On the twin designs with the Ionin–type parameters

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Dedicated to Professor Reza Khosrovshahi on the occasion of his 60th birthday

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Abstract

Let $4n^2$ be the order of a Bush-type Hadamard matrix with $q = (2n-1)^2$ a prime power. It is shown that there is a weighing matrix

$$W(4(q^m + q^{m-1} + \cdots + q + 1)n^2, 4q^m n^2)$$

which includes two symmetric designs with the Ionin–type parameters

$$\nu = 4(q^m + q^{m-1} + \cdots + q + 1)n^2, \quad \kappa = q^m(2n^2 - n), \quad \lambda = q^m(n^2 - n)$$

for every positive integer m. Noting that Bush–type Hadamard matrices of order $16n^2$ exist for all n for which an Hadamard matrix of order $4n$ exist, this provides a new class of symmetric designs.

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1 Introduction

Recently Ionin [3] introduced an elegant method to use a very special class of regular Hadamard matrix of order 36 in a class of balanced generalized weighing matrices to construct a large class of symmetric designs. The key to his construction is the existence of a class of balanced generalized weighing matrices $BGW(q^m+q^{m-1}+\cdots+q+1, q^m, q^m-q^{m-1})$ over a cyclic group of order t, where q is a prime power, m is a positive integer and t is a divisor of $q-1$. A balanced generalized weighing matrix $BGW(\nu,\kappa,\lambda)$ over a group G is a matrix $W=[w_{ij}]$ of order ν , with $w_{ij}\in G\cup\{0\}$ such that each row and column of W has κ non–zero entries and for each $k\neq l$, the multiset $\{w_{kj}w_{lj}^{-1} : 1\leq j\leq \nu, w_{kj}\neq 0, w_{lj}\neq 0\}$ contains $\lambda/|G|$ copies of every element of G. For his construction, Ionin starts with what he calls a starting regular Hadamard matrix. He then constructs a cyclic group from certain one to one maps on the starting Hadamard matrix. The final step is a clever use of the above known balanced generalized weighing matrices and the starting regular Hadamard matrix. See [3] for details.

In this paper we use a Bush-type Hadamard matrix. A Bush-type Hadamard matrix is a block matrix $H=[H_{ij}]$ of order $4n^2$ with block size $2n$, $H_{ii}=J_{2n}$ and $H_{ij}J_{2n}=J_{2n}H_{ij}=0$, $i\neq j$, $1\leq i\leq 2n$, $1\leq j\leq 2n$, where J_{2n} is the $2n$ by $2n$ matrix of all entries 1. We then introduce a cyclic group consisting of signed permutation matrices of order $4n^2$. Using the above balanced generalized weighing matrices over this cyclic group and a Bush-type Hadamard matrix, we will construct a very special weighing matrix (a weighing matrix is a $(0,\pm1)$-matrix with orthogonal rows and columns). By replacing certain blocks of this weighing matrix with zero blocks we get a $(0,\pm1)$-matrix which becomes a symmetric design with Ionin–type parameters;

$$\nu=4(q^m+q^{m-1}+\cdots+q+1)n^2,\quad \kappa=q^m(2n^2-n),\quad \lambda=q^m(n^2-n),$$

by replacing either all 1's or all -1's with zeros. The construction method in this paper works for any Bush–type Hadamard matrix and is potentially very useful for constructing many other designs. One of the differences between the construction here and that of Ionin is the way that the cyclic group being used. The cyclic group used in this paper acts as a multiplication by signed permutation matrix, while Ionin's cyclic group consists of column permutations. This makes our computations easier than that of Ionin.

Since Bush-type Hadamard matrices of order $16n^2$ is known to exist if $4n$ is the order of an Hadamard matrix, we get many more designs than Ionin. However, Ionin's method [3] is essential for orders $4n^2$ when n is an odd integer. The problem of investigating the existence of Bush–type Hadamard matrices of order $4n^2$, n an odd integer, is a tough one. It is quite interesting to note that if such matrices exist, then the construction method given in this paper simplifies Ionin's method substantially. On the other hand the non–existence of these matrices would imply the non–existence of projective planes of order $2n$, n an odd integer. The smallest order for which the existence of Bush–type

Hadamard matrices is unknown is 36. We conjecture here that Bush-type Hadamard matrices exist for all orders $4n^2$, n an odd integer.

