

Change History for The Graph Crossing Number and its Variants: A Survey

Marcus Schaefer
School of Computing
DePaul University
Chicago, Illinois 60604, USA
`mschaefer@cdm.depaul.edu`

Submitted: ?; Accepted: ?; Published:
Mathematics Subject Classifications: 05C62, 68R10

1 Changes Since Original Version

Newly added crossing number variants

- simple degenerate local crossing number, based on [33]
- book edge crossing number [11]
- monotone odd + crossing number [9]
- crossing number of abstract topological graph [47, 54, 63, 42]

Some bigger changes by topic:

- Precursors of the crossing number: David Eppstein [35] discovered a 1945 paper on crossing minimization in sociology [16], and there is more: [53, 16, 51, 52, 35, 14]; there now is a remark on “Forerunners of Crossing Minimization in Sociology”
- 1-planarity (a special case of the local crossing number), has been drawing increasing attention recently; newly added results include [17, 10, 37, 26, 20]
- some partial progress on Zarankiewicz’s conjecture: [24, 30]
- some progress on Guy’s conjecture for restricted layouts (monotone, cylindrical, shellable): [4, 3]
- current best upper bound on $\overline{cr}(K_n)$: [36]

- results on local rectilinear crossing number: [29, 57]
- results and questions about the relationship between cr_Σ and cr : [15, 21, 55]
- added Section on crossing lemma variants to introduction (for ocr , pcr , pcr_- , iocr)
- older papers, I'd so far overlooked: [13, 12, 64, 33, 49, 58, 65, 60]

2 Detailed Changes

- added [53, 16, 51, 52, 35, 14] with a remark on Forerunners of Crossing Minimization in Sociology
- added [5], drawings in which edges may pass through vertices
- added [34], footnote on curve drawing
- added [50], Fary's theorem
- added [25, 19], approximating the crossing number of bounded degree graphs
- added [17, 10], complexity of 1-planarity
- added [41, 40], Metro-line crossing number variant
- added [37, 26], edge bounds for 1-planarity
- added [46], edge bound for bipartite crossing number
- added [20], coloring 1-planar graphs
- added [33], suggests the (simple) degenerate local crossing number
- added [45], crossings in circuit layouts, related to bipartite crossing number
- added [12], alternative source for $\overline{\text{cr}}(K_8) = 19$.
- added [39] on bold drawing style
- added [24] on Zarankiewicz's conjecture
- added [13], alternative source on upper bound on $\text{cr}(K_n)$
- updated [1] to [2], updated [28] to [27]
- added [48, 64] on nested convex drawings of K_n
- added [64], alternative source on k -layer crossing number, also added open question to k -layer crossing number

- added [11], introduces the book edge crossing number
- added [8], results on local convex crossing number
- added [7], a crossing number for geometric hypergraphs; mentioned, not included
item added [4, 3, 9], Guy's conjecture for cylindrical and monotone layouts
- updated [17] to [18]
- added [43] introduce new name for local crossing number
- added [38, 49], work on $\text{bkcr}_2(Q_n)$ and $\overline{\text{cr}}(Q_n)$
- added [57, 29], results on rectilinear local crossing number
- added [22, 23], upward planarity and upward crossing minimization
- added [56] on 2-page crossing number of $K_{1,4,n}$
- added [59], crossing minimization with port constraints
- added [30], improved bounds for $\text{cr}(K_{7,n})$
- added [6], introduces the crossing cover number, not really a crossing number
- added crossing number of abstract topological graph, [47, 54, 63, 42]
- added [62], lower bounds on $\text{cr}_{S_g}(K_n)$, $\text{cr}_{S_g}(K_{n,m})$, and $\text{cr}_{S_g}(Q_n)$
- updated [31] to [32]
- added [15, 21, 55], relationships between cr and cr_Σ
- added [36], improved upper bound on $\overline{\text{cr}}(K_n)$
- added [58, 65] on relationship between $\overline{\text{cr}}$ and Sylvester's Four Point Problem
- added [60, 61], on genus g crossing number
- added [44] on pseudolinear crossing number, sheds some new light on the Bienstock/Dean examples separating $\overline{\text{cr}}$ from cr

