

Comment by Lawrence Sze, April 23, 1996

After this paper was written, Ken Ono (IAS) and I completed a paper on 4-cores entitled

"4-cores, characters of finite general linear groups, and class numbers."

This paper has now been submitted for publication. We show that 4-cores possess surprising arithmetical structure which is suggested by these congruences of Hirschhorn and Sellers.

Using *Abaci* theory, we obtain a structure theorem that classifies and parametrizes all 4-cores. In particular, the parts of any 4-core partition are explicitly determined. We obtain a map ϕ from the set of 4-core partitions of an integer n to the class group $CL(-32n-20)$. An application of the multiplicative properties of class numbers $h(D)$ produces the following property for $\alpha_1(n)$'s: If p is prime and $\text{ord}_p(N) \leq 1$ then

$$\alpha_1\left(\frac{Np^{2k}-5}{8}\right) = \left(1 + \frac{p^{k+1}-p}{p-1} - \frac{p^k-1}{p-1} \cdot \left(\frac{-N}{p}\right)\right) \cdot \alpha_1\left(\frac{N-5}{8}\right),$$

where $\left(\frac{-N}{p}\right)$ is the Legendre symbol and $\left(\frac{-N}{p}\right) = 0$ when $p|N$. This property clearly yields an infinite number of congruences in all primes to all positive powers on the $\alpha_1(n)$'s. Moreover, the map ϕ induces in a uniform way a structure on the set of 4-core partitions of n for each n . In particular, if $8n+5$ is prime, then ϕ induces a group structure on the set of 4-core partitions of n isomorphic to $CL^2(-32n-20)$.

The interested reader may request a preprint from sze@math.psu.edu or download one in *dvi* from <http://www.math.psu.edu/sze/>

Lawrence Sze

sze@math.psu.edu

<http://www.math.psu.edu/sze/>

April 12, 1996

Penn State University, Graduate Student