', C_4', C_5', C_6'\}$ of G' as above. For every $u \in V(G')$, set $L'(u) := L_{G'}(u)$, $L(u) := L_G(u)$, $d'(u) := d_{G'}(u)$ and $d(u) = d_G(u)$. As every vertex has even degree, $|L'(u)| = 1$ if $d'(u) = 4$ and $|L'(u)| = 0$ if $d'(u) \geqslant 6$. Also, as G' has at least six vertices, every vertex in G' has degree at least 4.

Without loss of generality, assume that $x \in C_1'$, $y \in C_2'$, and $z \in C_3'$. We may also assume that $L'(x) \subseteq \{4\}$, $L'(y) \subseteq \{5\}$, and $L'(z) \subseteq \{6\}$. We define a new coloring χ by maintaining the same colors to the vertices in G' and assigning color 5 to a, color 6 to b, and color 4 to c. Note that χ is in fact a proper coloring for G. We claim that χ is a coloring as desired. Indeed, note that $L(x) = L'(x) \backslash \{4, 6\} = \emptyset$. A similar argument holds for y and z. Thus every vertex in G of degree at least 6 sees all colors. Also, $L(a) = \{1\}$, $L(b) = \{2\}$, and $L(c) = \{3\}$. $\square$

Theorem 13. *For every recursive Eulerian triangulation G on $n \geqslant 9$ vertices, we have $\iota(G) < 13n/42$.*

Proof. Let V_4 be the set of degree-4 vertices of G.

Claim 14. $6\,\iota(G) \leqslant n + |V_4|$.

Proof. Let $\chi = \{C_1, C_2, C_3, C_4, C_5, C_6\}$ be a 5-dynamic 6-coloring of G as in Lemma 12. For each i, let U_i be the set of vertices that are not dominated by C_i. Observe that all vertices with degree at least 5 are dominated by every color of χ. Hence, $\cup_{i=1}^{6} U_i$ is precisely the set of degree-4 vertices. Moreover, a degree-4 vertex has exactly four different colors on its neighborhood. Thus, the set $\{U_1, U_2, U_3, U_4, U_5, U_6\}$ partitions V_4. Also, by the property of χ stated in Lemma 12, two adjacent degree-4 vertices are not in the same set U_i. Hence, for every i, the set $C_i \cup U_i$ is independent. Therefore, $6\,\iota(G) \leqslant \sum_i |C_i| + \sum_i |U_i| = n + |V_4|$. $\qquad\square$

Claim 15. $7\,|V_4| \leqslant 6n - 12$.

Proof. Let $G' = G - V_4$. As G' is a plane graph, the number f' of faces of G' is at most $2|V(G')| - 4 = 2(n - |V_4|) - 4$. By Proposition 11, if there are vertices of V_4 inside a face of G', then there are at most three such vertices. Hence, $|V_4| \leqslant 3f' \leqslant 3(2n - 2|V_4| - 4)$ and the proof follows. $\qquad\square$

By Claims 14 and 15, we deduce that $\iota(G) \leqslant \frac{13n-12}{42}$. $\qquad\square$

We conclude by observing that, if every vertex of a planar triangulation G on n vertices has odd degree, then every color class of a 4-coloring of G is dominating, so $\iota(G) \leqslant n/4$. We can extend this result as in the following theorem.

Theorem 16. *Let G be a planar triangulation on n vertices. If G has at least αn odd-degree vertices, then $\iota(G) \leqslant (2 - \alpha)n/4$.*

Proof. Let C_1, C_2, C_3, C_4 be the color classes of a 4-coloring of G. For each i, let U_i be the set of vertices in G that are not dominated by C_i. Let $U = U_1 \cup U_2 \cup U_3 \cup U_4$ and observe that $\{U_1, U_2, U_3, U_4\}$ partitions U. Note that each vertex in U_i, and thus in U, has even degree. Therefore the set of odd-degree vertices of G has size at most $n - |U|$, and we must have $n - |U| \geqslant \alpha n$, which implies that $|U| \leqslant n - \alpha n = (1 - \alpha)n$.

For each i, let S_i be an independent dominating set of $G[U_i]$. The set $C_i \cup S_i$ is an independent dominating set of G. Thus $\iota(G) \leqslant \min_i\{|C_i| + |U_i|\}$. Therefore,

$$4\iota(G) \;\leqslant\; \sum_{i=1}^{4}(|C_i| + |U_i|) \;=\; n + |U| \;\leqslant\; n + (1 - \alpha)n \;=\; (2 - \alpha)n,$$

as desired. $\qquad\square$

Theorem 16 improves the result in Theorem 3 when $\alpha > 1/2$. Also, Conjecture 2 holds for any n-vertex planar triangulation with at least $2n/3$ odd-degree vertices.

Acknowledgements

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