References

- [1] Bernardo M. Ábrego, Oswin Aichholzer, Silvia Fernández-Merchant, Pedro Ramos, and Gelasio Salazar. The 2-page crossing number of K_n . In Tamal K. Dey and Sue Whitesides, editors, *Symposium on Computational Geometry 2012, SoCG '12, Chapel Hill, NC, USA, June 17-20, 2012*, pages 397–404. ACM, 2012.
- [2] Bernardo M. Ábrego, Oswin Aichholzer, Silvia Fernández-Merchant, Pedro Ramos, and Gelasio Salazar. The 2-Page Crossing Number of K_n . *Discrete Comput. Geom.*, 49(4):747–777, 2013.
- [3] Bernardo M. Ábrego, Oswin Aichholzer, Silvia Fernández-Merchant, Pedro Ramos, and Gelasio Salazar. More on the crossing number of K_n : Monotone drawings. *Electronic Notes in Discrete Mathematics*, 44(0):411–414, 2013.
- [4] Bernardo M. Ábrego, Oswin Aichholzer, Silvia Fernández-Merchant, Pedro Ramos, and Gelasio Salazar. Shellable drawings and the cylindrical crossing number of K_n . *ArXiv e-prints*, September 2013.
- [5] O. Aichholzer, G. Araujo-Pardo, N. García-Colín, T. Hackl, D. Lara, C. Rubio-Montiel, and J. Urrutia. Geometric achromatic and pseudoachromatic indices. *ArXiv e-prints*, March 2013.
- [6] Michael O. Albertson. Chromatic number, independence ratio, and crossing number. *Ars Math. Contemp.*, 1(1):1–6, 2008.
- [7] Anurag Anshu and Saswata Shannigrahi. A Lower Bound on the Crossing Number of Uniform Hypergraphs. *ArXiv e-prints*, September 2013.
- [8] Christopher Auer, Christian Bachmaier, Franz J. Brandenburg, Andreas Gleißner, Kathrin Hanauer, Daniel Neuwirth, and Josef Reislhuber. Recognizing outer 1-planar graphs in linear time. In Stephen Wismath and Alexander Wolff, editors, *Proceedings of the 21st International Symposium (GD'13) September 23–25, 2013*, volume 8242 of *Lecture Notes in Computer Science*, pages 108–119. Springer-Verlag, Berlin, 2013.
- [9] Martin Balko, Radoslav Fulek, and Jan Kynčl. Crossing numbers and combinatorial characterization of monotone drawings of K_n . *ArXiv e-prints*, December 2013.
- [10] Michael J. Bannister, Sergio Cabello, and David Eppstein. Parameterized Complexity of 1-Planarity. *ArXiv e-prints*, April 2013.
- [11] Michael J. Bannister, David Eppstein, and Joseph A. Simons. Fixed parameter tractability of crossing minimization of almost-trees. *ArXiv e-prints*, August 2013.
- [12] Calvin Pascal Barton. *THE RECTILINEAR CROSSING NUMBER FOR COMPLETE SIMPLE GRAPHS IN E_2* . PhD thesis, The University of Texas at Austin, Austin, Texas, 1970.
- [13] J. Blažek and M. Koman. A minimal problem concerning complete plane graphs. In *Theory of Graphs and its Applications (Proc. Sympos. Smolenice, 1963)*, pages 113–117. Publ. House Czechoslovak Acad. Sci., Prague, 1964.

