A Small Trivalent Graph of Girth 14

Geoffrey Exoo
Department of Mathematics and Computer Science
Indiana State University
Terre Haute, IN 47809
g-exoo@indstate.edu

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Abstract

We construct a graph of order 384, the smallest known trivalent graph of girth 14.

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In this note we use a construction technique that can be viewed as a kind of generalized Cayley graph. The vertex set V of such a graph consists of the elements in multiple copies of some finite group $\mathcal{G}$. The action of $\mathcal{G}$ on V is determined by the regular action on each of the copies of $\mathcal{G}$. This induces an action on the edges of the complete graph on V. The edge set of the graph we construct is the union of certain of these orbits.

The particular graph we describe is a trivalent graph of girth 14 and order 384. It is constructed as above using a permutation group $\mathcal{G}$ of order 48: the group generated by the following two permutations.

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(1, 33, 37,  6,  9, 44,  8, 19, 23, 18, 22, 34)
(2,  5, 40,  4, 12, 16, 11, 15, 27, 48, 26, 30)
(3, 25, 41, 24, 14, 39, 13, 38, 29, 10, 28, 46)
(7, 32, 45, 31, 21, 43, 20, 42, 36, 17, 35, 47)

( 1, 26, 20, 25,  8, 12,  7, 38)( 2, 21, 24,  9, 11, 35, 10, 22)
( 3, 32, 48, 19, 13, 42,  4, 33)( 5,  6, 28, 31, 15, 18, 14, 17)
(16, 34, 41, 47, 30, 44, 29, 43)(23, 40, 45, 39, 37, 27, 36, 46)
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The permutation representation so generated is in fact a regular representation of the underlying abstract group, as can be verified using **GAP** [4].

Now let the vertex set of our graph be $V = \{i \mid 0 \le i < 384\}$ and define the action of $\mathcal{G}$ on V as follows. For each $\sigma \in \mathcal{G}$ define a permutation σ^* on V by

$$\sigma^*(v) = \sigma(v \bmod 48) + 48\lfloor v/48 \rfloor$$

for all $v \in V$. To construct our graph we choose a set of unordered vertex pairs, (u, v), and for each $\sigma \in \mathcal{G}$ we add the edge $(\sigma^*(u), \sigma^*(v))$ to the graph. To construct the required graph, we use the following 12 vertex pairs, which give us a set of $12 \times 48 = 576$ edges.

```
( 0, 115) ( 48, 140) (144, 199) (192, 362)
( 0, 126) ( 48, 173) (144, 261) (240, 289)
( 0, 282) ( 48, 339) (192, 312) (288, 360)
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The construction guarantees that the automorphism group of the graph will have order at least 48; in fact, it has order 96 [3]. Incidently, it is the only trivalent graph of girth 14 and order 384 or less on which $\mathcal{G}$ acts semiregularly. According to [1], this graph is the smallest known trivalent graph of girth 14, improving the previous bound of 406 [2]. An electronic copy of the adjacency list is available at

`http://isu.indstate.edu/ge/CAGES/g3.14.384.`

References

[1] N. Biggs, Constructions for cubic graphs of large girth. *Electron. J. Combin.*, 5 (1998) A1.

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[3] B. D. McKay, *nauty User's Guide* (version 1.5). Technical Report TR-CS-87-03 (Department of Computer Science, Australian National University, 1990)

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