For a $(0,\pm 1)$-matrix K, let $K = K^+ - K^-$, where K^+ and K^- are $(0,1)$-matrices. The Kronecker product of two matrices $A = [a_{ij}]$ and B, denoted $A \otimes B$ is defined, as usual, by $A \otimes B = [a_{ij}B]$. Throughout the paper P is the block matrix defined by $P = (J_{2n} - I_{2n}) \otimes J_{2n}$, where I_{2n} is the identity matrix of order $2n$. For a matrix $A = [a_{ij}]$, denote by $|A|$ the matrix $[|a_{ij}|]$. Throughout the paper $-$ represent -1.

2 Bush-type Hadamard matrices and twin designs

K. A. Bush [1] proved that if there exists a projective plane of order $2n$, then there is an Hadamard matrix H of order $4n^2$, such that:

1. H is symmetric.
2. $H = [H_{ij}]$, where H_{ij} are blocks of order $2n$, $H_{ii} = J_{2n}$ and $H_{ij}J_{2n} = J_{2n}H_{ij} = 0$, for $i \neq j$, $1 \leq i \leq 2n$, $1 \leq j \leq 2n$.

Bush's interest was mainly in the non-existence of such matrices. While there are different methods to construct matrices of above type of order $16n^2$, (see [5]), we are not aware of non–existence of matrices of this form of order $4n^2$ for a single odd value of n, $n > 1$.

Please note that in this paper by a *Bush-type Hadamard matrix* we mean an Hadamard matrix satisfying only condition 2 above, as we do not need to assume that H is symmetric for our construction. For completeness we include the following result of the author [5].

Theorem 1 *Let $4n$ be the order of an Hadamard matrix, then there is a Bush-type Hadamard matrix of order $16n^2$.*

PROOF.

Let K be a normalized Hadamard matrix of order $4n$. Let r_1, r_2, …, r_{4n} be the row vectors of K. Let $C_i = r_i^t r_i$, $i = 1, 2, \ldots, 4n$. Then it is easy to see that:

1. $C_i^t = C_i$, for $i = 1, 2, \ldots, 4n$.
2. $C_1 = J_{4n}$, $C_iJ_{4n} = J_{4n}C_i = 0$, for $i = 2, \ldots, 4n$.

3. $C_iC_j^t = 0$, for $i \neq j$, $1 \leq i, j \leq 4n$.
4. $\sum_{i=1}^{4n} C_iC_i^t = 16n^2 I_{4n}$.

Now let $H = \mathrm{circ}(C_1, C_2, \ldots, C_{4n})$, the block circulant matrix with first row $C_1C_2 \ldots C_{4n}$. Then H is a Bush-type Hadamard matrix of order $16n^2$. ∎

Example 2

Let

$$K = \begin{pmatrix} 1 & 1 & 1 & 1 \\ 1 & 1 & - & - \\ 1 & - & 1 & - \\ 1 & - & - & 1 \end{pmatrix}$$

Then,

$$r_1 = \begin{pmatrix} 1 & 1 & 1 & 1 \end{pmatrix}$$

$$r_2 = \begin{pmatrix} 1 & 1 & - & - \end{pmatrix}$$

$$r_3 = \begin{pmatrix} 1 & - & 1 & - \end{pmatrix}$$

$$r_4 = \begin{pmatrix} 1 & - & - & 1 \end{pmatrix}$$

$$C_1 = r_1^t r_1 = \begin{pmatrix} 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \end{pmatrix}$$

$$C_2 = r_2^t r_2 = \begin{pmatrix} 1 & 1 & - & - \\ 1 & 1 & - & - \\ - & - & 1 & 1 \\ - & - & 1 & 1 \end{pmatrix}$$

$$C_3 = r_3^t r_3 = \begin{pmatrix} 1 & - & 1 & - \\ - & 1 & - & 1 \\ 1 & - & 1 & - \\ - & 1 & - & 1 \end{pmatrix}$$

$$C_4 = r_4^t r_4 = \begin{pmatrix} 1 & - & - & 1 \\ - & 1 & 1 & - \\ - & 1 & 1 & - \\ 1 & - & - & 1 \end{pmatrix}$$