- [14] Edgar F. Borgatta. A diagnostic note on the construction of sociograms and action diagrams. *Group Psychotherapy*, 3:300–308, 1951.
- [15] Károly J. Böröczky, János Pach, and Géza Tóth. Planar crossing numbers of graphs embeddable in another surface. *Internat. J. Found. Comput. Sci.*, 17(5):1005–1015, 2006.
- [16] Urie Bronfenbrenner. *The measurement of sociometric status, structure and development*. Number 6 in Sociometry monographs. New York: Beacon House, 1945.
- [17] Sergio Cabello and Bojan Mohar. Adding one edge to planar graphs makes crossing number and 1-planarity hard. *ArXiv e-prints*, March 2012.
- [18] Sergio Cabello and Bojan Mohar. Adding one edge to planar graphs makes crossing number and 1-planarity hard. *SIAM Journal on Computing*, 42(5):1803–1829, 2013.
- [19] Chandra Chekuri and Anastasios Sidiropoulos. Approximation algorithms for Euler genus and related problems. *ArXiv e-prints*, April 2013.
- [20] Zhi-Zhong Chen and Mitsuharu Kouno. A linear-time algorithm for 7-coloring 1-planar graphs (extended abstract). In *Mathematical foundations of computer science 2003*, volume 2747 of *Lecture Notes in Comput. Sci.*, pages 348–357. Springer, Berlin, 2003.
- [21] Markus Chimani, Petr Hliněný, and Gelasio Salazar. Toroidal grid minors and stretch in embedded graphs. *ArXiv e-prints*, March 2014.
- [22] Markus Chimani and Robert Zeranski. Upward planarity testing via SAT. In Walter Didimo and Maurizio Patrignani, editors, *Graph Drawing*, volume 7704 of *Lecture Notes in Computer Science*, pages 248–259. Springer Berlin Heidelberg, 2013.
- [23] Markus Chimani and Robert Zeranski. An exact approach to upward crossing minimization. In *SIAM Meeting on Algorithm Engineering & Experiments (ALENEX’14)*, 2014.
- [24] Robin Christian, R. Bruce Richter, and Gelasio Salazar. Zarankiewicz’s conjecture is finite for each fixed m . *J. Combin. Theory Ser. B*, 103(2):237–247, 2013.
- [25] Julia Chuzhoy. An algorithm for the graph crossing number problem [extended abstract]. In *STOC’11—Proceedings of the 43rd ACM Symposium on Theory of Computing*, pages 303–312. ACM, New York, 2011.
- [26] Július Czap and Dávid Hudák. On drawings and decompositions of 1-planar graphs. *The Electronic Journal of Combinatorics*, 20(2):P54, 2013.
- [27] E. de Klerk, D. V. Pasechnik, and G. Salazar. Improved lower bounds on book crossing numbers of complete graphs. *SIAM J. Discrete Math.*, 27(2):619–633, 2013.
- [28] Etienne de Klerk, Dmitrii V. Pasechnik, and Gelasio Salazar. Improved lower bounds on book crossing numbers of complete graphs. *ArXiv e-prints*, July 2012.
- [29] Walter Didimo. Density of straight-line 1-planar graph drawings. *Inf. Process. Lett.*, 113(7):236–240, April 2013.
- [30] Cristian Dobre and Juan Vera. Exploiting symmetry in copositive programs via semidefinite hierarchies. *Optimization Online*, 2013.

- [31] Vida Dujmović, Pat Morin, and Adam Sheffer. Crossings in Grid Drawings. *ArXiv e-prints*, January 2013.
- [32] Vida Dujmović, Pat Morin, and Adam Sheffer. Crossings in grid drawings. *Electron. J. Combin.*, 21(1):P1.41, 2014.
- [33] Roger B. Eggleton. Rectilinear drawings of graphs. *Utilitas Math.*, 29:149–172, 1986.
- [34] David Eppstein. A brief history of curves in graph drawing. <http://11011110.livejournal.com/266622.html> (last accessed 2013/4/9).
- [35] David Eppstein. An early reference on crossing minimization. <http://11011110.livejournal.com/265544.html> (last accessed 2013/4/1).
- [36] Ruy Fabila-Monroy and Jorge López. Computational search of small point sets with small rectilinear crossing number. *ArXiv e-prints*, March 2014.
- [37] Igor Fabrici and Tomáš Madaras. The structure of 1-planar graphs. *Discrete Math.*, 307(7-8):854–865, 2007.
- [38] Luerbio Faria, Celina M. H. Figueiredo, R. Bruce Richter, and Imrich Vřto. The same upper bound for both: The 2-page and the rectilinear crossing numbers of the n -cube. In Andreas Brandstädt, Klaus Jansen, and Rüdiger Reischuk, editors, *Graph-Theoretic Concepts in Computer Science*, volume 8165 of *Lecture Notes in Computer Science*, pages 249–260. Springer Berlin Heidelberg, 2013.
- [39] Darya Filippova, Geet Duggal, Rob Patro, and Carl Kingsford. Chromo-vis: Feature-rich layouts of chromosome conformation graphs. Unpublished manuscript at <http://www.cs.cmu.edu/~dfilippo/chromovisGD2013.pdf> (last accessed 7/9/2013), 2013.
- [40] Martin Fink and Sergey Pupyrev. Metro-Line Crossing Minimization: Hardness, Approximations, and Tractable Cases. *ArXiv e-prints*, June 2013.
- [41] Martin Fink and Sergey Pupyrev. Ordering Metro Lines by Block Crossings. *ArXiv e-prints*, April 2013.
- [42] Jacob Fox, János Pach, and Andrew Suk. The number of edges in k -quasi-planar graphs. *SIAM J. Discrete Math.*, 27(1):550–561, 2013.
- [43] Alexander Grigoriev, Athanassios Koutsonas, and Dimitrios M. Thilikos. Nearly Planar Graphs and λ -flat Graphs. *ArXiv e-prints*, November 2013.
- [44] César. Hernández-Vélez, Jesús Leaños, and Gelasio Salazar. On the pseudolinear crossing number. *ArXiv e-prints*, April 2014.
- [45] Günter Hotz. *Schaltkreistheorie*. Walter de Gruyter, Berlin-New York, 1974.
- [46] Yasuaki Kobayashi, Hirokazu Maruta, Yusuke Nakae, and Hisao Tamaki. A linear edge kernel for two-layer crossing minimization. In Ding-Zhu Du and Guochuan Zhang, editors, *Computing and Combinatorics*, volume 7936 of *Lecture Notes in Computer Science*, pages 458–468. Springer Berlin Heidelberg, 2013.
- [47] Jan Kratochvíl. String graphs. I. The number of critical nonstring graphs is infinite. *J. Combin. Theory Ser. B*, 52(1):53–66, 1991.

