Improved identifying codes for the grid

Gérard Cohen
cohen@inf.enst.fr

Sylvain Gravier
Sylvain.gravier@imag.fr

Iiro Honkala
honkala@utu.fi

Antoine Lobstein
lobstein@inf.enst.fr

Michel Mollard
Michel.mollard@imag.fr

Charles Payan
Charles.payan@imag.fr

Gilles Zémor
zemor@infres.enst.fr

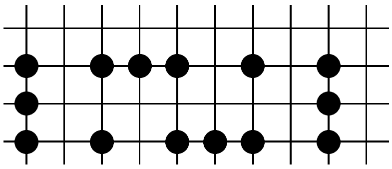
Abstract

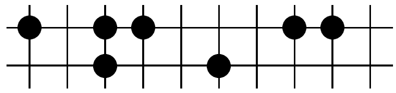
Let $G = (V, E)$ be an undirected graph and C a subset of its vertices. For any vertex $v \in V$, the neighbouring set $N(v, C)$ is the set of vertices of C at distance at most one from v. We say that C is an *identifying code* of G if the neighbouring sets $N(v, C), v \in V$, are all nonempty and different. What is the smallest size of an identifying code C ? We give improved constructions when G is the two-dimensional square lattice that we conjecture are optimal.

AMS subject classification: 05C70, 68R10, 94B99, 94C12.

G. Cohen, A. Lobstein and G. Zémor are with ENST and CNRS URA 820, Computer Science and Network Dept., Paris, France, S. Gravier, M. Mollard and C. Payan are with CNRS, Laboratoire Leibniz-IMAG, Grenoble, France, I. Honkala is with Turku University, Mathematics Dept., Turku, Finland

This short note is meant as an addendum to [1] on identifying codes in the infinite rectangular grid. For complete definitions and motivation we refer to that paper. The following two tiles





generate periodic tilings of the plane with periods $(10, 0)$ and $(1, 4)$ in the first case, and periods $(10, 0)$ and $(3, 2)$ in the second case. They are represented on figure 1. Both yield identifying codes with density $14/40 = 7/20 = 0.35$. This reduces the gap between upper and lower bounds on the smallest density of an identifying code in the infinite rectangular grid to less than 0.002. We conjecture that 0.35 is the exact value.

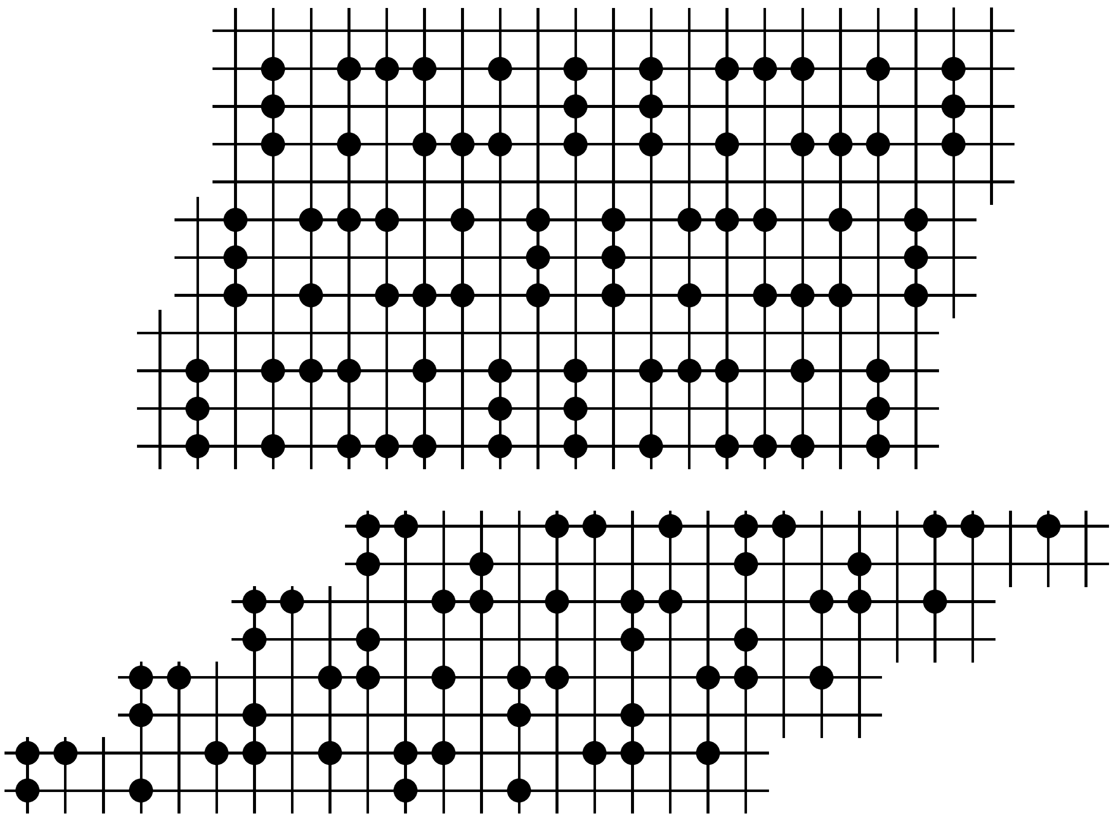


Figure 1: Two identifying codes of density 0.35

References

[1] G. Cohen, I. Honkala, A. Lobstein and G. Zémor: New bounds for codes identifying vertices in graphs, *Electronic Journal of Combinatorics*, vol. 6(1), R19, 1999.