Then $H = \mathrm{circ}(C_1, C_2, C_3, C_4)$ is the following matrix,

$$\begin{bmatrix}
1 & 1 & 1 & 1 & 1 & 1 & - & - & 1 & - & 1 & - & 1 & - & - & 1 \\
1 & 1 & 1 & 1 & 1 & 1 & - & - & - & 1 & - & 1 & - & 1 & 1 & - \\
1 & 1 & 1 & 1 & - & - & 1 & 1 & 1 & - & 1 & - & - & 1 & 1 & - \\
1 & 1 & 1 & 1 & - & - & 1 & 1 & - & 1 & - & 1 & 1 & - & - & 1 \\
1 & - & - & 1 & 1 & 1 & 1 & 1 & 1 & 1 & - & - & 1 & - & 1 & - \\
- & 1 & 1 & - & 1 & 1 & 1 & 1 & 1 & 1 & - & - & - & 1 & - & 1 \\
- & 1 & 1 & - & 1 & 1 & 1 & 1 & - & - & 1 & 1 & 1 & - & 1 & - \\
1 & - & - & 1 & 1 & 1 & 1 & 1 & - & - & 1 & 1 & - & 1 & - & 1 \\
1 & - & 1 & - & 1 & - & - & 1 & 1 & 1 & 1 & 1 & 1 & 1 & - & - \\
- & 1 & - & 1 & - & 1 & 1 & - & 1 & 1 & 1 & 1 & 1 & 1 & - & - \\
1 & - & 1 & - & - & 1 & 1 & - & 1 & 1 & 1 & 1 & - & - & 1 & 1 \\
- & 1 & - & 1 & 1 & - & - & 1 & 1 & 1 & 1 & 1 & - & - & 1 & 1 \\
1 & 1 & - & - & 1 & - & 1 & - & 1 & - & - & 1 & 1 & 1 & 1 & 1 \\
1 & 1 & - & - & - & 1 & - & 1 & - & 1 & 1 & - & 1 & 1 & 1 & 1 \\
- & - & 1 & 1 & 1 & - & 1 & - & - & 1 & 1 & - & 1 & 1 & 1 & 1 \\
- & - & 1 & 1 & - & 1 & - & 1 & 1 & - & - & 1 & 1 & 1 & 1 & 1
\end{bmatrix}$$

Lemma 3 *Let $H = [H_{ij}]$ be a Bush-type Hadamard matrix of order $4n^2$. Let $M = H - I_{2n} \otimes J_{2n}$. Then each of M^+ and M^- is a symmetric $(4n^2, 2n^2 - n, n^2 - n)$–design.*

PROOF.

The row sums of H are all $2n$. Thus the negative entries in H (hence in M) can be viewed as the incidence matrix of a symmetric $(4n^2, 2n^2 - n, n^2 - n)$-design. Since negating all the off diagonal blocks of H leaves it Bush-type Hadamard, the positive entries of M form another symmetric $(4n^2, 2n^2 - n, n^2 - n)$-design. ∎

ALTERNATE PROOF.

We need this proof for use in the proof of our main result. First note that,

$$M = M^+ - M^-,$$

$$P = M^+ + M^-.$$

So, $2M^+ = M + P$. Now it is easy to see that, $MP^t = PM^t = 0$ and thus $4M^+M^{+t} = MM^t + PP^t$ and $M^+M^{+t} = M^-M^{-t}$.
Now,

$$\begin{aligned}
MM^t + PP^t &= (H - I_{2n} \otimes J_{2n})(H^t - I_{2n} \otimes J_{2n}) + ((J_{2n} - I_{2n}) \otimes J_{2n})((J_{2n} - I_{2n}) \otimes J_{2n}) \\
&= HH^t - 2nI_{2n} \otimes J_{2n} + 4n(n-1)J_{2n} \otimes J_{2n} + 2nI_{2n} \otimes J_{2n} \\
&= 4n^2 I_{4n^2} + 4n(n-1)J_{2n} \otimes J_{2n}.
\end{aligned}$$

Therefore, $M^+M^{+t} = M^-M^{-t} = n^2 I_{4n^2} + n(n-1)J_{2n} \otimes J_{2n}$. This means that both M^+ and M^- are symmetric $(4n^2, 2n^2-n, n^2-n)$–designs. ∎

Note that the two designs may not be isomorphic in general (and the problem of finding out when the two are equivalent is not an easy one). Following [4] We call the matrix M a twin design.