- [48] Lorena Mercedes López. Rectilinear crossing numbers of complete graphs with specific nested sequence of convex hulls. Master's thesis, California State University, Northridge, 2013.
- [49] Tom Madej. Bounds for the crossing number of the N -cube. *Journal of Graph Theory*, 15(1):81–97, 1991.
- [50] Bojan Mohar. Drawing graphs in the hyperbolic plane. In Jan Kratochvíl, editor, *Proceedings of the 7th International Symposium (GD'99) held at Štířín Castle, September 15–19, 1999*, volume 1731 of *Lecture Notes in Computer Science*, pages 127–136. Springer-Verlag, Berlin, 1999.
- [51] Jacob Levy Moreno. *Who shall survive?: A new approach to the problem of human interrelations*. Nervous and Mental Disease Publishing Co, Washington, D.C., 1934.
- [52] Jacob Levy Moreno. *Who shall survive?: A new approach to the problem of human interrelations*. Beacon House Inc, Beacon, N.Y., 1934.
- [53] Mary L. Northway. A method for depicting social relationships obtained by sociometric testing. *Sociometry*, 3(2):pp. 144–150, April 1940.
- [54] János Pach, Rom Pinchasi, Micha Sharir, and Géza Tóth. Topological graphs with no large grids. *Graphs Combin.*, 21(3):355–364, 2005.
- [55] János Pach and Géza Tóth. Crossing number of toroidal graphs. In *Topics in discrete mathematics*, volume 26 of *Algorithms Combin.*, pages 581–590. Springer, Berlin, 2006.
- [56] Dharna Satsangi, Kamal Srivastava, and Gursaran Srivastava. K -page crossing number minimization problem: An evaluation of heuristics and its solution using GESAKP. *Memetic Computing*, 5(4):255–274, 2013.
- [57] Marcus Schaefer. Picking Planar Edges; or, Drawing a Graph with a Planar Subgraph. *ArXiv e-prints*, November 2013.
- [58] Edward R. Scheinerman and Herbert S. Wilf. The rectilinear crossing number of a complete graph and Sylvester's "four point problem" of geometric probability. *Amer. Math. Monthly*, 101(10):939–943, 1994.
- [59] Christoph Daniel Schulze, Miro Spönemann, and Reinhard von Hanxleden. Drawing layered graphs with port constraints. *Journal of Visual Languages & Computing*, 2013.
- [60] Farhad Shahrokhi, O. Sýkora, László A. Székely, and Imrich Vrto. The crossing number of a graph on a compact 2-manifold. *Adv. Math.*, 123(2):105–119, 1996.
- [61] Farhad Shahrokhi, László A. Székely, O. Sýkora, and Imrich Vrto. Drawings of graphs on surfaces with few crossings. *Algorithmica*, 16(1):118–131, 1996.
- [62] Ondrej Sýkora and Imrich Vrto. Edge separators for graphs of bounded genus with applications. *Theoret. Comput. Sci.*, 112(2):419–429, 1993.
- [63] Gábor Tardos and Géza Tóth. Crossing stars in topological graphs. *SIAM J. Discrete Math.*, 21(3):737–749, 2007.

- [64] John N. Warfield. Crossing theory and hierarchy mapping. *IEEE Trans. Systems Man Cybernet.*, SMC-7(7):505–523, 1977.
- [65] Herbert S. Wilf. On crossing numbers, and some unsolved problems. In *Combinatorics, geometry and probability (Cambridge, 1993)*, pages 557–562. Cambridge Univ. Press, Cambridge, 1997.