Example 4

Let C_2, C_3, C_4 be the matrices of the previous example. Then $M = \mathrm{circ}(0, C_2, C_3, C_4)$ is the following matrix,

$$\begin{bmatrix}
0 & 0 & 0 & 0 & 1 & 1 & - & - & 1 & - & 1 & - & 1 & - & - & 1 \\
0 & 0 & 0 & 0 & 1 & 1 & - & - & - & 1 & - & 1 & - & 1 & 1 & - \\
0 & 0 & 0 & 0 & - & - & 1 & 1 & 1 & - & 1 & - & - & 1 & 1 & - \\
0 & 0 & 0 & 0 & - & - & 1 & 1 & - & 1 & - & 1 & 1 & - & - & 1 \\
1 & - & - & 1 & 0 & 0 & 0 & 0 & 1 & 1 & - & - & 1 & - & 1 & - \\
- & 1 & 1 & - & 0 & 0 & 0 & 0 & 1 & 1 & - & - & - & 1 & - & 1 \\
- & 1 & 1 & - & 0 & 0 & 0 & 0 & - & - & 1 & 1 & 1 & - & 1 & - \\
1 & - & - & 1 & 0 & 0 & 0 & 0 & - & - & 1 & 1 & - & 1 & - & 1 \\
1 & - & 1 & - & 1 & - & - & 1 & 0 & 0 & 0 & 0 & 1 & 1 & - & - \\
- & 1 & - & 1 & - & 1 & 1 & - & 0 & 0 & 0 & 0 & 1 & 1 & - & - \\
1 & - & 1 & - & - & 1 & 1 & - & 0 & 0 & 0 & 0 & - & - & 1 & 1 \\
- & 1 & - & 1 & 1 & - & - & 1 & 0 & 0 & 0 & 0 & - & - & 1 & 1 \\
1 & 1 & - & - & 1 & - & 1 & - & 1 & - & - & 1 & 0 & 0 & 0 & 0 \\
1 & 1 & - & - & - & 1 & - & 1 & - & 1 & 1 & - & 0 & 0 & 0 & 0 \\
- & - & 1 & 1 & 1 & - & 1 & - & - & 1 & 1 & - & 0 & 0 & 0 & 0 \\
- & - & 1 & 1 & - & 1 & - & 1 & 1 & - & - & 1 & 0 & 0 & 0 & 0
\end{bmatrix}$$

3 A cyclic subgroup of signed permutation matrices

Let SP_m be the set of all signed permutation matrices of order m. Let $U = \mathrm{circ}(0, 1, 0, \ldots, 0)$ be the circulant shift permutation matrix of order $2n$ (this is a circulant matrix of order $2n$ with first row $010\ldots0$) and $N = \mathrm{diag}(-, 1, 1, \ldots, 1)$ be the diagonal matrix of order $2n$ with -1 at the $(1,1)$-position and 1 elsewhere on the diagonal. Let $E = UN$, then E is in SP_{2n}. Let $G_{4n} = \{\gamma^i = E^i \otimes I_{2n} : i = 1, 2, \ldots, 4n\} = \prec \gamma \succ$.

Lemma 5 *G_{4n} is a cyclic subgroup of SP_{4n^2} of order $4n$.*

Proof.

For $1 \le r \le 2n$, $(UN)^r$ is U^r with its first r columns negated. Thus $\gamma^{2n} = E^{2n} \otimes I_{2n} = -I_{2n} \otimes I_{2n} = -I_{4n^2}$. It follows now that G_{4n} is a cyclic subgroup of SP_{4n^2} of order $4n$. ∎

Note that G_{4n} is a (signed) group subgroup of SP_{4n^2} and $\sum_{\gamma \in G_{4n}} \gamma = 0$.

Example 6

Let $n = 2$ in lemma 5,

$$\gamma = E \otimes I_4 = \begin{pmatrix} 0 & I_4 & 0 & 0 \\ 0 & 0 & I_4 & 0 \\ 0 & 0 & 0 & I_4 \\ -I_4 & 0 & 0 & 0 \end{pmatrix}$$

$$\gamma^2 = E^2 \otimes I_4 = \begin{pmatrix} 0 & 0 & I_4 & 0 \\ 0 & 0 & 0 & I_4 \\ -I_4 & 0 & 0 & 0 \\ 0 & -I_4 & 0 & 0 \end{pmatrix}$$

$$\gamma^3 = E^3 \otimes I_4 = \begin{pmatrix} 0 & 0 & 0 & I_4 \\ -I_4 & 0 & 0 & 0 \\ 0 & -I_4 & 0 & 0 \\ 0 & 0 & -I_4 & 0 \end{pmatrix}$$

$$\gamma^4 = E^4 \otimes I_4 = \begin{pmatrix} -I_4 & 0 & 0 & 0 \\ 0 & -I_4 & 0 & 0 \\ 0 & 0 & -I_4 & 0 \\ 0 & 0 & 0 & -I_4 \end{pmatrix}$$

$$\gamma^{4+j} = E^{(4+j)} \otimes I_4 = -E^j \otimes I_4 = -\gamma^j, j = 1, 2, 3, 4.$$

So for this example, $G_8 = \{\gamma^i = E^i \otimes I_4 : i = 1, 2, \ldots, 8\}$ is the cyclic subgroup of SP_{16} of order 8.

Lemma 7 *Let $q = (2n-1)^2$ be a prime power. Then there is a balanced weighing matrix $BGW(q^m + q^{m-1} + \cdots + q + 1, q^m, q^m - q^{m-1})$ over the cyclic group G_{4n} for each positive integer m.*

Proof.

Note that $4n$ is a divisor of $q - 1$ and apply [2], IV.4.22. ∎

4 Symmetric designs with the Ionin–type parameters

We are now ready for the main result of the paper.

Theorem 8 *Let H be a Bush-type Hadamard matrix of order $4n^2$ and $q = (2n-1)^2$ a prime power. Then there is a weighing matrix*

$$W(4(q^m + q^{m-1} + \cdots + q + 1)n^2, 4q^m n^2)$$

which contains a twin design with Ionin–type parameters,

$$\nu_m = 4(q^m + q^{m-1} + \cdots + q + 1)n^2, \quad \kappa_m = q^m(2n^2 - n), \quad \lambda_m = q^m(n^2 - n),$$

for each positive integer m.

Proof.

Let m be a positive integer. Let $W = [w_{ij}]$ be the balanced generalized weighing matrix $BGW(\nu, \kappa, \lambda)$ of the lemma 7, where $\nu = q^m + q^{m-1} + \cdots + q + 1$, $\kappa = q^m$, $\lambda = q^m - q^{m-1}$. Consider the block matrix $A = [Hw_{ij}]$ of order $4\nu n^2$. Let $AA^t = [B_{kl}]$.

For $k \neq l$,

$$\begin{aligned} B_{kl} &= \sum_{j=1}^{\nu} Hw_{kj}(Hw_{jl})^t \\ &= \sum_{j=1}^{\nu} H(w_{kj}{w_{lj}}^t)H^t \\ &= H(\sum_{j=1}^{\nu} w_{kj}{w_{lj}}^t)\, H^t \\ &= H(\sum_{\gamma \in G_{4n}} \frac{\lambda}{4n}\gamma)\, H^t = O. \end{aligned}$$

For $k = l$,

$$\begin{aligned} B_{kl} &= H\sum_{j=1}^{\nu}(w_{kj}{w_{lj}}^t)\, H^t \\ &= \kappa HH^t \\ &= 4n^2\kappa I_{2n}. \end{aligned}$$

So A is a weighing matrix $W(4n^2\nu, 4n^2\kappa)$. Now, let $D = [Mw_{ij}]$, where M is the matrix of lemma 3, obtained from H by replacing all the diagonal blocks with the zero matrix.

This matrix contains two symmetric designs with the Ionin–type parameters. To see this, let

$$D = D^+ - D^- = [Mw_{ij}].$$

Now note that $M = H - I_{2n} \otimes J_{2n}$ and it is easy to see that

$$D^+ + D^- = [P|w_{ij}|].$$

Therefore we have,

$$2D^+ = [P|w_{ij}| + Mw_{ij}].$$

First note that for every i, j, k, l, $(P|w_{ij}|)(Mw_{kl})^t = (Mw_{kl})(P|w_{ij}|)^t = 0$. Also note that a calculation on $D = [Mw_{ij}]$ akin of that on A above would give,

$$[Mw_{ij}][Mw_{ij}]^t = \kappa(I_\nu \otimes MM^t).$$

So, we have,

$$\begin{aligned} 4D^+D^{+t} &= [P|w_{ij|}][P|w_{ij}|]^t + [Mw_{ij}][Mw_{ij}]^t \\ &= [P|w_{ij}|][P|w_{ij}|]^t + \kappa(I_\nu \otimes MM^t). \end{aligned}$$

For $k \neq l$,

$$\begin{aligned} \sum_{j=1}^{\nu} P|w_{kj}|(P|w_{jl}|)^t &= \sum_{j=1}^{\nu} P(|w_{kj}||w_{lj}|^t)P^t \\ &= \frac{\lambda}{\frac{q-1}{2(n-1)}} P(J_{2n} \otimes I_{2n})P^t \\ &= \frac{2\lambda(n-1)}{q-1}((J_{2n} - I_{2n}) \otimes J_{2n})(J_{2n} \otimes I_{2n})((J_{2n} - I_{2n}) \otimes J_{2n}) \\ &= \frac{2\lambda(n-1)}{q-1} 2n(2n-1)^2 J_{2n} \otimes J_{2n} \\ &= \frac{2n-1}{q-1} 2nq^{m-1}(q-1)qJ_{2n} \otimes J_{2n} \\ &= 4n(n-1)q^m J_{2n} \otimes J_{2n}. \end{aligned}$$

Therefore, all the (k, l), $1 \leq k \neq l \leq \nu$ blocks of the matrix D^+D^{+t} consist of the matrix $n(n-1)q^m J_{2n} \otimes J_{2n}$.
For $k = l$, it follows from the alternate proof of lemma 3 that,

$$\begin{aligned} \sum_{j=1}^{\nu} P|w_{kj}|(P|w_{kj}|)^t + \sum_{j=1}^{\nu} Mw_{kj}(Mw_{kj})^t &= \kappa(PP^t + MM^t) \\ &= q^m(4n^2 I_{4n^2} + 4n(n-1)J_{2n} \otimes J_{2n}). \end{aligned}$$

From this we conclude that all the $(k, k), 1 \leq k \leq \nu$ blocks of the matrix D^+D^{+t} consist of the matrix $q^m(n^2 I_{4n^2} + n(n-1)J_{2n} \otimes J_{2n})$. Therefore, D^+ is a symmetric $(\nu_m, \kappa_m, \lambda_m)$–design.

Noting that,

$$2D^- = [P|w_{ij}| - Mw_{ij}]$$

an almost identical argument shows that D^- is also a symmetric design with Ionin–type parameters. ∎

Corollary 9 *Let $4n$ be the order of an Hadamard matrix with $q = (4n-1)^2$ a prime power. Then there is a weighing matrix $W(16(q^m + q^{m-1} + \cdots + q + 1)n^2, 16q^m n^2)$ which includes two symmetric designs with the Ionin–type parameters*

$$\nu = 16(q^m + q^{m-1} + \cdots + q + 1)n^2, \quad \kappa = q^m(8n^2 - 2n), \quad \lambda = q^m(4n^2 - 2n)$$

for every positive integer m.

Proof.

This follows from theorems 1 and 8. ∎

Remark 10

Corollary 9 provides a new class of symmetric designs. If a Bush–type Hadamard matrix of order $4n^2$ exists for odd values of n, then the construction methods in this paper simplifies Ionin's method significantly. It is also interesting to note that there is a similarity between the way that we get twin designs from Bush–type Hadamard matrices and the way that the twin designs with the Ionin–type parameters are obtained from the weighing matrices.

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